sense. For this purpose we introduce

$$\varphi(x)=\partial j^{\mu}(x)/\partial x^{\mu},\quad \pi(x)=\partial\varphi(x)/\partial t \tag{13}$$

and find, using (8) and (12),

$$\frac{d}{d\tau}\langle 0\,|\pi_{\tau,t}(0)\,|0\rangle\Big|_{\tau=0} = 0 = i\langle 0\,|[\textstyle\int_V j^0(\vec{x},t)d^3x,\ \pi(0)]\,|0\rangle, \tag{14}$$

and therefore, since boundary terms drop because of local commutativity,

$$\langle 0\,|[\textstyle\int_V \varphi(\vec{x},0)d^3x,\ \pi(0)]\,|0\rangle=0, \tag{15}$$

which using the Lehmann-Källen representation[7,8]

$$\langle 0\,|[\varphi(x),\varphi(y)]\,|0\rangle=\int_0^\infty \rho(\kappa^2)\Delta(x-y,\kappa^2)d\kappa^2,$$

$$\rho(\kappa^2)\geqslant 0, \tag{16}$$

implies

$$\int_0^\infty \rho(\kappa^2)d\kappa^2=0 \tag{17a}$$

and, therefore,

$$\rho(\kappa^2)=0. \tag{17b}$$

With (17b) one has that

$$\langle 0\,|\varphi(x)\varphi(y)\,|0\rangle=\int_0^\infty \rho(\kappa^2)\Delta^{(+)}(x-y,\kappa^2)d\kappa^2=0, \tag{18}$$

and since the metric in the Hilbert space is positive definite, Eq. (18) gives

$$\varphi(x)\,|0\rangle=0. \tag{19}$$

and by the Johnson-Federbush[9] theorem,

$$\varphi(x)=\partial j^{\mu}/\partial x^{\mu}=0. \tag{20}$$

From Eq. (20) one deduces now

$$[Q(t)-Q(0),A]\ \text{ for all } A, \tag{21}$$

and Eq. (21) together with the irreducibility of the algebra of local operators and (11) implies

$$Q(t)=Q(0). \tag{22}$$

Therefore, assumptions (A), (B), and (C) imply the existence of an exact symmetry which commutes with the space-time translations. It is thus impossible to set up an algebra for the "generators" of approximate symmetries in the sense of Gell-Mann,[10,11] since those generators do not exist. However, the algebra of the currents integrated over a finite but arbitrarily large volume might exist and lead to the same consequences as the formal Gell-Mann algebra.[12]

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POLARIZATION IN PION-PROTON SCATTERING FROM 670 TO 3750 MeV/c*

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Using a polarized proton target, we have measured the polarization parameter $P(\theta)$ in pion-proton scattering for both positive and negative pions. Because there seems to be a great deal of current interest in the analysis of pion-proton scattering, we wish to present these experimental results at this time, even though we have not yet completed their analysis. The measurement consisted of scattering pions from polarized target protons and observing the asymmetry in scattered intensity, $I(\theta)$, as the spin directions of the target protons were reversed. The intensity for scattering from a target of polarization, P_T, is

$$I(\theta)_{\text{pol}}=I(\theta)_{\text{unpol}}[1+P(\theta)P_T],$$

where the parameter $P(\theta)$ is the same as the recoil-proton polarization in scattering pions from unpolarized protons, under the assumption that parity is conserved in the process.

The pion beam was momentum analyzed to within ±1% by a counter hodoscope, and, in the case of π^+, separation of protons was achieved by time-of-flight requirements and a gas Cherenkov threshold counter. The beam was focused on the one-inch-square target, and the entrance angles in both planes were measured by counter hodoscopes in the beam. Detection of final-state particles was made with a pair of crossed counter hodoscopes—one above and one below the emergent beam. Acceptable events were required to show coincidence among elements of the momentum, beam, and final-state hodoscopes, as well as with a small counter just below the polarized target crystals. In the case of kinematical ambiguity between π^+ and p in the final state, distinction was made with a liquid Cherenkov counter beneath the lower hodoscope.

The polarized target[1] consisted of 7 g/cm^2 of $La_2Mg_3(NO_3)_{12}\cdot 24H_2O$ in which the protons of the water of hydration (3% by weight) could be polarized by dynamic nuclear orientation.[2] The average polarization during the experiment was 50% and was reversed in sign about once every two hours.

Characterization of each accepted event was made by an on-line PDP-5 computer, summaries were displayed, and a record was written on magnetic tape. In the subsequent analysis, the requirement that the beam and final-state momenta lie in the same plane removed a large fraction of the background from scattering on heavy elements in the target. When attention was restricted to events with a final-state particle hitting a small region of the upper-counter array, a plot of numbers of counts versus lower-counter-array position showed a clear peak corresponding to elastic scattering from free protons. Once the background had been subtracted, the number of counts in the elastic peak could be used to determine the asymmetry in pion-proton scattering.

The background under the peak was evaluated by using events which failed the coplanarity requirement. For each element of the upper hodoscope a conjugate set of elements in the lower hodoscope was chosen in a way which was identical to the choice for coplanar elements—except it was displaced perpendicular to the plane of scattering. The set of events selected by these criteria is due to quasielastic scattering from bound protons with a transverse component of Fermi momentum and to inelastic scattering. It was verified that the distribution of these events with angle is the same as that for coplanar events outside the elastic-scattering-peak regions. In addition, data were taken at some beam momenta with a dummy target which contained elements similar to those of the crystal but no free protons. These dummy data gave results which substantiated those from the noncoplanar events.

In order to verify the validity of our method we measured the polarization parameter in p-p scattering at 1400 MeV/c using essentially the same beam and detection conditions as were used in the $\pi^\pm p$ scattering experiment reported here. The results are in good agreement with previous measurements.[3]

The lower limit in momentum transfer for which measurements could be made was imposed by the requirement that the recoil proton have a momentum of at least 350 MeV/c, so it could easily escape the target and penetrate the detector array. The minimum differential cross section for which polarization measurements were possible was approximately 50 μb/sr (center-of-mass system).

It was discovered during the run that relatively small amounts of electron contamination in the beam could lead to serious background caused by bremsstrahlung and subsequent production of electron-positron pairs in the one-third radiation length of the polarized target crystals. The resulting pairs had momenta which closely paralleled the beam momentum. The polarized-target magnet then separated the e^+ and e^- and directed one into each of the final-state hodoscopes. These "events" had good coplanarity and tended to obscure the pion-proton elastic peak. The remedy chosen was to insert approximately one radiation length of Pb at the first focus of our doubly focused beam.

The results of this experiment are shown in Figs. 1 and 2.[4] The errors shown are statistical only and do not include a ±10% uncertainty in scale due to inaccurate knowledge of target polarization. At those energies where previous measurements have been made, the agreement is good.[5-7] It is seen that the polarization is not small even at the highest energies of this experiment and there is considerable

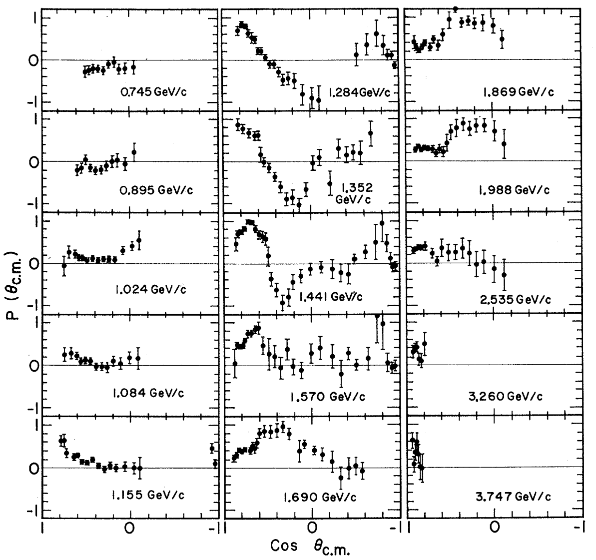

FIG. 1. Plots of the polarization parameter P versus cosine of the pion c.m. scattering angle for π^+p scattering. The errors shown are statistical only and do not inclued a ±10% uncertainty in scale due to inaccurate knowledge of target polarization.

structure in angular dependence. In particular the variation in the polarization with energy near the 1924-MeV $I=\frac{3}{2}$ resonance ($P\simeq 1500$ MeV/c, $T_\pi \simeq 1350$ MeV) is very striking.

Figure 3 is a plot of the momentum dependence of the coefficients in the Legendre expansion

$$I_0P=\sum C_i P_i^{\,1}(\cos\theta_{\text{c.m.}}),$$

fitted to the π^+ polarization P presented here and the π^+ differential cross section I_0 of Duke et al.[6] Preliminary analysis of these fits indicates that they are consistent with the assignment of $J^P=\frac{7}{2}^+$ for the 1924 resonance as reported by Duke et al.[6] on the basis of $\pi^\pm$-p cross-section and π^--p polarization data.

More extensive analysis of these data has been initiated; in the lower energy region a phase-shift search is in progress, and at higher energies attempts are being made to explain the data in terms of interference of Regge-exchange amplitudes with direct channel resonances.

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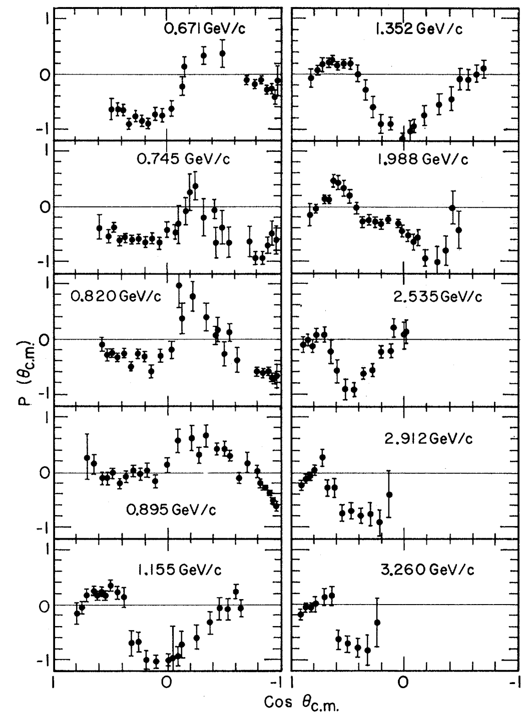

FIG. 2. Plots of the polarization parameter P versus cosine of the pion c.m. scattering angle for π^-p scattering. The errors shown are statistical only and do not include a ±10 % uncertainty in scale due to inaccurate knowledge of target polarization.

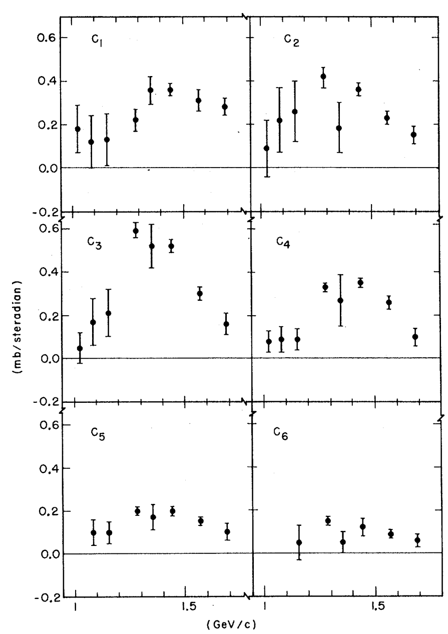

FIG. 3. Coefficients in the associated Legendre expansion $I_0P=\sum C_i P_i^1(\cos\theta_{\text{c.m.}})$ versus lab momentum of the pion for π^+p scattering.

course of this experiment. Finally, we are grateful to the Bevatron operating crew for their constant support.

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FORM FACTORS AT FINITE MOMENTA*

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The assumption[1,2] that the charge operators V_i and A_i of the vector and axial-vector current octets obey the algebra of SU(3)⊗SU(3) under equal time commutation has been extended[3-5] to include the entire algebra of moments of the charge-density operators $V_{i0}(\vec{x})$ and $A_{i0}(\vec{x})$ by the extra assumption that the equal-time commutators of these densities contain only the spatial δ function and not its derivatives. The algebra of moments may be expressed most clearly in terms of the Fourier components

$$V_i(\vec{q}) = \int e^{i\vec{q}\cdot\vec{x}} V_{i0}(\vec{x}) d_3x, \qquad (1)$$

$$A_i(\vec{q}) = \int e^{i\vec{q}\cdot x} A_{i0}(\vec{x}) d_3x, \qquad (2)$$

in the form

$$[V_i(\vec{q}), V_j(\vec{q}\,')] = if_{ijk} V_k(\vec{q}+\vec{q}\,'), \qquad (3)$$

$$[A_i(\vec{q}), A_j(\vec{q}\,')] = if_{ijk} V_k(\vec{q}+\vec{q}\,'), \qquad (4)$$

$$[V_i(\vec{q}), A_j(\vec{q}\,')] = if_{ijk} A_k(\vec{q}+\vec{q}\,'). \qquad (5)$$

The purpose of this Letter is to give a formal solution to this algebra, and to demonstrate how this solution may be employed in practice by use of the $p\to\infty$ technique.[6-8] For clarity the specific case of the vector current with isospin as the internal algebra will be taken, although the method generalizes in an obvious manner. In this case the algebra has the familiar form

$$[V^+(q'), V^-(q)] = V^3(q+q'), \qquad (6)$$

$$[V^3(q'), V^\pm(q)] = \pm V^\pm(q+q'), \qquad (7)$$

where the further specialization has been taken that q and q' are in the x direction, and all other commutators are zero. If Eq. (7) is now differentiated with respect to q', and q' is then set equal to zero, there results

$$[V^{3\prime}(0), V^\pm(q)] = V^{\pm\prime}(q), \qquad (8)$$

with the obvious formal solution

$$V^\pm(q) = \exp[\pm qV^{3\prime}(0)]V^\pm(0)\exp[\mp qV^{3\prime}(0)]. \qquad (9)$$