*Work supported by Texas Atomic Energy Research Foundation.

[1]L. Spitzer, Physics of Fully Ionized Gases (Interscience Publishers, Inc., New York, 1955).

[2]F. R. Scott and H. G. Voorhies, Phys. Fluids 4, 600 (1961).

PARAMETRIC COUPLING BETWEEN ELECTRON-PLASMA AND ION-ACOUSTIC OSCILLATIONS

R. A. Stern and N. Tzoar
Bell Telephone Laboratories, Whippany, New Jersey
(Received 19 August 1966)

The simultaneous excitation of electron-plasma and ion-acoustic oscillations by means of a single electromagnetic wave at $\omega \cong \omega_p$ has been observed experimentally and analyzed theoretically, with consistent results.

In the linear approximation, an electron-ion plasma supports two distinct, independent modes of oscillation: the electron-plasma and the ion-acoustic waves. We have observed a strong nonlinear process in which these two basic collective modes are coupled and simultaneously excited by means of a single high-frequency electromagnetic field. This process makes use of the nonlinear properties of large-amplitude, resonant plasma oscillations which can be generated in a bounded plasma,[1] and appears to be considerably more efficient than theoretically proposed schemes for parametric excitation of plasma oscillations using transverse electromagnetic fields.[2]

In the experiment, a microwave signal of frequency ω_0 was transversely beamed at a cylindrical plasma column whose average electron-plasma frequency $\omega_p \cong \omega_0$. Under these resonant conditions, strong coupling between the transverse microwave field and longitudinal electron-plasma oscillations takes place.[3] Figure 1 shows the spectrum of radiation reflected and emitted by the plasma for various levels of incident microwave power at resonance. For low values of incident power, the plasma reflected radiation at ω_0 only, as shown in Fig. 1(a). This illustrates the familiar condition in which a single electron-plasma oscillation mode at ω_0 is excited. When the incident power at ω_0 was increased above a very sharp threshold level, the plasma was found to emit coherent radiation at two additional frequencies $\omega_0-\Omega$ and $\omega_0+\Omega$, equally spaced about ω_0, as shown in Fig. 1(b). The frequency difference Ω fits very closely just above threshold to the value of the ion-acoustic oscillation with a wavelength given by the inside diameter of the tube.[4] Emission from the plasma at these high frequencies ($\cong\omega_0$) corresponds to the excitation of electron-plasma modes: This was independently verified by means of the incoherent-microwave-scattering technique.[5] High-frequency microwaves at $\omega_{\rm in} \gg \omega_p$ were beamed at the plasma, and the scattered spectrum was analyzed. Figure 1(c) shows the scattered spectrum in the vicinity of $\omega_{\rm in}-\omega_0$. For incident microwave power at ω_0 above the threshold, this exhibited components at $\omega_{\rm in}-\omega_0$ and $\omega_{\rm in}-\omega_0\pm\Omega$, showing that three electron-plasma oscillations were simultaneously excited by the microwaves at ω_0.

Above the threshold, the scattered spectrum also exhibited components at $\omega_{\rm in}\pm\Omega$, showing that an ion-acoustic oscillation at Ω was excited. This was borne out by independent low-frequency probe measurements. A negatively biased probe was introduced into the plasma near the region of maximum microwave field, so that it drew a current proportional to the ion density near its tip. The fluctuations in probe current were displayed on a low-frequency spectrum analyzer, as shown in Figs. 1(d) and 1(e). Above the threshold, a large ion-density component fluctuating at Ω appeared, and its amplitude increased with incident microwave power at ω_0.

For incident microwave power at and above the threshold (field strengths up to 15 V/cm), the spectrum described above did not occur when the microwave and plasma frequencies were not adjusted for resonance, or when the plasma column axis was parallel to the electric field vector, i.e., for low ratios of longitudinal to transverse field in the plasma.

We regard the three electron-plasma oscillations at ω_0 and $\omega_0\pm\Omega$ and the ion-acoustic oscillation at Ω as the components of the spectrum of a four-frequency parametric amplifier driven to oscillate by the application of a longitudinal "pump" field at ω_0 in excess of the

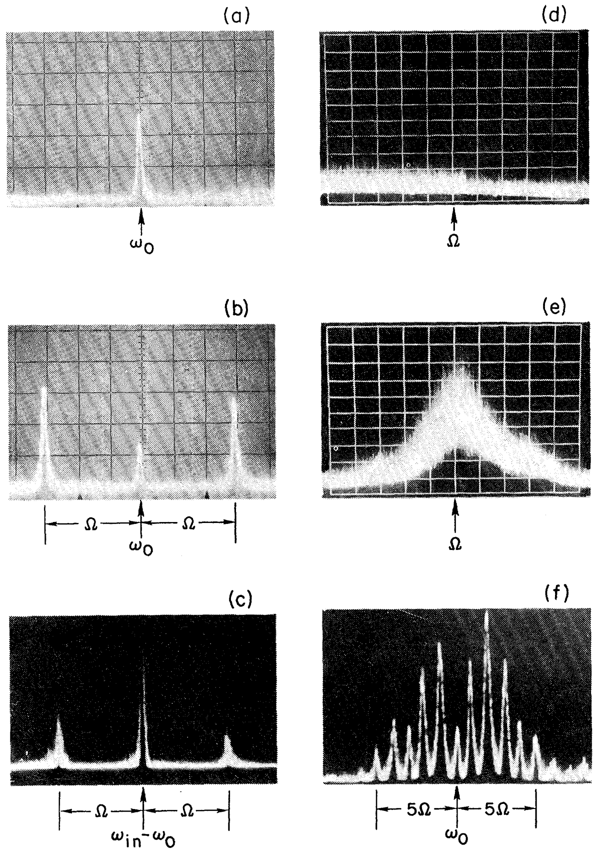

FIG. 1. Spectral components of parametrically excited plasma oscillations. Ordinate: amplitude. Abscissa: frequency. Curves (a) and (b) have emission near $\omega_0 = 4.4$ Gc/sec, dispersion 40 kc/cm, resolution 1 Kc/sec. (a) Below threshold, incident power 2 W (field strength ~10 V/cm). (b) Above threshold, incident power 3 W (field strength ~12 V/cm). (c) Incoherent scattering near $\omega_{in}-\omega_0$, with $\omega_{in} = 11.4$ Gc/sec. Curves (d) and (e) have ion-current-fluctuation spectrum near $\Omega = 120$ kc/sec, dispersion 10 kc/cm, resolution 10 cps. (d) Below threshold, incident power 2 W. (e) Above threshold, incident power 3 W. (f) Emission near $\omega_0 = 4.4$ Gc/sec from multiple ion-acoustic oscillations excited at maximum power (4 W into cavity with $Q = 700$). All data for 0.8-cm-i.d. mercury dc discharge at 1 mTorr, $\omega_0 = \omega_p = 4.4$ Gc/sec, $\Omega = 120$ kc/sec.

threshold level.[6] The sharp onset characteristic of parametric excitation can be checked for consistency with the interpretation. Extending Silin's analysis[2] of parametric excitation by transverse electric fields through the inclusion of a temperature term, we found the initial growth rate for parametrically excited electron-plasma and ion-acoustic oscillations to be $s = \Lambda\omega_{pe}\omega_{pi}/(\omega\Omega)^{1/2}$, where $\Lambda = (eE/m\omega_0^2)$, e and m are the electron charge and mass, E is the amplitude of the applied electric field at the frequency $\omega_0 = \omega + \Omega$, and ω and Ω are the eigenvalues of the electron-plasma and ion-acoustic modes, respectively. Introducing typical observed frequencies and local electric field strength at threshold, s is found to be about two orders of magnitude less than known damping rates.[7] In order to estimate the effect of the large-amplitude density fluctuations associated with longitudinal applied fields, we considered an alternate model. In the hydrodynamic description the inertial term $\vec{V}\cdot\nabla\vec{V}$ was neglected, but typical driven density-fluctuation terms which are first order in the applied field were retained. Using the same perturbation technique, the initial growth rate for small Λ ($<10^{-3}$) now turned out to be $s' = \Lambda\omega_{pe}^2/(\omega\Omega)^{1/2}$, $s'/s = (M/m)^{1/2}$, a large number. Introducing the value of an observed threshold, for instance $E = 5$ V/cm at 4.4 Gc/sec in a 0.5-Torr, 0.2-cm-i.d. helium discharge, the growth rate $s' \cong 10^6$ sec^{-1}, large enough to overcome damping of the ion-acoustic oscillations.[7]

Our simplified model, therefore, seems to confirm the experimental observations, that longitudinal field coupling is more efficient than transverse field excitation in the parametric generation of plasma oscillations.

The threshold value of the ion-acoustic mode frequency should be close to the value given by the linear dispersion relation. Table I shows that the agreement is very good over a range of almost two orders of magnitude obtained by varying both wavelength and ion mass. This also suggests that the spectrum of parametrically induced oscillations may be used as a simultaneous diagnostic of the electron density and temperature in bounded plasmas.

At field strengths greatly in excess of the threshold, e.g., with a standing-wave field of about 200 V/cm, the plasma was also found

Table I. Calculated and observed ion-acoustic oscillation frequency at threshold for parametric excitation.

Ion mass (a.u.)	Tube i.d. (cm)	Calculated frequency	Observed frequency
200 (Hg)	0.6	122 kc/sec	120 kc/sec
200 (Hg)	0.15	490 kc/sec	500 kc/sec
4 (He)	0.2	3.9 Mc/sec	3.6 Mc/sec

to emit over a wide spectrum about ω_0, with sharp peaks in the vicinity of the multiples of Ω and of the ion-plasma frequency. The latter may represent instabilities, in which the screening effect of the electrons on the ions is lost.

In summary, we have described a strong nonlinear coupling between electron-plasma and ion-acoustic oscillations. The properties of the process are in qualitative agreement with theory.

It is a pleasure to acknowledge very valuable discussions and communications with S. J. Buchsbaum, A. Yariv, and D. Montgomery.

[1]R. A. Stern, Phys. Rev. Letters 14, 517 (1965).

[2]Coupling between plasma-electron and ion oscillations generated spontaneously in beam-plasma systems has been reported by J. M. Jones and K. G. Emeleus, Phys. Letters 12, 187 (1964). The parametric excitation of plasma oscillations by means of transverse electromagnetic radiation has been discussed by A. Yariv, Bell Telephone Laboratories, Murray Hill, Technical Memorandum No. MM 59-124-28, 6 August 1959 (unpublished), and in Proceedings of the Seventh International Conference on Ionization Phenomena in Gases, Belgrade, 1965 (Gradjevinska Knjiga Publishing House, Beograd, 1966), Vol. 3, paper 4.4.5; V. P. Silin, Zh. Eksperim. i Teor. Fiz. 48, 1679 (1965) [translation: Soviet Phys.–JETP 21, 1127 (1965)]; D. F. DuBois and M. V. Goldman, Phys. Rev. Letters 14, 544 (1965); E. Atlee Jackson, Bull. Am. Phys. Soc. 11, 577 (1966), and University of Illinois Coordinated Science Laboratory Report No. R-298, 1966 (unpublished); D. Montgomery and I. Alexeff, Phys. Fluids 9, 1362 (1966); and Y. C. Lee and C. H. Su, to be published. Parametric excitation of electron-plasma oscillations at ω_p by means of an electromagnetic wave at $2\omega_p$ has been reported by Kiyoe Kato, Morihiro Yoseli, Satoru Kiyama, and Shinsuke Watanabe, J. Phys. Soc. Japan 20, 2097 (1965).

[3]The positive column of a low-pressure dc discharge tube was inserted across a wave guide with the tube axis normal to the direction of propagation and the electric field vector of microwaves in the TE_{01} mode. The coupling mechanism has been described by L. Tonks, Phys. Rev. 37, 1458 (1931); A. Dattner, Phys. Rev. Letters 10, 205 (1963); P. Weissglass, ibid. 10, 206 (1963); J. Nucl. Energy: Pt. C 6, 251 (1964). J. C. Nickel, J. V. Parker, and R. W. Gould, Phys. Rev. Letters 11, 183 (1963); Phys. Fluids 7, 1489 (1964). F. C. Hoh, Phys. Rev. 133, A1016 (1964); P. E. Vandenplas and R. W. Gould, J. Nucl. Energy: Pt. C 6, 449 (1964).

[4]In Table I are shown observed and calculated values for Ω. The latter were obtained from the formula for the frequency of radial electrostatic sound waves (ion-acoustic waves) given by F. W. Crawford, Phys. Rev. Letters 6, 663 (1961), as $f=(2.405/\pi d)(\gamma kTe/M)^{1/2}$, with d taken as the discharge-tube i.d. The values of $(\gamma kTe/M)^{1/2}$ used were taken from the measurements of I. Alexeff and W. D. Jones, Phys. Rev. Letters 15, 286 (1965); and Proceedings of the International Conference on Ionization Phenomena in Gases, Paris, 1963, edited by P. Hubert (S. E. R. M. A., Paris, 1964), Vol. 3, p. 121.

[5]Incoherent microwave scattering from driven plasma oscillations is described in R. A. Stern and N. Tzoar, Phys. Rev. Letters 15, 485 (1965); V. D. Fedorchenko, V. I. Muratov and B. N. Rutkevich, Zh. Tekh. Fiz. 35, 2021 (1965) [translation: Soviet Phys.–Tech. Phys. 10, 1549 (1966)]; M. Iannuzzi, Phys. Rev. 145, 81 (1966); and P. F. Little and S. M. Hamberger, Nature 209, 972 (1966).

[6]That such a process is possible follows intuitively from previous work, where it was shown that, when resonant electron-plasma oscillations at discrete frequencies ω and $\omega+\Delta\omega$ are simultaneously generated, appreciable oscillations at the difference frequency $\Delta\omega$ are excited.[1] By extension, if $\Delta\omega$ is made equal to the ion-acoustic frequency Ω, then strong coupling between electron-plasma and ion-acoustic oscillations can be expected. The coexistence of such nonlinearly coupled oscillations suggested that, above a threshold level, it may be sufficient to "pump" the system at the frequency of a single linear oscillation mode in order to elicit the entire spectrum, as borne out by our observations.

[7]An upper bound for damping of ion-acoustic oscillation is given by ionization damping, cf. F. W. Crawford, Phys. Letters 4, 135 (1963). For our conditions, using $\alpha=10^{-2}$ ion pairs/cm Torr and $V_D=2\times10^7$ cm/sec for the first Townsend coefficient and electron drift velocities, respectively, the ionization frequency $\nu_1=4\times10^5\ \mathrm{sec}^{-1}<s'$.

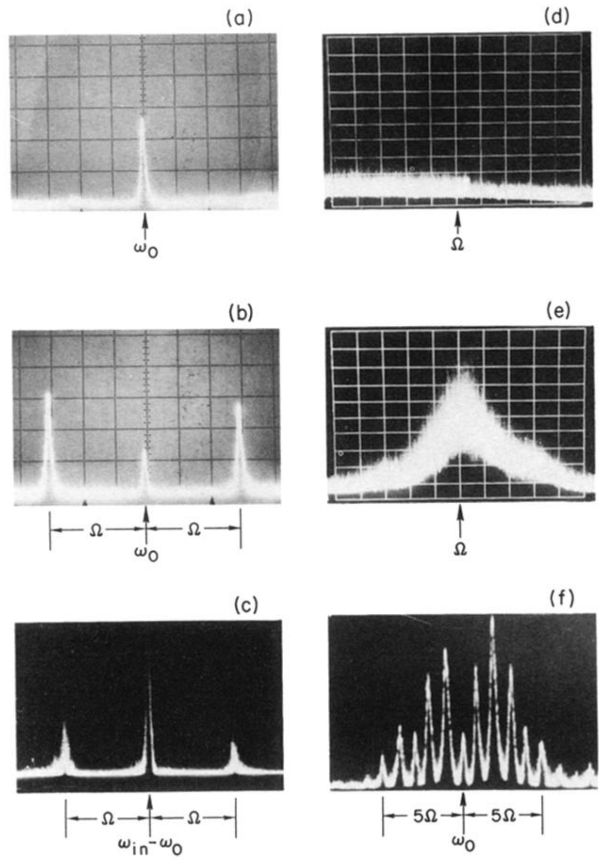

FIG. 1. Spectral components of parametrically excited plasma oscillations. Ordinate: amplitude. Abscissa: frequency. Curves (a) and (b) have emission near $\omega_0 = 4.4$ Gc/sec, dispersion 40 kc/cm, resolution 1 Kc/sec. (a) Below threshold, incident power 2 W (field strength ~10 V/cm). (b) Above threshold, incident power 3 W (field strength ~12 V/cm). (c) Incoherent scattering near $\omega_{in}-\omega_0$, with $\omega_{in} = 11.4$ Gc/sec. Curves (d) and (e) have ion-current-fluctuation spectrum near $\Omega = 120$ kc/sec, dispersion 10 kc/cm, resolution 10 cps. (d) Below threshold, incident power 2 W. (e) Above threshold, incident power 3 W. (f) Emission near $\omega_0 = 4.4$ Gc/sec from multiple ion-acoustic oscillations excited at maximum power (4 W into cavity with $Q = 700$). All data for 0.8-cm-i.d. mercury dc discharge at 1 mTorr, $\omega_0 = \omega_p = 4.4$ Gc/sec, $\Omega = 120$ kc/sec.