[17]The absolute rates such as $\Gamma(\tau^+)$ can be replaced by branching ratios $\Gamma(\tau^+)/\Gamma(K^+\to \text{all})$ or $\Gamma(\tau^+)/\Gamma(K^+\to\mu+\nu)$, etc., since CPT invariance ensures $\Gamma(K^+\to\text{all}) = \Gamma(K^-\to\text{all})$, etc. We note that the equality $\Gamma(\tau^+)+\Gamma(\tau'^+)=\Gamma(\tau^-)+\Gamma(\tau'^-)$ is also guaranteed by CPT invariance.

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ISOBAR PRODUCTION IN $p+p\to p+p+\pi^++\pi^-$ AT 6.6 GeV/c*

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At 6.6 GeV/c, the reaction $p+p\to p+\pi^++p+\pi^-$ proceeds dominantly through the $N^{*++}p\pi^-$ channel. When peripheral N^{*++}'s are selected, the $p\pi^-$ angular distribution reproduces the angular distributions of free π^-p scattering in the c.m. energy range from threshold to 2.0 GeV. The diffraction scattering at the upper end of this energy band can account for the 1.4-GeV N^* deduced in recoil-proton spectrum studies.

The spectrum of masses recoiling against a final-state proton in proton-proton collisions has been measured in several recent counter and spark-chamber experiments[1-5] for incident momenta from 2 to 26 GeV/c. The "missing mass" spectra have demonstrated the formation of various baryon isobars. Of particular interest is the enhancement near 1.43 GeV which appears with a full width of ~0.2 GeV. These experiments,[1-5] although performed at a variety of incident momenta and proton recoil angles, all essentially agree on the existence of this enhancement, particularly for low momentum transfers (≤ 0.1 GeV^2) to the "isobar." It has been tempting to associate this enhancement with a P_{11} resonance at approximately the same mass which has been inferred[6,7] from phase-shift analyses of pion-proton elastic scattering. However, the study of the recoil proton spectrum alone in the above experiments cannot rule out the possibility that an observed enhancement is, in fact, due to a kinematic effect.

We have investigated this question in an analysis of 2097 events of the type

$$p+p\to p+p+\pi^++\pi^-, \qquad (1)$$

produced by 6.6-GeV/c incident protons in the Lawrence Radiation Laboratory 72-inch hydrogen bubble chamber. The inelasticity of the P_{11} resonance is apparently large[8]; therefore a detailed analysis of Reaction (1) appears to be a suitable way to investigate the relation of the P_{11} resonance[9] to the 1.43-GeV enhancement reported in Refs. 1-5. Our results lead to the following conclusions: (a) Reaction (1) is dominated by one-pion exchange with pion-nucleon elastic scattering at each vertex. (b) A large enhancement of the $p\pi^+\pi^-$ mass spectrum in the 1.38- to 1.58-GeV range is observed; this enhancement can be understood as a kinematic reflection of the $p\pi^-$ elastic scattering at one vertex.

Thus we find no evidence in our data for the formation of a resonant state in the 1.38- to 1.58-GeV mass range which decays into the $p\pi^+\pi^-$ channel. Furthermore, the enhancement which we do observe can account quantitatively for the low-mass enhancement observed in counter experiments at nearby momenta.[4,5]

The dominance of peripheral $N^{*++}(1238)$ production in Reaction (1) is illustrated in the following manner. Two $p\pi^+$ effective-mass combinations can be formed from the final-state particles of Reaction (1); furthermore, for each mass combination the invariant momentum transfer from either projectile or target proton may be considered. However, for each $p\pi^+$ combination only the smaller of the two possible momentum transfers is plotted on a Chew-Low graph. The clustering of events in the N^{*++} mass range and at low momentum transfer is strikingly apparent. A $p\pi^+$ mass projection of events which have an N^{*++} production angle θ_A in the over-all c.m. system limited by $|\cos\theta_A| > 0.965$ demonstrates that Reaction (1) proceeds dominantly through

$$p + p \to N^{*++} + p + \pi^-. \qquad (2)$$

If the reaction proceeded fully through channel (2), one half of the area in the total projection (without selection on θ_A) would be in the N^{*++} peak, the remainder of the area being due to the "wrong" $p\pi^+$ mass combination for each event. This is found to be approximately the case. The selection $|\cos\theta_A| > 0.965$ greatly reduces the background due to the "wrong" $p\pi^+$ mass combinations and thus tends to isolate channel (2). At the central position of the $N^*(1238)$ the nonisobar background level is $\lesssim 10\%$ of the peak value.

The highly peripheral production of the N^{*++} system invites further analysis of the data in terms of the one-pion exchange process. We proceed as follows: The combination of the π^- and one proton, $p_j\pi^-$, is considered only if the combination of the π^+ and the other proton, $p_i\pi^+$, is within the range $1.16 < M(p_i\pi^+) < 1.28$ GeV, which we define to be N^{*++} band. We confine our study to the $|\cos\theta_A| > 0.965$ $N^{*++}p\pi^-$ events; this sample consists of 560 events (of which 45 have two $p\pi^+$ combinations which satisfy the selection criteria). These events are divided into 12 $p\pi^-$ mass intervals. The quasielastic $p\pi^-$ scattering in each of these intervals is studied separately. The scattering angle is defined as the angle between the two protons at the $p\pi^-$ vertex calculated in the rest frame of the outgoing $p\pi^-$ system. As usual, we express the differential cross section as a series in Legendre polynomials of the scattering angle,

$$\frac{d\sigma}{d\Omega} = \lambda^2 \sum A_l P_l = \frac{\sigma}{4\pi} \sum \frac{A_l}{A_0} P_l. \qquad (3)$$

The shape parameters (A_l/A_0) of the angular distributions are determined by evaluating for each l the average value of the Legendre polynomial P_l over the events in the sample:

$$\frac{A_l}{A_0} = (2l+1)\langle P_l\rangle; \quad \sigma\left(\frac{A_l}{A_0}\right) = \frac{2l+1}{\sqrt{N}} \{\langle P_l^2\rangle - \langle P_l\rangle^2\}^{1/2}, \qquad (4)$$

where N is the total number of events in the $p\pi^-$ mass bin. The (A_l/A_0) parameters obtained in our analysis are shown in Fig. 1; the solid

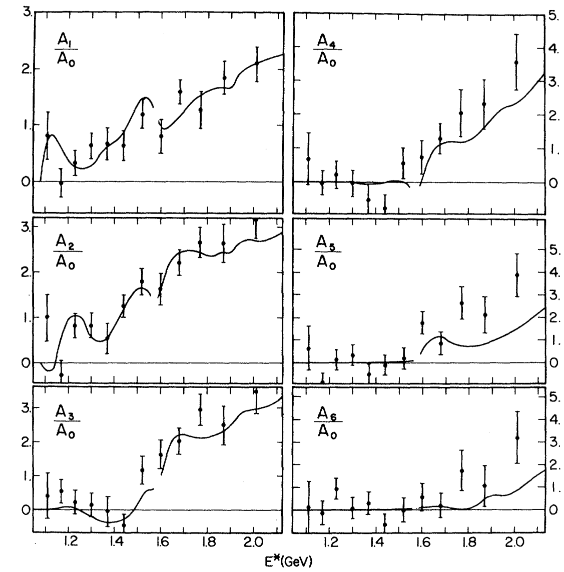

FIG. 1. Shape parameters for the $p_j\pi^-$ vertex of events with $|\cos\theta_A| > 0.965$, provided $p_i\pi^+$ is in the N^* band. Smooth curves are the shape parameters derived from free $p\pi^-$ elastic scattering.

curves which appear in this figure are derived entirely from existing free $p\pi^-$ elastic scattering data; the curves were drawn through points calculated from the 0-700 MeV laboratory pion energy fit of Roper, Wright, and Feld[10]; the higher mass region is based on the results of Duke et al.[11] and Esterling, Hill, and Booth.[12] The agreement between our results and the free $p\pi^-$ elastic-scattering data seems good at least to ~1.8 GeV.[13] Beyond ~1.8 GeV, and especially for large l, our results tend to exceed the values derived from free $p\pi^-$ scattering. Events with $p\pi^-$ mass in excess of 1.8 GeV correspond to a larger momentum transfer, and the departures of the shape parameters may be related to this fact.

The corresponding (A_l/A_0) results for the $p\pi^+$ vertex are shown[14] in Fig. 2. The moments are given for $M(p\pi^+) < 1.55$ GeV only. It is observed that the background-to-signal ratio becomes significant for $1.4 < M(p\pi^+) < 1.16$ GeV. The background (i.e., "wrong" combinations which occur at low $p\pi^+$ mass) is characterized by a pronounced forward quasiscattering angle. The discrepancy between the lowest (A_l/A_0) points and the curve may be attributed to this cause. Otherwise the agreement is generally good.

In view of the well defined and relatively clean N^{*++} peak, it is possible to represent the data

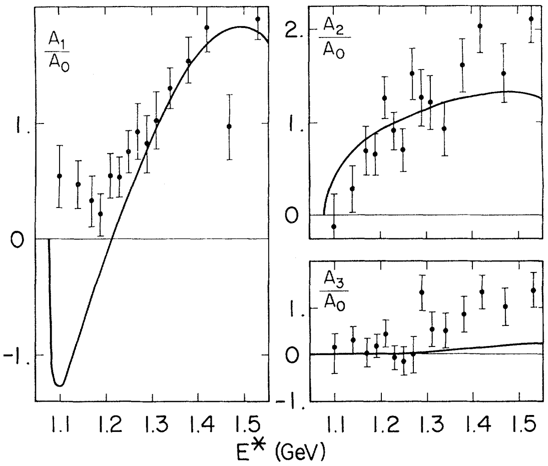

FIG. 2. Shape parameters for the $p_i\pi^+$ vertex for events with $|\cos\theta_A|>0.965$. Smooth curves are the shape parameters derived from free $p\pi^+$ elastic scattering.

as an $N^{*++}p\pi^-$ Dalitz plot. Such Dalitz plots for all the data satisfying the N^* condition $1.16 < M(p\pi^+) < 1.28$ GeV and for the data which also satisfy $|\cos\theta_A| > 0.965$ are shown in Figs. 3(a) and 3(b), respectively. The points tend to cluster along the low $p\pi^-$ and the low $N^*\pi^-$ mass boundaries; this effect is especially strong for the peripheral events. The $N^*\pi^-$ mass spectra for all events and for the $|\cos\theta_A| > 0.965$ events are shown in Fig. 3(c). The peripheral events exhibit a strong enhancement centered at 1.45 GeV. For extremely peripheral events, values of $M^2(N^*\pi^-)$ on lines of constant $M^2(p\pi^-)$ correspond linearly to the $p\pi^-$ c.m. scattering angle cosine (+1 at the left-hand boundary, −1 at the right). Therefore the events which contribute to the low $(N^*\pi^-)$ mass enhancement can be identified with those events which lie in the $p\pi^-$ diffraction-scattering peak. Since the events on the Dalitz plot of Fig. 3(b) are precisely those which gave shape parameters which were in good agreement with those obtained in the free $p\pi^-$ elastic-scattering experiments, it follows that the $N^*\pi^-$ enhancement at 1.45 GeV can be accounted for as due to a kinematic reflection of the diffraction scattering at the $p\pi^-$ vertex. Such kinematic reflections have been considered by Deck and Maor and O'Halloran,[15] in connection with pion-nu-

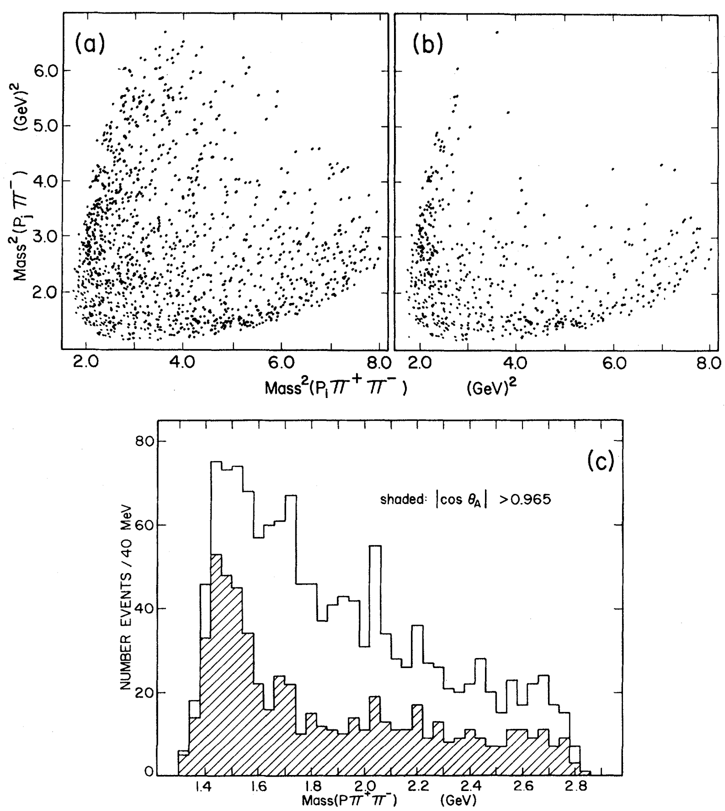

FIG. 3. (a) Dalitz plot of $p_i\pi^+\pi^-$ vs $p_j\pi^-$ for events with the $p_i\pi^+$ mass in the N^* band; 1211 events, 164 plotted twice. (b) Dalitz plot of $p_i\pi^+\pi^-$ vs $p_j\pi^-$ for events with the $p_i\pi^+$ mass in the N^* band and with $|\cos\theta_A|>0.965$; 560 events, 45 plotted twice. (c) Projections of the Dalitz plots of (a) and (b) onto the $M^2(p\pi^+\pi^-)$ axis.

cleon scattering leading to the A_1 enhancement, and have been investigated experimentally[16] by means of techniques similar to ours.

The total cross section for Reaction (1) is 2.6 ± 0.3 mb, a result which is in satisfactory agreement with 2.92 ± 0.16 mb recorded at 5.5 GeV/c incident momentum.[17] The enhancement of the shaded events at 1.45 GeV in Fig. 3(c) then corresponds to a cross section of 0.22 $\pm$ 0.05 mb. A subsidiary study showed that for most of the events in the enhancement, the momentum transfer to the proton recoiling against the $p\pi^+\pi^-$ system is <0.1 GeV^2. To relate these results to the recoil proton measurements we assume $p \to N^* + \pi$ dissociation at one vertex and $p\pi$ elastic scattering (averaged over our $p\pi^-$ mass band) at the other vertex. Under these assumptions, isospin conservation leads to the prediction that of all the events recorded in a system sensitive only to recoil protons, $\sim\frac{1}{2}$ would appear in our $N^{*++}p\pi^-$ channel. The present experiment would therefore predict a 0.44 ± 0.1 mb enhancement for the mass band near 1.45 GeV if only the recoil proton is observed. This prediction may be compared with the result of Anderson et al.[4] who quote a 0.544 $\pm$ 0.090 mb cross section at 10 GeV/c for their enhancement. A rough extrapolation of their result to 6.6 GeV/c leads to $\sim 0.4 \pm 0.1$ mb. Blair et al.[5] report cross sections of 0.65 ± 0.18 and 0.45 ± 0.09 mb at 6.06 and 7.88 GeV/c, respectively. All three values are in good agreement with ours.

On the other hand, if we assume that the enhancement of Anderson et al.[4] is due to the P_{11} resonance and that the inelasticity of this channel[8] is due to the $N2\pi$ decay, either through $N^*\pi$ or through $N(2\pi)$ with the di-pion in the $T = 0$ channel,[9] then the results of Anderson et al.[4] may be used to estimate the enhancement which we should see in the $N^{*++}\pi^-$ channel. This estimate leads to $(0.4 \pm 0.1) \times (0.34 \pm 0.04) \times (\frac{5}{9}$ or $\frac{2}{3}) \simeq 0.09 \pm 0.03$ mb, which does not appear consistent with our result of 0.22 ± 0.05 mb. This is also the case if one uses the cross sections of Blair et al.[5] at nearby momenta.

Lastly, we may speculate on the relationship of our findings to higher momentum "missing mass" experiments,[3,4] assuming the $N^*p\pi^-$ channel is produced similarly at these momenta. For example, at 18 GeV/c the left-hand boundary of the Dalitz plot of Fig. 3(b) intersects $M(N^*\pi) = 1.5$ GeV at $M(p\pi^-) = 3.3$ GeV (compared with 2.1 GeV at 6.6 GeV/c). Consequently, one would expect further peaking in the 1.4- to 1.5-GeV region arising from diffraction scattering in the higher $p\pi^-$ mass interval. This also would occur primarily for momentum transfer to the proton ≤ 0.1 GeV^2.

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[8]At the resonance, $\sigma_{\rm el}/\sigma_{\rm inel} = (1+\eta)^2/(1-\eta^2)$, where η is the absorption parameter. From the graphs of Roper (Ref. 6) and Bareyre et al. (Ref. 7) we take $\eta = 0.25$ and 0.40, respectively, giving $\sigma_{\rm inel}/\sigma_{\rm tot} = 0.37$ and 0.30, respectively.

[9]R. H. Dalitz and R. G. Moorhouse [Phys. Letters 14, 159 (1965)] have pointed out that the phase shift analyses do not necessarily lead one to the conclusion that a P_{11} resonance exists in this mass region, and that the effect may be the result of the known large $I = 0$ π-π interaction at low energy.

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[13]We have also examined the Y_l^m moments for $m \neq 0$. Using the standard t-channel coordinates ($\hat{y} = \hat{n} \propto \hat{p}_{\rm incident} \times \hat{N}^*$) the $\langle {\rm Im} Y_l^m \rangle$ are all zero within statistics as required by conservation of parity. For $p\pi^+$, $\langle {\rm Re} Y_1^1 \rangle \sim -0.025$; for $p\pi^-$, $\langle {\rm Re} Y_3^1 \rangle \sim -0.05$ in the entire E^* range, and $\langle {\rm Re} Y_2^1 \rangle \sim -0.06$ above $E^* \sim 1.3$ GeV. These moments should vanish for the unadorned one-pion-exchange model. We defer a detailed consideration of these moments to a forthcoming high-statistics study.

[14]The (A_l/A_0) moments for $l=4,5,6$ have also been evaluated and are consistent with zero throughout this mass range, in agreement with $p\pi^+$ scattering data.

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CALCULATION OF THE BRANCHING RATIO $(\eta\to 2\pi+\gamma)/(\eta\to 2\gamma)$*

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Considerable interest attaches to the problem of the relative frequencies of the decay modes of the η meson. Since G-conjugation invariance requires the intercession of at least one photon (real or virtual), the observed decay modes turn out to be

$$\eta\to 3\pi\ (46.4\%);\ 2\gamma\ (33.5\%);$$

$$\pi^0+\gamma+\gamma\ (14.9\%);\ \pi^++\pi^-+\gamma\ (5.1\%).$$

Curiously enough, the first three decays, which are of order α^2 (α is the fine structure constant), dominate over the fourth, which is of order α. Previous attempts to understand this situation, even allowing for phase-space factors, have led to the introduction of a new quantum number.[1] The recent elegant extension by Weinberg[2] of the current algebra method to processes involving the emission of two "soft" pions makes it possible to understand the branching ratio $(\eta\to 2\pi+\gamma)/(\eta\to 2\gamma)$ on a dynamical basis and without the need of new quantum numbers.

The calculation proceeds as follows. Using the notation of Ref. 2 wherever possible, we expand the time-ordered product

$$T\{\partial_\mu A_a^{\ \mu}(x),\partial_\nu A_b^{\ \nu}(y),V_d^{\ \lambda}(0)\},\tag{1}$$

where $A_a^{\ \mu}$ is the $\Delta S=0$ axial-vector current and $V_d^{\ \lambda}$ is the $\Delta I=1$ vector current. In this time-ordered product, $V_d^{\ \lambda}(0)$ has replaced the $\Delta I=\frac{1}{2},\Delta S=-1$ axial-vector current $A_\eta^{\ \lambda}(0)$ considered in Ref. 2. The expansion of (1) leads to seven terms, similar to Eq. (4) of Ref. 2, of which the first three are disregarded for the same reasons as in Ref. 2. The four terms which remain are

$$-\frac{1}{2}\left(\frac{\partial}{\partial y^\nu}-\frac{\partial}{\partial x^\nu}\right)\delta(x^0-y^0)T\{[A_a^{\ 0}(x),A_b^{\ \nu}(y)],V_d^{\ \lambda}(0)\}$$
$$-\tfrac{1}{2}\delta(x^0)\delta(y^0)\{[A_b^{\ 0}(y),[A_a^{\ 0}(x),V_d^{\ \lambda}(0)]]+[A_a^{\ 0}(x),[A_b^{\ 0}(y),V_d^{\ \lambda}(0)]]\}$$
$$-\delta(y^0)T\{[A_b^{\ 0}(y),V_d^{\ \lambda}(0)],\partial_\mu A_a^{\ \mu}(x)\}-\delta(x^0)T\{[A_a^{\ 0}(x),V_d^{\ \lambda}(0)],\partial_\nu A_b^{\ \nu}(y)\}.\tag{2}$$

In general, all four terms in (2) must be considered. But if we specialize to the matrix element $\langle 0|T\{\partial_\mu A_a^{\ \mu}(x),\partial_\nu A_b^{\ \nu}(y),V_d^{\ \lambda}(0)\}|P\rangle$ (where P is a pseudoscalar meson), then the last three terms in (2) do not contribute: The second term in (2) vanishes because the (double) commutators yield vector currents which are taken between $|0\rangle$ and $|P\rangle$, while the third and fourth terms vanish because the single commutators give rise to axial-vector currents taken between $|\pi\rangle$ and $|P\rangle$. We are left with only the first term in (2):

$$-i\epsilon_{abc}\left(\frac{\partial}{\partial y^\nu}-\frac{\partial}{\partial x^\nu}\right)\langle 0|T\{V_c^{\ \nu}(x),V_d^{\ \lambda}(0)\}|P\rangle\delta^4(x-y).\tag{3}$$

Let us choose $P=\eta$ and define

$$\langle\pi_{qa}\pi_{pb}|V_d^{\ \lambda}(0)|\eta_k\rangle=i(2\pi)^{-9/2}(8q^0p^0k^0)^{-1/2}(e/m_\eta^{\ 3})F_\eta\epsilon_{\lambda\alpha\beta\gamma}k^\alpha q^\beta p^\gamma,\tag{4}$$