ed by meson resonances, but it may be hoped that a small number of suitable baryonic states can be found which approximately form a representation of the algebra.[10] If this is the case, the resulting form factors should be a superposition of oscillating functions[11] which behave reasonably (i.e., like the measured experimental quantities) for small q^2, and the oscillations should become serious only at higher q^2 where the neglect of higher baryon resonances is more important.

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[10]The authors have considered the inclusion of spin-$\frac{3}{2}$ states, and this work will be published in some more suitable place.
[11]J. Paton and the present authors have established that, for certain classes of representations, the resulting form factors are superpositions of Bessel functions such as those in Eq. (16).

VALUE OF Reξ FROM MEASUREMENT OF A TRANSVERSE COMPONENT OF MUON POLARIZATION IN $K_L^0 \to \pi^- + \mu^+ + \nu$ †

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It has been shown[1] that for a given kinematic configuration, e.g., for fixed E_μ and E_π, muons from $K_{\mu 3}$ decay are completely polarized along some direction such that $\vec{P} = \vec{B}(E_\mu, E_\pi, \xi)/|\vec{B}|$, where B depends on ξ, the ratio of the form factors (f_-/f_+) in the hadronic current describing the semileptonic decays of kaons. Further, after integration over E_π, assuming $f_\pm(q^2) = f_\pm(0)$, the magnitude of the polarization still approaches unity. A useful choice of coordinate axes in which to describe $\vec{P}$ is the following: $\hat{k}$, a unit vector in the direction of the muon momentum; $\hat{t} = \hat{k} \times \vec{p}_\pi / |\hat{k} \times \vec{p}_\pi|$; and $\hat{n} = \hat{t} \times \hat{k}$. We have measured the component P_n which, after integration over E_π, is given in kaon center-of-mass quantities by

$$P_n = \left(\frac{1}{H}\right)\left\{-F(E_\mu)\frac{\pi}{4}\frac{m_\mu}{M_K}\left(1 + \frac{m_\mu^2}{M_K^2} - 2\frac{E_\mu}{M_K}\right)^{1/2}\left[1 + \frac{m_\mu^2}{4M_K^2}(1 - 2\,\mathrm{Re}\xi + |\xi|^2) - \frac{E_\mu}{M_K}(1 - \mathrm{Re}\xi)\right]\right\},$$

where H and F are given by

$$H = F(E_\mu)\left\{\frac{E_\mu}{M_K}\left(1 - 2\frac{E_\mu}{M_K}\right) + \frac{m_\mu^2}{M_K^2}\left[1 - \left(1 - \frac{E_\mu}{M_K}\right)(1 - \mathrm{Re}\xi) + \frac{1}{4}\left(\frac{E_\mu}{M_K} - \frac{m_\mu^2}{M_K^2}\right)(1 - 2\,\mathrm{Re}\xi + |\xi|^2)\right]\right\},$$

and

$$F(E_\mu) = \left[\frac{M_K^2 + m_\mu^2 - m_\pi^2 - 2M_K E_\mu}{M_K^2 + m_\mu^2 - 2M_K E_\mu}\right]^2.$$

Unlike P_t which disappears when $\mathrm{Im}\xi = 0$, P_n (and also P_k) depends on both Reξ and Imξ. Curves of P_n (and also P_k for comparison) as a function of $(E_\mu - m_\mu)$ for various values of Reξ, assuming[2] $\mathrm{Im}\xi = 0$, are plotted in Fig. 1.

The two experimental arrangements that were

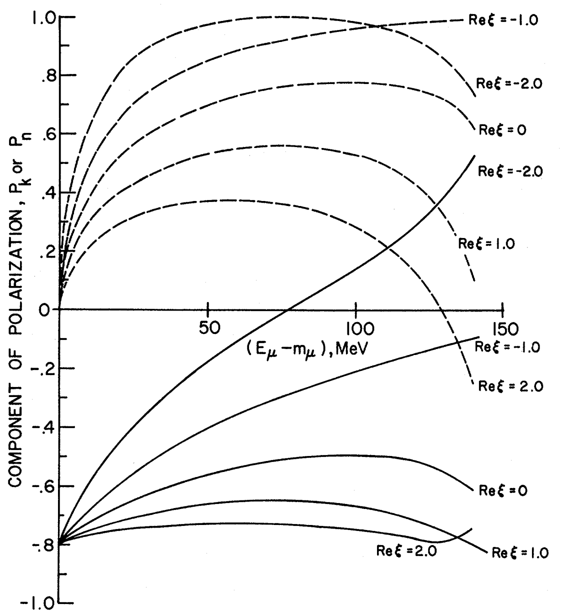

FIG. 1. Plots of P_k (dashed curves) and P_n (solid curves) as functions of muon kinetic energy $(E_\mu - m_\mu)$ for various values of Reξ.

used are shown in Fig. 2. The 13° neutral beam at the Princeton-Pennsylvania Accelerator (PPA) was cleared of photons with 3 in. of lead, collimated, swept magnetically, and then traversed the 7-ft-long decay region between points A and B which was bounded top and bottom by scintillation counters to detect one of the charged products from K_L^0 decay. The other (positively) charged particle from the decays $\pi^\pm \mu^\mp \nu$, $\pi^\pm e^\mp \nu$, and $\pi^+\pi^-\pi^0$ of K_L^0 was bent through a mean angle of 26.5° at a mean momentum of 700 MeV/c by the first bending magnet (BM1) and, in arrangement (a), passed through the threshold gas Cherenkov counter and emerged from the second bending magnet (BM2) with a parallel displacement of its line of flight. Some of the emerging particles traversed the aluminum absorber and stopped and decayed in the carbon stopping region, where the direction of the decay product was observed in scintillation counters at the top and bottom of that region.

In these arrangements the formation of the unit vector $\hat{t}$ in the rest frame of the kaon was made as follows: The azimuthal angle of the π^-, in the coordinate system in which the K_L^0 line of flight is the polar axis, was determined by the scintillation counters in the region A and B. In the same coordinate system, the

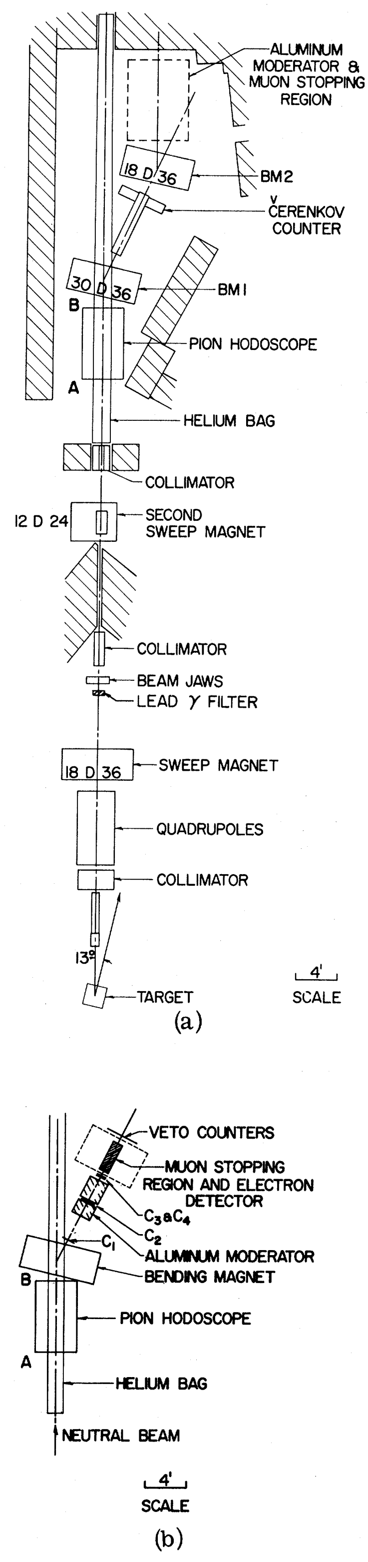

FIG. 2. Sketches of the two experimental arrangements employed.

average laboratory angle made by a μ^+ that ultimately stops in the carbon was $\langle\cos\theta^{\rm lab}\rangle \simeq 0.99$. For the incident $K_L{}^0$ momentum spectrum[3] at 13° at PPA these forwardly directed muons were also forwardly directed in the rest system of the $K_L{}^0$ and had $\langle\cos\theta^{\rm c.m.}\rangle \simeq 0.925$, where the exact value of $\langle\cos\theta^{\rm c.m.}\rangle$ depends to some extent on the $K_L{}^0$ momentum distribution. Hence for pion scintillation counters of reasonable size, the hemisphere in which $\hat{t}$ lay was uniquely specified. The determination of $\langle\hat{\sigma}_\mu\cdot\hat{n}\rangle$ was accomplished by observing the direction of the e^+ from μ^+ decay with scintillation counters above and below the carbon region, so that the average directions of $\hat{n}$ and $\vec{p}_e$ were collinear.

In both arrangements (a) and (b), muons with laboratory momenta between 560 and 840 MeV/c were stopped in the carbon block. The measured value of P_n was not sensitive to the details of the two experimental arrangements as demonstrated by the results of Monte Carlo calculations of P_n as a function of $\mathrm{Re}\xi$ for arrangements (a) and (b) and also for a fictitious arrangement that would accept all muon laboratory momenta uniformly. For the two experimental arrangements, the observed asymmetry $A=(N_+-N_-)/(N_++N_-)$, where N_+ = number of events with both electron and pion emitted in the same direction and N_- = number of events with electron and pion oppositely directed, is related to P_n by $A=(0.225\pm0.015)P_n$. The factor 0.225 ± 0.015 was obtained from the distribution in positron energy and angle predicted by the $V-A$ theory after integrating over all free parameters using the known detection efficiencies of our apparatus for positrons, muons, and pions; the error arises from uncertainty in the spatial distribution of the muons in the carbon stopping region.

The principal sources of background were decays in flight of π^+ from $K_L{}^0\to\mu^-+\pi^++\nu$ and accidental coincidences between π and μ counters (resolving time 30 nsec) and between $(\pi\mu)$ and e counters (4.7-μsec gate); a secondary source was neutron interactions in the helium along the neutral beam path. Experimental arrangement (a) was used to explore these and other backgrounds and to acquire about 30% of the data. The Cherenkov counter counted muons of $p_\mu>560$ MeV/c but did not count pions of $p_\pi<740$ MeV/c. By running with and without the Cherenkov counter requirement, it was determined that the background from π^+ traversing the entire apparatus and decaying at rest in the carbon was small. This was checked in arrangement (b) using negative particles stopping in the carbon and yielded a background of stopping π^+ less than 6% of the total count rate. The calculated fractional background in configuration (a) from decays in flight of π^+ from $K_L{}^0$ decay was 0.32 ± 0.03 of the total event rate, and the measured decays in flight of π^+ from interactions in He was 0.075 ± 0.025 of the total rate. In configuration (b) with the pion flight path shortened by about a factor of 2, the fraction of π^+ decays in flight from $K_L{}^0$ decay was 0.180 ± 0.015 and from neutron interactions was negligible. An additional reason for the two experimental configurations was to insure against an inadvertently large spin-flip transition probability in the fringing fields of the bending magnets, although the calculated transition probability in each of the configurations was less than 1%.

To insure against the introduction of apparatus-induced asymmetries, the carbon region was constructed to rotate about an axis parallel to the incident muon direction and data were taken in two positions separated by 180°. The data were analyzed for correlations of each of these positron-detecting positions with each of the two pion-detecting assemblies, and no statistically significant correlation was found. In addition, in 1086 events obtained with μ^- (from $K_L{}^0\to\pi^++\mu^-+\nu$) stopping in the carbon, an asymmetry of $+0.004\pm0.03$ was found. On the average, the horizontal component of the stray magnetic field in the carbon region was 0.5 ± 0.3 G, leading to a dilution of the muon polarization of less than 1%. The distribution of observed decay times of muons was used to estimate the random "π-μ-e" background.

The observed total number of events was 4197, of which 780 ± 46 were from π^+ decays in flight, 364 ± 11 were from accidental "π-μ" coincidences, 343 ± 21 were accidental "π-μ-e" coincidences, and 102 ± 13 were from interactions in helium. The observed value of $\Delta N=N_+-N_-$ was 167 events, yielding an asymmetry $A=-0.064\pm0.032$, where the negative sign indicates that the e^+ from the μ^+ decay is, on the average, oppositely directed to the π^+ from the $K_L{}^0$ decay. The corresponding transverse component of polarization is

$$P_n=-0.28\pm0.13,$$

from which one obtains

$$\mathrm{Re}\xi = -1.2 \pm 0.5,$$

assuming $f_\pm(q^2) = f_\pm(0)$ and $\mathrm{Im}\xi = 0$.

The previously determined values of $\mathrm{Re}\xi$ are $\mathrm{Re}\xi = +1.2 \pm 0.8$ from measurement[4] of the muon energy spectrum from $K_L^0(\mu 3)$ and $\mathrm{Re}\xi = +0.8 \pm 0.8$ from two values[5,6] of the ratio $K_L^0(\mu 3)/K_L^0(e3)$. Both of these results assume $f_\pm(q^2) = f_\pm(0)$ and $\mathrm{Im}\xi = 0$. However, the assumption $f_\pm(q^2) = f_\pm(0)$ may lead to differences in evaluating $\mathrm{Re}\xi$ from spectrum and branching-ratio data and from polarization data. [For example, it is possible to fit the muon energy spectrum from K_L^0 with $\mathrm{Re}\xi = -0.8 \pm 0.3$ as was done in Ref. 4 by assuming $f_\pm(q^2)$ to be determined by a single $J=1$ intermediate K-π state.] If this explanation is valid, a re-examination of K^+ semileptonic-decay data will be required before a meaningful comparison of $\mathrm{Re}\xi_0$ for K_L^0 and $\mathrm{Re}\xi_+$ for K^+ can be made.

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SUM RULES FOR P-WAVE PION-NUCLEON SCATTERING LENGTHS FROM CURRENT ALGEBRAS*

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From current commutation relations and the hypothesis of partial conservation of axial-vector current, sum rules are derived relating πN P-wave scattering lengths to the S-wave scattering lengths, the nucleon isovector magnetic moment, g_A, and $G_{\pi NN}$.

From current commutation relations and the hypothesis of partial conservation of axial-vector current (PCAC), we derive in this note sum rules relating the πN P-wave scattering lengths to g_A, the nucleon isovector magnetic moment $\mathfrak{M}^V$, the πNN coupling constant G, and the πN S-wave scattering lengths, and compare these sum rules with available experimental data.

Consider the matrix amplitude $M_{AA}{}^{\mu\nu}$ defined by

$$i(2\pi)^4\delta(p_f+q_2-p_i-q_1)M_{AA}{}^{\mu\nu} = \int d^4x\,d^4y\,d^4z\,d^4w \exp[i(-q_1\cdot x+q_2\cdot y-p_i\cdot w+p_f\cdot z)]$$
$$\times(-\gamma\cdot p_f+m)\langle 0|T[\mathfrak{A}_{1\alpha}{}^{\mu}(x)\mathfrak{A}_{2\beta}{}^{\nu}(y)\psi_\gamma(z)\bar{\psi}_\sigma(w)]|0\rangle(-\gamma\cdot p_i+m), \qquad (1)$$

and the amplitude M_{PP} obtained by replacing $\mathfrak{A}_{1\alpha}{}^{\mu}(x)$ and $\mathfrak{A}_{2\beta}{}^{\nu}(y)$ in (1) by $\varphi_{1\alpha}(x)$ and $\varphi_{2\beta}(y)$ and operating on the vacuum expectation value by $(\Box_x{}^2+\mu^2)(\Box_y{}^2+\mu^2)$. Here $\psi_\gamma(z)$ and $\varphi_\alpha(x)$ are nucleon and pion-field operators, $\mathfrak{A}_\alpha{}^\mu(x)$ is the axial-vector current density, μ and m are the pion and nucleon masses, and $\alpha, \beta, \gamma, \cdots$ are isospin indices. $\bar{u}(p_f)M_{PP}u(p_i)$ is the transition amplitude for πN scattering.

Using the PCAC hypothesis, $\partial_\mu\mathfrak{A}_\alpha{}^\mu(x) = C\varphi_\alpha(x)$, and the commutation relation (c.r.)

$$[\mathfrak{A}_\alpha{}^0(x), \mathrm{A}_\beta{}^\nu(y)]_{x_0=y_0} = i\epsilon_{\alpha\beta\beta'}\mathfrak{V}_{\beta'}{}^\nu(x)\delta(\vec{x}-\vec{y}) + \text{a possible Schwinger term}, \qquad (2)$$