RADAR VERIFICATION OF THE DOPPLER FORMULA

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Frequencies of radar echoes from the planets Mercury and Venus have recently been measured to about 1 part in 10^{10} at times when the line-of-sight component of the relative velocity between Earth and target was as large as $10^{-4}c$, thus in principle permitting the second-order "longitudinal" term in the Doppler formula to be tested at about the 1% level.

Under the assumption that the "fixed" stars and the Newtonian center of mass of the solar system determine an inertial frame, the Doppler shift Δf, derived according to special relativity, is given by[1,2]

$$\Delta f = f\left(\frac{1-\beta_1^2}{1-\beta_3^2}\right)^{1/2}\left(\frac{1-\vec{\beta}_2\cdot\vec{e}_{12}}{1-\vec{\beta}_1\cdot\vec{e}_{12}}\right)\left(\frac{1-\vec{\beta}_3\cdot\vec{e}_{23}}{1-\vec{\beta}_2\cdot\vec{e}_{23}}\right)-f, \qquad (1)$$

where f is the transmitted frequency; $\vec{\beta}_1$, $\vec{\beta}_2$, and $\vec{\beta}_3$ denote the velocities (expressed as fractions of the speed of light c) of the radar antenna at the time of transmission (t_1), of the target planet at reflection (t_2), and of the antenna at echo reception (t_3), respectively; and $\vec{e}_{12}$, $\vec{e}_{23}$ are unit vectors pointing from the antenna at t_1 to the position of the planet at t_2 and from the planet at t_2 to the antenna at t_3, respectively. All coordinates are given with respect to the inertial frame. A "classical" derivation leads to the same result, except that the first parenthetical term is absent. Because the change in speed $\beta_3-\beta_1$, due mostly to the earth's rotation, is small, this first ("transverse") contribution never deviates from unity by more than a few parts in 10^{12}, and the difference is therefore unobservable.[2] (Similarly the small change in speed of the terrestrial observer implies that the variations in his clock rate with respect to the inertial-frame observer may be neglected.) If the motion of the antenna between t_1 and t_3 is ignored, we may set $\vec{\beta}_3=\vec{\beta}_1$, $\vec{e}_{23}=-\vec{e}_{12}$ and obtain

$$\Delta f = -2f[(\vec{\beta}_2-\vec{\beta}_1)\cdot\vec{e}_{12}\{1-(\vec{\beta}_2-\vec{\beta}_1)\cdot\vec{e}_{12}\}+O(\beta^3)]. \qquad (2)$$

The velocities of the earth and target planet must obviously be determined with sufficient accuracy by some independent means to test meaningfully the second-order term in Eq. (1). Traditionally their orbits are deduced from Newton's laws of motion and gravity and from observations of the angular positions of the planets with respect to the stars. First-order aberration and propagation-time corrections to these data are made routinely[3]; hence the same theory of light propagation that leads to Eq. (1) is intimately interwoven in the orbit-determination process. Interplanetary Doppler-shift measurements that verify Eq. (1) through terms of second order in β (Fig. 1) are, therefore, a test to this accuracy of the consistency of Newton's laws of planetary motion and the theory of light propagation.[8] The limiting factor in the comparison in Fig. 1 is the inaccuracy in the orbital determinations.[5] Using interplanetary time-delay measurements as well as optical observations to improve the orbits of Mercury, Venus, and Earth[9] leads

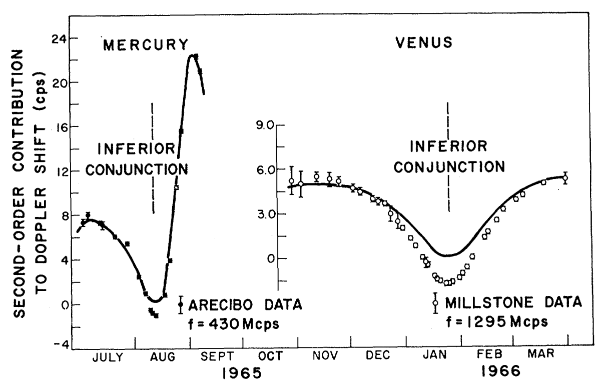

FIG. 1. The solid curves represent the theoretical second-order velocity contributions to the Doppler shift [see Eq. (1)] calculated from the standard Newcomb orbits for the inner planets.[4] The data points were obtained from the measurements by subtracting the first-order theoretical contributions. The rather large discrepancies are caused mainly by the greatly enhanced sensitivity near inferior conjunction of the longitudinal velocity component to the (known) errors in the Newcomb predictions of the relative angular orientations of the sun-planet vectors.[5] The larger differences apparent for Venus stem from the three-fold higher Millstone frequency and from the lower angular motion of Venus as seen from Earth. The magnitudes of the second-order contributions peak near elongation and vanish at inferior conjunction (as do the first-order parts); the asymmetry in the first curve is due to the large eccentricity (0.2) of Mercury's orbit. The Mercury data were obtained at Cornell's Arecibo Ionospheric Observatory[6] and the Venus data at the Massachusetts Institute of Technology Lincoln Laboratory Millstone Hill Facility.[7]

to the results shown in Fig. 2. Of course, the interpretation of the delay data also involves the theory of light, and, in fact, for propagation in a nondispersive medium, a simple "wave-counting" argument[10] shows that

$$\Delta f = -f\, d\tau/dt, \tag{3}$$

where Δf and τ, respectively, are the Doppler shift and delay associated with a signal received at t.

To separate quantitatively the above verifications of consistency into their component theoretical parts would presumably require an ad hoc parametrization of the basic theoretical structure followed by, say, a maximum-likelihood estimate of the parameter values and probable errors from a reanalysis of all available data. Such a procedure, which would of necessity include the simultaneous estimate of all other unknown parameters of the physical system, has not yet been carried out. (See, however, Ref. 9.)

The effects of general relativity on the Doppler formula may in principle be verified similarly. To investigate their expected magnitude, we consider corrections of first order in the gravitational radius r_0 of the sun to be of second order in β and find that the modifications of Eq. (1) are essentially of third order in β and, hence, are negligible,[11] except perhaps near superior conjunction when the radar signal passes close to the sun. (For X band and higher frequencies the solar corona is not expected to have any practical effect on the Doppler shift aside from possible frequency broadening.[12,13]) Equation (3) may be used to calculate the two-way Doppler formula from

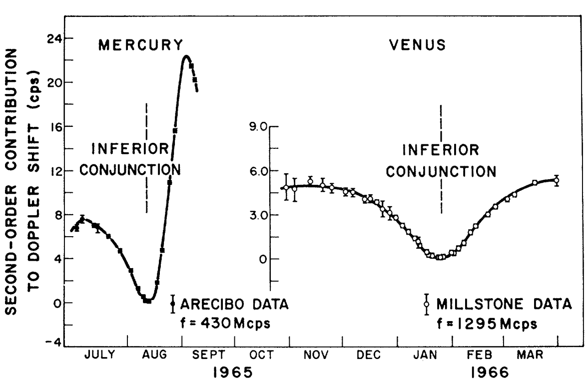

FIG. 2. Same as Fig. 1, except that the first-order theoretical contributions subtracted from the Doppler-shift measurements were obtained from recently improved orbits of the inner planets.[9]

the corresponding time-delay result for the Schwarzschild metric.[12,14] Near superior conjunction the important additional term Δf_{gr} can be approximated in the limit of circular coplanar orbits by[12]

$$\Delta f_{gr} \approx \pm f \frac{8r_0}{d(r_1+r_2)} |r_1\beta_2 - r_2\beta_1|; \quad d \ll r_1, r_2, \tag{4}$$

where d is the distance of closest approach of the radar wave to the center of the sun, and r_1, r_2 are the orbital radii of the earth and target planet, respectively. The minus sign applies for the presuperior-conjunction configuration and the plus sign for the post-conjunction configuration. The absolute-value term becomes $|r_2\beta_2 + r_2\beta_1|$ if one object orbits in the opposite sense; hence this contribution to the Doppler formula would be maximized if the target were an artificial planet in a retrograde orbit.[13]

Although not derived operationally, Eq. (4) does yield the approximate magnitude of the significant modification of the Doppler formula by general relativity.[15,16] With d equal to three solar radii (the closest approach feasible with present radar systems), the right-hand side of Eq. (4) has a value of only about $5\times10^{-10}f$ for Earth-Mercury parameters and $1.5\times10^{-10}f$ for Earth-Venus parameters.[12,17] Moreover, since the orbits are not coplanar, the interplanetary line of sight will not in general approach the sun along the path of "steepest descent" and Δf_{gr} will be less than indicated by Eq. (4).[18] Because of the weakness of the echo near superior conjunction and the frequency broadening introduced by the rotating planet (and perhaps augmented by the solar corona), it is doubtful that the general relativistic modification to the Doppler formula can be verified reliably from presently planned planetary observations. In any event, Eq. (3) shows that a test of any nondispersive theory of light propagation (e.g., general relativity with a static metric) utilizing a two-way Doppler shift will be equivalent to a test involving the time dependence of the time delay.[14,19]

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[1] I. I. Shapiro, Bull. Astron. 25, 178 (1965).

[2] See also D. O. Muhleman, D. B. Holdridge, and N. Block, Astron. J. 67, 191 (1962).

[3]U. S. Naval Observatory, Explanatory Supplement to the American Ephemeris and Nautical Almanac (U. S. Government Printing Office, Washington, D. C., 1960).

[4]More precisely, we used the Jet Propulsion Laboratory ephemeris tapes that were based directly on Newcomb's orbits [see P. R. Peabody, J. F. Scott, and E. G. Orozco, Jet Propulsion Laboratory Technical Memorandum 33-167, 1964 (unpublished)].

[5]See, for example, R. L. Duncombe, Astron. J. 61, 266 (1956); and D. K. Kulikov, Bull. Astron. 25, 139 (1965).

[6]G. H. Pettengill, R. B. Dyce, and D. Campbell, to be published.

[7]J. V. Evans, R. A. Brockelman, E. N. Dupont, L. B. Hanson, and W. A. Reid, to be published.

[8]A preliminary comparison based only on the much less accurate 1961 Earth-Venus data [W. B. Smith, Astron. J. 68, 15 (1963)] is given in Ref. 1. See also J. E. B. Ponsonby, J. K. Thomson, and K. S. Imrie, Bull. Astron. 25, 217 (1965).

[9]M. E. Ash, I. I. Shapiro, and W. B. Smith, to be published.

[10]Let $f_\gamma(t)$ and $\tau(t)$ be the frequency and round-trip time delay, respectively, of a signal whose echo is received at t. Successive "crests" of the echo detected at t and $t+f_\gamma^{-1}$ were transmitted, respectively, at $t-\tau$ and approximately $t+f_\gamma^{-1}-\tau(t+f_\gamma^{-1})$, with the difference of the latter being simply f^{-1}. Since the instantaneous frequency is the time derivative of phase, it follows exactly that $f^{-1}=f_\gamma^{-1}-\dot{\tau}f_\gamma^{-1}$ and hence that $\Delta f \equiv f_\gamma - f = -f\dot{\tau}$ with all times and frequencies as measured by the observer. If the transmitter and receiver are physically separated (one-way effect), this derivation is still valid provided that the "same" clock is available at both locations (e. g., provided that a universal coordinate time exists).

[11]Expected improvements in frequency standards may make such terms experimentally accessible by means of phase-coherent radio communications maintained between Earth and an interplanetary spacecraft.

[12]I. I. Shapiro, Lincoln Laboratory Technical Report No. 368, 1964 (unpublished).

[13]I. I. Shapiro, Phys. Rev. 145, 1005 (1966).

[14]I. I. Shapiro, Phys. Rev. Letters 13, 789 (1964).

[15]C. R. Smith and I. I. Shapiro, to be published.

[16]Neglect throughout of the differences between Newtonian and relativistic orbits has not significantly affected our conclusions.

[17]See also J. P. Richard, Bull. Am. Phys. Soc. 11, 708 (1966).

[18]A formula valid for arbitrary orbits is given by M. J. Tausner, Lincoln Laboratory Technical Report No. 425, 1966 (unpublished).

[19]If highly accurate frequency standards were placed in interplanetary orbits, the one-way Doppler shift could be monitored; this feature would, in addition, allow the "red-shift" effect to be studied.

COSMIC ELECTRONS ABOVE 10 GeV AND THE UNIVERSAL BLACK-BODY RADIATION AT 3°K

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The increasing number of observations made during recent years on the cosmic-ray electrons, whose abundance is only about 1% of that of cosmic-ray protons, has focused importance on the important role they can play in helping to understand some of the astrophysical properties associated with cosmic space traversed by them; of these the most important ones are the magnetic field strength and the radiation energy density. The potentiality of this method arises from the basic fact that the rates of energy loss suffered by electrons, through synchrotron radiation in magnetic fields and inverse Compton scattering in radiation fields, are both essentially proportional to the square of the energy, resulting in a progressively rapid depletion of electrons of high energy. The radiation field due to the universal black-body radiation at 3°K suggested on the basis of recent evidence[1-5] is expected to become so important compared to visible light in cosmic space (such as galactic halo and intergalactic space) that the energy loss suffered by electrons through inverse Compton scattering in this field would seriously affect their energy spectrum at high energies.

Until recently, all measurements on the cosmic-ray electrons have been made at energies <10 GeV; of these seven are between 1 and 10 GeV.[6-12] At these energies the importance of the deductions that can be made of the type described earlier is severely limited because of the following two reasons: (i) At energies below a few GeV, solar modulation considerably modifies the energy spectrum of the electrons reaching the vicinity of the earth. Hence, in order to infer the spectrum in interstellar space, it is necessary to make corrections for the solar modulation which are not known well enough yet. (ii) It is now generally believed