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CALCULATION OF THE BRANCHING RATIO $(\eta\to 2\pi+\gamma)/(\eta\to 2\gamma)$*

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Considerable interest attaches to the problem of the relative frequencies of the decay modes of the η meson. Since G-conjugation invariance requires the intercession of at least one photon (real or virtual), the observed decay modes turn out to be

$$\eta\to 3\pi\ (46.4\%);\ 2\gamma\ (33.5\%);$$

$$\pi^0+\gamma+\gamma\ (14.9\%);\ \ \pi^++\pi^-+\gamma\ (5.1\%).$$

Curiously enough, the first three decays, which are of order α^2 (α is the fine structure constant), dominate over the fourth, which is of order α. Previous attempts to understand this situation, even allowing for phase-space factors, have led to the introduction of a new quantum number.[1] The recent elegant extension by Weinberg[2] of the current algebra method to processes involving the emission of two "soft" pions makes it possible to understand the branching ratio $(\eta\to 2\pi+\gamma)/(\eta\to 2\gamma)$ on a dynamical basis and without the need of new quantum numbers.

The calculation proceeds as follows. Using the notation of Ref. 2 wherever possible, we expand the time-ordered product

$$T\{\partial_\mu A_a{}^\mu(x), \partial_\nu A_b{}^\nu(y), V_d{}^\lambda(0)\}, \tag{1}$$

where $A_a{}^\mu$ is the $\Delta S=0$ axial-vector current and $V_d{}^\lambda$ is the $\Delta I=1$ vector current. In this time-ordered product, $V_d{}^\lambda(0)$ has replaced the $\Delta I=\frac{1}{2}, \Delta S=-1$ axial-vector current $A_\eta{}^\lambda(0)$ considered in Ref. 2. The expansion of (1) leads to seven terms, similar to Eq. (4) of Ref. 2, of which the first three are disregarded for the same reasons as in Ref. 2. The four terms which remain are

$$-\frac{1}{2}\left(\frac{\partial}{\partial y^\nu}-\frac{\partial}{\partial x^\nu}\right)\delta(x^0-y^0)T\{[A_a{}^0(x),A_b{}^\nu(y)], V_d{}^\lambda(0)\}$$
$$-\tfrac{1}{2}\delta(x^0)\delta(y^0)\{[A_b{}^0(y),[A_a{}^0(x),V_d{}^\lambda(0)]]+[A_a{}^0(x),[A_b{}^0(y),V_d{}^\lambda(0)]]\}$$
$$-\delta(y^0)T\{[A_b{}^0(y),V_d{}^\lambda(0)],\partial_\mu A_a{}^\mu(x)\}-\delta(x^0)T\{[A_a{}^0(x),V_d{}^\lambda(0)],\partial_\nu A_b{}^\nu(y)\}. \tag{2}$$

In general, all four terms in (2) must be considered. But if we specialize to the matrix element $\langle 0|T\{\partial_\mu A_a{}^\mu(x),\partial_\nu A_b{}^\nu(y),V_d{}^\lambda(0)\}|P\rangle$ (where P is a pseudoscalar meson), then the last three terms in (2) do not contribute: The second term in (2) vanishes because the (double) commutators yield vector currents which are taken between $|0\rangle$ and $|P\rangle$, while the third and fourth terms vanish because the single commutators give rise to axial-vector currents taken between $|\pi\rangle$ and $|P\rangle$. We are left with only the first term in (2):

$$-i\epsilon_{abc}\left(\frac{\partial}{\partial y^\nu}-\frac{\partial}{\partial x^\nu}\right)\langle 0|T\{V_c{}^\nu(x),V_d{}^\lambda(0)\}|P\rangle\delta^4(x-y). \tag{3}$$

Let us choose $P=\eta$ and define

$$\langle\pi_{qa}\pi_{pb}|V_d{}^\lambda(0)|\eta_k\rangle=i(2\pi)^{-9/2}(8q^0p^0k^0)^{-1/2}(e/m_\eta{}^3)F_\eta\epsilon_{\lambda\alpha\beta\gamma}k^\alpha q^\beta p^\gamma, \tag{4}$$

where F_η is the form factor involved in the decay $\eta \to 2\pi + \gamma$. Following Weinberg,[2] the off-shell amplitude for both pions in terms of (4) is defined by

$$F_\pi^{-2} m_\pi^{-4} (q^2 + m_\pi^2)(p^2 + m_\pi^2) \int d^4x\, d^4y\, e^{-iq\cdot x} e^{ip\cdot y} \langle 0 | T\{\partial_\mu A_a^\mu(x), \partial_\nu A_b^\nu(y), V_d^\lambda(0)\} | \eta\rangle$$
$$= i(2\pi)^{-3/2} (2k^0)^{-1/2} (e/m_\eta^3) F_\eta \epsilon^{\lambda\alpha\beta\gamma} k_\alpha q_\beta p_\gamma. \quad (5)$$

It should be noted that the transition amplitude (4) vanishes if one contracts one pion at a time and uses the current commutation relations instead of treating the two pions symmetrically as is done in (5). Using (3), the left-hand side of (5) can be rewritten as

$$F_\pi^{-2} m_\pi^{-4} (q^2 + m_\pi^2)(p^2 + m_\pi^2)(q-p)_\nu \epsilon_{abc} M_{cd}^{\nu\lambda}(q+p, k), \quad (6)$$

where

$$M_{cd}^{\nu\lambda} = \int d^4x\, e^{-i(q+p)\cdot x} \langle 0 | T\{V_c^\nu(x), V_d^\lambda(0)\} | \eta\rangle. \quad (6a)$$

The expression (6a) can now be related to the amplitude for η to decay into two isovector photons as follows; the amplitude for the decay $\eta \to 2\gamma$ (real photons) is

$$\langle \gamma_{k_1} \gamma_{k_2}^{\text{out}} | \eta_k^{\text{in}} \rangle = i(2\pi)^{-9/2} (8k_{10} k_{20} k_0)^{-1/2} e^2 \epsilon_{1\nu}^* \epsilon_{2\lambda}^*$$
$$\times i(2\pi)^4 \delta^4(k - k_1 - k_2) \int \langle 0 | T\{V^\nu(x) V^\lambda(0)\} | \eta\rangle e^{-ik_1\cdot x} d^4x$$
$$= i(2\pi)^{-9/2} (8k_{10} k_{20} k_0)^{-1/2} e^2 i(2\pi)^4 \delta^4(k - k_1 - k_2) \epsilon_{1\nu}^* \epsilon_{2\lambda}^* M^{\nu\lambda}(k_1, k), \quad (7)$$

where $V^\nu(x)$ is the electromagnetic current and ϵ^* is the photon polarization. On grounds of covariance, we can next write

$$M^{\nu\lambda}(k_1, k) = M \epsilon^{\nu\lambda\rho\sigma} k_{1\rho} k_\sigma, \quad (8)$$

$$M_{cd}^{\nu\lambda}(q+p, k) = M_{cd} \epsilon^{\nu\lambda\rho\sigma} (q+p)_\rho k_\sigma, \quad (9)$$

where M (M_{cd}) is the amplitude for the decay of η into two real (isovector) photons. The relation between M and M_{cd}, using SU(3) invariance, is

$$M_{cd} = \tfrac{3}{2} M \delta_{cd}. \quad (10)$$

Combining Eqs. (5), (6), and (10), we obtain

$$F_\eta = F_\pi^{-2} 3 M M_\eta^3 \quad (11)$$

from which $\Gamma(\eta \to 2\pi)/\Gamma(\eta \to 2\gamma) = 0.19$ in excellent agreement with the experimental value of 0.15 ± 0.03.

From a physical point of view, one may understand the relative weakness of the mode $\eta \to 2\pi + \gamma$ as follows. There is no bremsstrahlung term because of parity conservation. The transition rate depends on the fourth power of the pion momentum so that the high-frequency part of the photon spectrum is suppressed. The close agreement of our result with experiment shows that the kinematical dependence of the matrix element in pion momenta makes it possible to ignore the final-state interaction[3] of the pions even though they are in a P state.

A previous dynamical calculation[4] of the branching ratio $(\eta \to 2\pi + \gamma)/(\eta \to 2\gamma)$ required the hypothesis of "ρ-pole" dominance, the questionable assumption[5] of nonvanishing of the $(\eta\rho\gamma)$ vertex, and a knowledge of certain parameters. In this calculation, neither assumption is made and there are no free parameters.

The above method is easily extended to the vector amplitude in K_{l4} which can be related to the amplitude for the decay of K into an isovector photon and a $\Delta S = 1$. This vector amplitude makes an insignificant contribution to the K_{l4} decay rate but can affect—through its interference with the axial-vector amplitude—the "up-down" asymmetry.[6] Using SU(3) invariance and the $\eta \to 2\gamma$ decay rate, we find a 5% effect.[7]

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Note added in proof.—After this paper was completed, a recent paper by Ademollo and Gatto[8] came to our attention which derives essentially the same result for the η branching ratio by a somewhat different procedure. The method used here is more general and will yield different results for most other applications.

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[2]S. Weinberg, Phys. Rev. Letters 17, 336 (1966).

[3]The axial-vector amplitude in K_{l4} leads to both S- and P-wave pions; however, the decay rate is dominated by the S-wave part [cf. Ref. 2 and C. G. Callan and S. B. Treiman, Phys. Rev. Letters 16, 153 (1966)].

[4]L. M. Brown and P. Singer, Phys. Rev. Letters 8, 460 (1962).

[5]Recently, it has been shown that a dispersion treatment of photopion production gives agreement with low-energy experiments if the $\pi\rho\gamma$ vertex (and a fortiori $\eta\rho\gamma$) is set equal to zero (A. Donnachie and G. Shaw, to be published).

[6]N. Cabibbo and A. Maksymowicz, Phys. Rev. 137, B438 (1965).

[7]It is possible to relate $\eta \to \pi+\pi+\gamma$ and the vector amplitude in K_{l4}, using SU(3) invariance; considering the pseudoscalar octet to be degenerate (i.e., equal mass for π and η). V. I. Zakharov, Yadern. Fiz. 1, 1053 (1965), obtains a 10% "up-down" asymmetry.

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MASS SPECTRUM OF BOSONS FROM 500 TO 2500 MeV IN THE REACTION $\pi^- + p \to p + X^-$ OBSERVED BY A MISSING-MASS SPECTROMETER

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We summarize here the combined data on mass spectrum of charged bosons X^- (isospin 1 or 2) produced in the reaction $\pi^- + p \to p + X^-$ at incident laboratory pion momenta $p_1 = 3$, 3.5, 4.5, 5, 6, 7, and 12 GeV/c, observed by the missing-mass spectrometer. The original data, containing a total of 180 000 processed events, have been published.[1-7] In each event, the instrument measures (1) the mass of X^-, M, with a full-width resolution $\Gamma_{\rm res} = 26 \pm 6$ MeV, nearly independent of M (by adjustment of p_1 and t); (2) the number of charged decay products of X^- which can decay into 1, 3, or 5 charged particles plus possible neutral(s) (denoted by 1c, 3c, and 5c); (3) the four-momentum transfer square, $-t$, in each event with an accuracy of $\Delta t = \pm 0.006$, 0.025, and 0.07, for $t = 0.1$, 0.25, and 0.5 (GeV/c)^2, respectively. We also measure the differential cross section, $d\sigma/dt$, in a band of t whose width can be adjusted between 0.05 and 0.15 (Gev/c)^2. The total cross section obtained from our $d\sigma/dt$ data by integration over t is strongly model-dependent and therefore of limited usefulness.

In order to optimize the mass resolution, each of the observed peaks has, in principle, been investigated in different conditions of background, incident momentum, and momentum transfer; further, in order to avoid possible instrumental biases, each run has been repeated at several different positions of the spectrometer angle and, when possible, at more than one incident pion momentum. The full efficiency of the system extends over a mass bite of 500-1000 MeV, depending on p_1. Thus, to present the full spectrum in the mass range 1900 MeV wide with a smooth background line in one diagram, we have taken only the events above the background and so compiled the data from all the runs. For the authors and the detailed discussion of each peak, we refer the reader to the publications listed in the last column of Table I.

The compiled spectrum is given in Fig. 1, and the most important information on each object seen in Fig. 1 is given in Table I. In addition, we would like to make the following remarks on the δ, the A_1 region, the A_2, and