series. The last term is the contribution due to surface states. In this expression $R_r(i)$ is the residue of the integrand in Eq. (4) at the ith pole which occurs in the upper half complex plane to the right of the contour ζ, while $R_l(i)$ is the residue to the left of ζ in the upper half plane. (Poles on the contour are treated by letting $\Omega \to \Omega + i0^+$.) The ratio of the change in the density of states due to the first Born term to the unperturbed BCS density of states, $N(\omega)$, is

$$\frac{\delta N_{\mathrm{T}}^{(1)}(\omega)}{N(\omega)} = \frac{\varphi^2 t}{2\Omega E_{\mathrm{F}}} \mathrm{Si}\left(\frac{2\Omega}{\hbar v_f} d\right), \quad \frac{\Omega}{\varphi} \gg \left(\frac{\varphi}{E_{\mathrm{F}}}\right) t,$$

$$N(\omega) = \frac{\omega}{\Omega} \frac{m k_{\mathrm{F}}}{\pi^2 \hbar^2}. \tag{10}$$

We may estimate t by requiring this ratio to be of the order of 10^{-2} at $(2\Omega/\hbar v_{\mathrm{F}})d = \pi$ (~1% Tomasch effect in dV/dI). Taking $(\varphi/E_{\mathrm{F}}) \sim 3 \times 10^{-4}$ and $k_{\mathrm{F}} d \sim 10^5$, one obtains $t \sim 40$. For this value of t, the second term in Eq. (8) contributes

$$\frac{\delta N_{\mathrm{T}}^{(2)}(\omega)}{N(\omega)} \approx \left(\frac{\varphi^2 t}{2\Omega E_{\mathrm{F}}}\right)\left[\frac{t}{k_{\mathrm{F}} d}\left(\frac{\varphi}{\Omega}\right)^2\right] \sin\left(\frac{2\Omega d}{\hbar v_{\mathrm{F}}}\right),$$

$$\frac{\Omega}{\varphi} \gg \left(\frac{\varphi}{E_{\mathrm{F}}}\right) t. \tag{11}$$

This term is also of the order of 1% at its first maximum and thus comparable to the first Born term and shifted in phase by $\pi/2$. One observes the total δN, and it may not be possible to decompose this into the two terms. A careful study of the amplitude of δN with Ω should, however, reveal the presence of $\delta N_{\mathrm{T}}^{(2)}$ since it varies as Ω^{-4}. One would expect δN to vary initially as Ω^{-4} and change to Ω^{-3} as the energy increases.

It is also interesting to note for $t = 40$ that the surface-states band edge occurs according to Eq. (7) at $\omega_m \approx 0.8\varphi$ so that such a system has a double gap. If these surface states exist in real films, it may be possible to observe their effects in a tunneling experiment in which the edges of two films are separated by an insulating barrier.

In conclusion we would like to make a simple observation concerning the effect of an ordinary potential perturbation. If one considers a perturbation $V\tau_3\delta(x-d)$ at the film surface, it is readily seen that this induces a localized gap perturbation at the surface whose magnitude in the first approximation will be linear in V and of opposite sign. This gap perturbation will then induce oscillations in the density of states whose amplitude is linear in V, so that to first order, an ordinary potential perturbation can give rise to a Tomasch effect.

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[4]See Ref. 2 and also W. J. Tomasch, to be published, and Bull. Am. Phys. Soc. 11, 190 (1966).
[5]Y. Nambu, Phys. Rev. 17, 648 (1960).
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OBSERVATION OF RESONANT STRUCTURES AT 20- TO 23-MeV EXCITATION IN Si^{30} THROUGH THE REACTION $Mg^{26}(\alpha,\alpha)Mg^{26}$†

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Recently, large fluctuations in nuclear cross sections have been observed for many reactions at high excitations where the ratio of average compound nucleus width to average compound nucleus spacing, $\langle\Gamma_{CN}\rangle/\langle D_{CN}\rangle$, is larger than unity. Such data have been extensively analyzed in terms of the Ericson fluctuation theory[1] to extract such nuclear parameters as $\langle\Gamma_{CN}\rangle$. Block and Feshbach[2] and Kerman, Rodberg, and Young[3] have suggested that some of the structure might be due to particularly simple modes of excitation of the nucleus, e.g., two-particle, one-hole (2p1h) states. These states would have unique angular momentum and parity and would have widths which are intermediate between those of the states of the compound nucleus and those of the states of single-particle resonances. Some attempts to isolate such structures have been made.[4] In this study, we report the discovery of resonance structures in Si^{30} having nuclear properties which are not easily explainable in terms of the compound nucleus model. These structures are consistant with their being states of a simple configuration.

An alpha beam from the Argonne tandem Van de Graaff generator was used to bombard 70-μg/cm^2 thick self-supporting foils of enriched Mg^{26}. The scattered particles were detected simultaneously in six solid-state detectors mounted on a movable plate at 20, 45, 70, 95, 135, and 160° to the incident beam direction. Yield curves at 40, 65, 90, 115, and 140° were obtained in 25-keV steps from 9.0- to 13.75-MeV bombarding energy. Detailed angular distributions (shown in Fig. 1), from 20 to 160° in steps of 5°, were obtained in the above energy range in steps of 25 keV in some regions and 50 and 100 keV in the rest. The lines in Fig. 1 are smooth curves through the data points.

The large fluctuations found in the yield curves have only random correlation among the yields at different angles. In general, fluctuations are much larger at backward angles than at forward angles. The angular distributions show a typical diffraction pattern with forward maxima and minima smoothly shifting to smaller angles with increasing α energy. Over a large energy range, at backward angles, the angular pattern loses this smooth variation, but, in general, changes smoothly over a small energy interval.

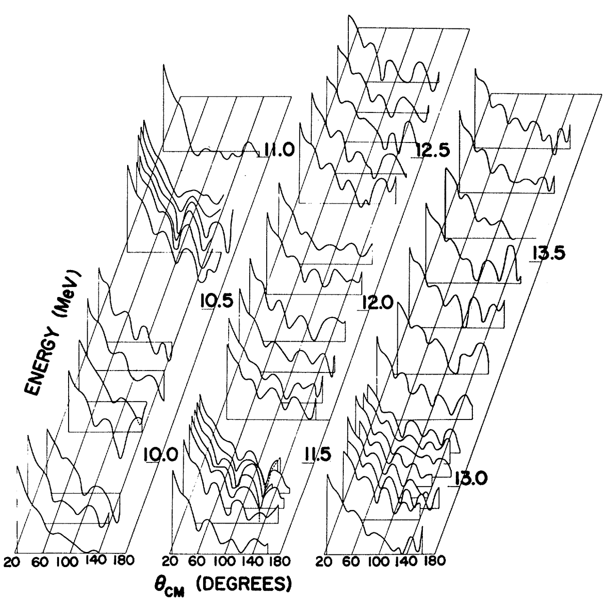

FIG. 1. Isometric plot of measured angular distributions for elastic α scattering by Mg^{26} as a function of α energy. The lines are smooth curves through experimental points.

Differential cross sections were first fitted with an optical model. On one hand, fits were not very good. On the other hand, the optical model parameters so obtained did not show a smooth variation as a function of energy. Optical-model fits have also been obtained by the Florida State group[5] to their α-scattering data for Mg^{24}.

Next, a serious attempt was made to fit the data with the smooth-cutoff model in which the scattering amplitude is represented by

$$f(\theta)=f_C-\frac{1}{2ik}\sum(2l+1)\exp(2i\sigma_l)(1-S_l)P_l(\cos\theta). \quad (1)$$

Here

$$S_l=\eta_l\exp(2i\delta_l),$$

$$\eta_l=\{1+\exp[-(l-l_\Delta)/\Delta]\}^{-1},$$

$$\delta_l=\delta_0\{1+\exp[(l-l_a)/\Delta_a]\}^{-1},$$

and f_C and σ_l are the Coulomb amplitude and phase shifts, respectively. The S matrix, S_l, was parametrized in terms of δ_0, l_Δ, Δ, l_a, and Δ_a. Fits obtained by this approach suffered from the same defects as the optical-model calculations. Up to three resonance terms of a Breit-Wigner form were introduced into (1) to simulate resonating partial waves. Fits to data improved; but, still, these were not altogether satisfactory and the resonance parameters for best fits were not easy to interpret.

Finally, a phase-shift analysis of the observed differential cross section was performed using (1). Starting phase shifts for the analysis were those obtained by fitting the data with the smooth-cutoff model. δ_l and η_l of partial waves with $l=10$ to 18 were fixed and were set equal to those so obtained. η_l and δ_l for these partial waves were very close to unity and zero degrees, respectively, over the entire energy range. At each energy η_l and δ_l for $l=0$ to 9 were varied independently to obtain best fits to the angular distributions. In Fig. 2 are shown a few typical fits (solid line) to the observed differential cross sections (solid circles). In all cases the fits reproduced the measured cross sections within experimental errors (~3 %).

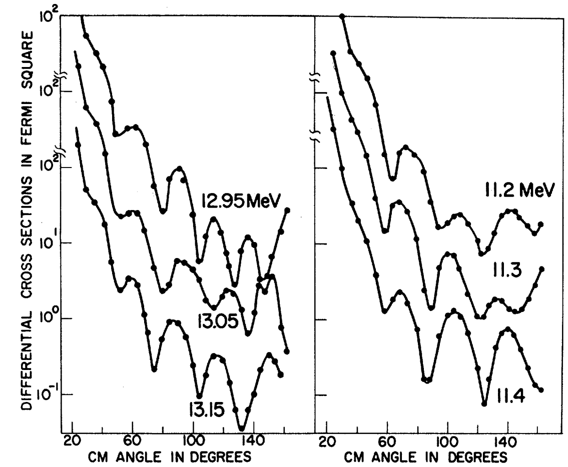

FIG. 2. Angular distribution of elastic alpha scattering by Mg^{26}. The differential cross sections are in units of square Fermi/unit solid angle. Solid circles are experimental points and solid lines represent the theoretical fits.

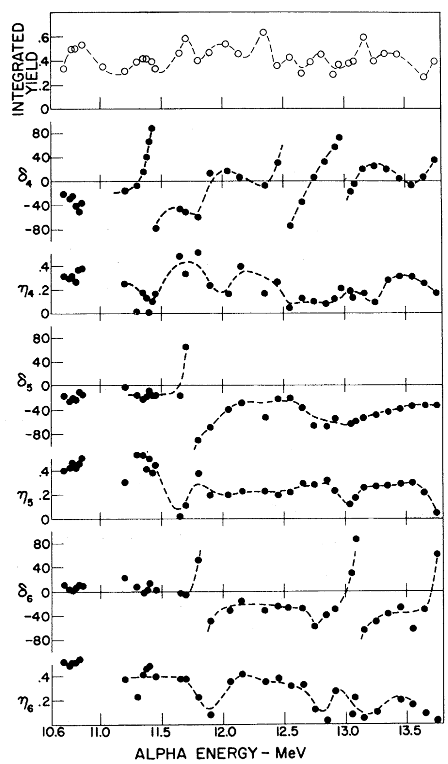

FIG. 3. Phase shifts, δ_l, and damping amplitude, η_l, for $l=4$, 5, and 6 as a function of alpha-particle energy. The topmost curve gives the sum of the ratio of the differential cross section to the Rutherford cross section over the measured angular range.

The fact that η_l and δ_l for $l=8$ and 9 were found to be close to unity and zero, respectively, over the entire energy range implies that enough partial waves were taken into account and also justifies fixing the parameters for the partial waves $l=10$ to 18.

The results of the phase-shift analysis for $l=4$, 5, and 6 are shown in Fig. 3 where δ_l and η_l are plotted as a function of incident energy. The topmost curve in the figure, integrated yield, is the sum of the ratio of the differential cross section to Rutherford cross section over the measured angular range. Both δ_l and η_l seem to change smoothly with energy; where δ_l goes through $\pi/2$, the resonance condition, η_l shows a dip as expected. η_l also shows dips at a few other energies where the phase shifts are going through zero, although the excursions around zero are not very large. For the case of an isolated resonance, doorway or compound-nucleus, Kerman[6] and Monahan[4] have shown that

$$\tan 2\delta = \frac{(E-E_d)\Gamma_d^{\uparrow}}{(E-E_d)^2+\frac{1}{4}\Gamma d(\Gamma_d-2\Gamma_d^{\uparrow})}$$

and

$$\eta = \left[\frac{(E-E_d)^2+\frac{1}{4}(\Gamma_d-2\Gamma_d^{\uparrow})^2}{(E-E_d)^2+\frac{1}{4}\Gamma_d^{\,2}}\right]^{1/2},$$

where E_d is the resonance energy, $\Gamma_d^{\uparrow}$ is the width for decay into the continuum, and Γ_d is the total width. In the vicinity of the resonance η would show a dip, the depth being dependent upon the ratio $\Gamma_d^{\uparrow}/\Gamma_d$; δ would go through either $\pi/2$ or zero depending upon whether $\Gamma_d^{\uparrow}/\Gamma_d$ is greater or less than 0.5, respectively. Presence of this systematic behavior of η_l and δ_l in our results clearly establishes the existence of resonant structures in Si^{30} at 20- to 23-MeV excitation. Because of the zero spin of the α particle and of Mg^{26}, the spins and parities of these structures are determined by the resonating partial wave.

In total we find about 14 resonances for which η_l and δ_l follow the predicted behavior. Their locations[7] are 10.81 (2^+), 11.30 (3^-), 11.43 (4^+), 11.65 (5^-), 11.85 (6^+), 11.95 (3^-), 12.15 (1^-), 12.20 (4^+), 12.4 (2^+), 12.5 (4^+), 12.95 (4^+), 13.1(6^+), 13.45 (4^+), and 13.75 (6^+) MeV. There seems to be strong correlation between these resonance positions and bumps in the integrated yield (Fig. 3). The widths[8] of these resonances determined from the variation of δ_l and η_l as a function of energy lie between 150 and 200 keV. The values of Γ_α/Γ for these resonances lie between 0.3 and 0.7. Between resonances, δ_l and η_l vary smoothly and show some fluctuation. The values of η_l off resonance are always less than unity, contrary to the expectation for an isolated resonance. With 10- to 13-MeV alpha energies many reaction channels, e.g., (α,α'), (α,n), and (α,p), are open. Direct interaction with finite absorption will reduce the η_l below unity. The observed off-resonance values of η_l imply the presence of direct interaction. In such a case, strictly speaking, the observed $\eta_l(E)=\eta_l^{\,\mathrm{di}}(E)\eta_l^{\,\mathrm{res}}(E)$.

Average spacing of compound nucleus levels at this excitation in Si^{30} was calculated[9] to be 2-4 keV. The average compound nucleus width as determined by fluctuation analysis of reaction cross sections in this excitation and mass region is ~50 keV.[10] Average spacing of 2p1h states calculated according to Le Couteur's[11] recipe comes to 100 keV, whereas Kerman, Young, and Rodberg[3] estimated the widths of the 2p1h states to be of the order of a few hundred kilovolts. The average spacing of the resonances observed in this study, about 150 keV, and their widths, 150-200 keV, cannot be easily reconciled with their being compound nucleus levels of the statistical model. However, their nuclear properties are consistent with these resonances in Si^{30} being states of a more simple configuration, such as 2p2h, than those of the compound nucleus. Further, we find that the observed resonances of a given spin almost exhaust among themselves the Wigner limit for alpha emission. For example, in the case of $J=4$ resonances the sum of their partial widths, Γ_α, for the elastic channel is 90% of the Wigner limit (for $J=6$ resonances it is 70%). It seems as if the strength of a single alpha-particle state of a given spin and parity is distributed among the observed resonances. This in itself indicates the simple nature of these resonances.

In summary, the phase-shift analysis of the differential cross section for the elastic scattering of 10.7- to 13.75-MeV alpha particles by Mg^{26} reveals that nearly all the partial waves up to $l=6$ are resonanting at certain energies though the majority of the resonances occur for $l=4$ and 6. 14 resonances at 20- to 23-MeV excitation in Si^{30} have been assigned spins and parities. The width and spacing of these resonances are indicative of their being of simple configuration.

A detailed report of this work will be published elsewhere shortly. Extremely valuable discussions with Dr. J. E. Monahan and Dr. J. P. Schiffer of Argonne National Laboratory and Dr. J. G. Wills of Indiana University are thankfully acknowledged. We also wish to thank Mr. F. J. Karasek of Argonne National Laboratory for preparing thin magnesium foils, and Dr. W. W. Eidson and Dr. B. B. Bardin for the help in taking part of the data.

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[7]Some of the locations are slightly uncertain because for a few cases, especially for 4^+ levels, the resonances are close enough to interfere. The effect of such an interference has not been taken into account.

[8]Phase shift, conventionally, is parametrized [J. W. Olness, W. Haeberli, and H. W. Lewis, Phys. Rev. 112, 1702 (1958)] in terms of $\gamma=\Gamma_\alpha/\Gamma$ and $\beta=\tan^{-1}[\Gamma/2(E-E_R)]$. The representation used here and the conventional one are equivalent; for example, γ and β can be expressed in terms of δ and η. In the γ and β parametrization one can get Γ_α/Γ and Γ a little more directly. However, the effect of the presence of direct interaction, as seems to be the case here, would increase the number of parameters to be fitted in the γ,β representation.

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