

0.8 and 1.4 and between 2.0 and 2.6 MeV. These structures can be due to several things, such as particle-vibration coupling, or noncollective (2p-1h) states. In order to distinguish among the various possibilities, it is essential to measure the spins of the resonances in proton scattering and also look at the (d, p) experiments to the corresponding regions in Sn¹¹⁹. To separate noncollective (2p-1h) strength, it is necessary to measure inelastic scattering to noncollective particle-hole states in Sn¹¹⁸.

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PHENOMENOLOGICAL ANALYSIS OF CP NONINVARIANCE IN $K^\pm \rightarrow 3\pi$ DECAY*

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We analyze the effect of CP noninvariance in the decay $K^\pm \rightarrow 3\pi$ under the assumption that the violation of CP invariance, first observed by Christenson et al.,¹ occurs in the weak interactions.² It has been pointed out^{3,4} that the violation of CP invariance may produce a difference between the Dalitz plots and partial rates for $K^+ \rightarrow 3\pi$ and its CPT conjugate state; if CPT invariance is valid, these effects also require the existence of strong final-state interaction between the pions.⁵ The following

analysis differs from a previous calculation by Ueda and Okubo³ in that (a) we use a different parametrization of the CP nonconserving effect, and (b) a pure $I=1$ final state is not assumed. We shall examine in some detail the possibility of testing CP noninvariance with $\Delta I > \frac{1}{2}$ as suggested earlier.^{6,7}

We analyze the τ and τ' decays in terms of the four independent isospin states of three pions which can contribute to a matrix element linear in the energy of the odd pion. Assuming CPT invariance, we write

$$M(\tau^\pm) = 2\lambda \exp(i\delta_\lambda \pm i\varphi_\lambda) + b \exp(i\delta_b \pm i\varphi_b)(s_3 - s_0)/\mu^2 + c \exp(i\delta_c \pm i\varphi_c)(s_3 - s_0)/\mu^2 + d \exp(i\delta_d \pm i\varphi_d), \quad (1)$$

$$M(\tau'^\pm) = -\lambda \exp(i\delta'_\lambda \pm i\varphi'_\lambda) + b \exp(i\delta'_b \pm i\varphi'_b)(s_3 - s_0)/\mu^2 - c \exp(i\delta_c \pm i\varphi_c)(s_3 - s_0)/\mu^2 + 2d \exp(i\delta_d \pm i\varphi_d), \quad (2)$$

where $s_i = (P_K - P_i)^2 = (m - \mu)^2 - 2mT_i$, $i = 1, 2, 3$, and the index 3 refers to the odd pion; the symmetry point is $s_0 = \frac{1}{3}(s_1 + s_2 + s_3)$, T_i is the kinetic energy of the i th pion in the rest frame of the K meson, and m and μ are, respectively,

the K and π masses. The δ are the phase shifts due to strong (CP -conserving) pion-pion interaction, the φ are the CP -nonconserving phases, assumed for simplicity to be independent of the

s_i , and the magnitudes λ , b , c , and d contribute to $I=1$, 1, 2, and 3 states, respectively.⁸ We see immediately that in the absence of strong final-state interactions, i.e., if all the δ are zero (or equal), there cannot be any manifestation of CP violation since the matrix elements for K^+ and K^- decay are simply complex conjugates (within an over-all phase) of each other.⁹

In general, because of the final-state interaction, the energy dependence of τ and τ' decay can have a form much more complicated than that used above. The justification for using the linear energy dependence lies in its simplicity and in its good fit to the experimental data.¹⁰ Since the $\Delta I = \frac{1}{2}$ rule is fairly well satisfied, c and d must be small compared with b and λ , respectively. The c term must arise from intrinsic structure, but the b term which gives rise to most of the dependence on the energy of the odd pion might be due entirely to the final-state interaction of the pions; in that case there would be no justification for putting φ_b different from φ_λ since the strong interaction is CP conserving to a good accuracy. However, if one assumes that the S -wave $\pi\pi$ interaction in the $I=0$ state is attractive and larger than in the $I=2$ state, then intrinsic structure for the b term must be assumed in order to give the correct slope.¹¹ If such an intrinsic structure is due for example to the $K\rho\pi$ coupling, then the introduction of a CP -nonconserving phase $\varphi_b \neq \varphi_\lambda$ is justified.

The phase shift δ can in principle be calculated from the two-body $\pi\pi$ interaction by, for

example, using the unsubtracted dispersion relation of Khuri and Treiman or Sawyer and Wali.¹² We shall include only the S -wave $I=0$ $\pi\pi$ scattering, neglecting the smaller S -wave $I=2$ and low-energy P -wave interactions, so only δ_λ and δ_λ' are appreciable. To estimate the effect of final-state interactions for the total $I=1$ state, we write¹³

$$M_1(\tau^+) = \lambda \left[\frac{5}{3} \frac{1}{D(s_1)} - \frac{2}{3} \right] + \lambda \left[\frac{5}{3} \frac{1}{D(s_2)} - \frac{2}{3} \right],$$

$$M_1(\tau'^+) = -\lambda \left[\frac{5}{3} \frac{1}{D(s_3)} - \frac{2}{3} \right],$$

where

$$\frac{1}{D(s)} = \exp\left(\frac{s-4\mu^2}{\pi}\right) \int_{4\mu^2}^{\infty} \frac{\delta_0(s') ds'}{\mu^2 (s'-4\mu^2)(s'-s-i\epsilon)} \approx \frac{1}{1-ika}$$

in the scattering length approximation for the S -wave $I=0$ $\pi\pi$ interaction. An estimate of the average δ_λ is obtained by taking

$$\begin{aligned} \tan \delta_\lambda' &\approx \tan \delta_\lambda = \left. \frac{\text{Im} M_1(\tau^+)}{\text{Re} M_1(\tau^+)} \right|_{s_1=s_2=s_3=s_0} \\ &\approx (5/3)k_0 a_0, \text{ where } k_0 = \left(\frac{s_0}{4} - \mu^2\right)^{1/2} \approx 0.6\mu, \\ &\approx a_0\mu. \end{aligned}$$

Since the strength of the S -wave $\pi\pi$ interaction is not clear,¹⁴ we shall assume that $a_0\mu \approx \frac{1}{2}$ which yields $\tan \delta_\lambda \approx \sin \delta_\lambda \approx \frac{1}{2}$.

In order to use all available experimental data on $K \rightarrow 3\pi$ decays, we also write the matrix elements for $K_2^0 \rightarrow 3\pi$ in the same approximation for the final-state interaction,

$$M(K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0) = \tilde{\lambda} \exp(i\delta_\lambda) \cos \tilde{\varphi}_\lambda - \tilde{b} \cos \varphi_b (s_3 - s_0)/\mu^2 + (i\tilde{c}/\sqrt{3}) \sin \tilde{\varphi}_c (s_2 - s_1)/\mu^2 + \tilde{d} \cos \tilde{\varphi}_d, \quad (3)$$

$$M(K_2^0 \rightarrow 3\pi^0) = -3\tilde{\lambda} \exp(i\delta_\lambda) \cos \tilde{\varphi}_\lambda + 2\tilde{d} \cos \tilde{\varphi}_d, \quad (4)$$

(index 3 labels the π^0). We discuss below the extent to which the parameters $\tilde{\lambda} \cdots \tilde{d}$ can be related to $\lambda \cdots d$.

The decay rates and spectra are¹⁵ as follows:

$$\gamma(\tau^\pm) = \frac{4\lambda^2}{2!} \left[1 + \frac{d}{\lambda} \cos(\delta_\lambda \pm \varphi_\lambda \delta) + \frac{(1.22)^2}{16} \frac{b^2}{\lambda^2} \right], \quad (5a)$$

$$\gamma(\tau'^\pm) = \frac{\lambda^2}{2!} \left[1 - \frac{4d}{\lambda} \cos(\delta_\lambda \pm \varphi_\lambda \delta) + \frac{(1.35)^2}{4} \frac{b^2}{\lambda^2} \right], \quad (5b)$$

$$\gamma(K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0) = \tilde{\lambda}^2 \cos^2 \tilde{\varphi}_\lambda \left[1 + \frac{2\tilde{d} \cos \tilde{\varphi}_d}{\tilde{\lambda} \cos \tilde{\varphi}_\lambda} \cos \delta_\lambda + \frac{(1.39)^2}{4} \frac{\tilde{b}^2 \cos^2 \varphi_b}{\tilde{\lambda}^2 \cos^2 \tilde{\varphi}_\lambda} \right], \quad (5c)$$

$$\gamma(K_2^0 \rightarrow 3\pi^0) = \frac{9\tilde{\lambda}^2 \cos^2 \tilde{\varphi}_\lambda}{3!} \left[1 - \frac{4}{3} \frac{\tilde{d} \cos \tilde{\varphi}_d}{\tilde{\lambda} \cos \tilde{\varphi}_\lambda} \cos \delta_\lambda \right]; \quad (5d)$$

$$a(\tau^\pm) = -\frac{1}{2} \frac{b}{\lambda} \cos(\delta_\lambda \pm \varphi_{\lambda b}) - \frac{1}{2} \frac{c}{\lambda} \cos(\delta_\lambda \pm \varphi_{\lambda c}), \quad (6a)$$

$$a(\tau'^\pm) = \frac{b}{\lambda} \cos(\delta_\lambda \pm \varphi_{\lambda b}) - \frac{c}{\lambda} \cos(\delta_\lambda \pm \varphi_{\lambda c}), \quad (6b)$$

$$a(K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0) = \frac{\bar{b} \cos \bar{\varphi}_b}{\bar{\lambda} \cos \bar{\varphi}_\lambda} \cos \delta_\lambda, \quad (6c)$$

where the γ 's are the rates divided by phase space (with the statistical factor for like pions included in γ), and the slopes a are defined¹⁶ by $|M|^2 = 1 + 2a\alpha\gamma$. We have neglected terms of order bc , c^2 , or d^2 compared with λ^2 . The detection of any of the following quantities would demonstrate violation of CP invariance: (i) Differences in rates or branching ratios¹⁷:

$$\Delta\Gamma(\tau) = \frac{\Gamma(\tau^+) - \Gamma(\tau^-)}{\Gamma(\tau^+)} = -2 \frac{d}{\lambda} \sin \delta_\lambda \sin \varphi_{\lambda d}, \quad (7a)$$

$$\Delta\Gamma(\tau') = \frac{\Gamma(\tau'^+) - \Gamma(\tau'^-)}{\Gamma(\tau'^+)} = -4\Delta\Gamma(\tau), \quad (7b)$$

$$\Delta R(\tau, \tau') = \frac{\gamma(\tau^+)}{4\gamma(\tau'^+)} - \frac{\gamma(\tau^-)}{4\gamma(\tau'^-)} = -10 \frac{d}{\lambda} \sin \delta_\lambda \sin \varphi_{\lambda d}. \quad (7c)$$

(ii) Differences in spectra:

$$\Delta a(\tau) = \frac{a(\tau^+) - a(\tau^-)}{a(\tau^+)} = \frac{-2[b \sin \varphi_{\lambda b} + c \sin \varphi_{\lambda c}] \sin \delta_\lambda}{b \cos(\delta_\lambda + \varphi_{\lambda b}) + c \cos(\delta_\lambda + \varphi_{\lambda c})}, \quad (8a)$$

$$\Delta a(\tau') = \frac{a(\tau'^+) - a(\tau'^-)}{a(\tau'^+)} = \frac{-2[b \sin \varphi_{\lambda b} - c \sin \varphi_{\lambda c}] \sin \delta_\lambda}{b \cos(\delta_\lambda + \varphi_{\lambda b}) - c \cos(\delta_\lambda + \varphi_{\lambda c})}, \quad (8b)$$

$$\Delta N(\tau) = \left(\frac{N_u - N_l}{N} \right)_{\tau^+} - \left(\frac{N_u - N_l}{N} \right)_{\tau^-} = \frac{8\alpha}{3\pi} \left[\frac{b}{\lambda} \sin \varphi_{\lambda b} + \frac{c}{\lambda} \sin \varphi_{\lambda c} \right] \sin \delta_\lambda, \quad (8c)$$

$$\Delta N(\tau') = \left(\frac{N_u - N_l}{N} \right)_{\tau'^+} - \left(\frac{N_u - N_l}{N} \right)_{\tau'^-} = -\frac{16\alpha}{3\pi} \left[\frac{b}{\lambda} \sin \varphi_{\lambda b} - \frac{c}{\lambda} \sin \varphi_{\lambda c} \right] \sin \delta_\lambda, \quad (8d)$$

where N_u and N_l denote the number of events in the upper and lower halves, respectively, of the Dalitz plot and $N = N_u + N_l$. (iii) Asymmetry in $K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0$:

$$\begin{aligned} \Delta N(K_2^0) &= [N_\gamma(T_+ > T_-) - N_l(T_+ < T_-)]/N, \\ &= \frac{8\alpha \bar{c} \sin \bar{\varphi}_c}{3\pi \bar{\lambda} \cos \bar{\varphi}_\lambda} \sin \delta_\lambda, \end{aligned} \quad (9)$$

where N_γ and N_l denote the number of events in the right and left halves, respectively, of the Dalitz plot, and $N = N_\gamma + N_l$.

We now note the restrictions placed on the parameters involved in our description of the decay $K \rightarrow 3\pi$ by the present experimental information. The observed slopes¹⁰ $a(\tau^+) = 0.107$

± 0.01 , $a(\tau'^+) = -0.25 \pm 0.02$, and¹⁸ $a(\tau^-) = 0.094 \pm 0.016$ require that

$$(b/\lambda) \cos(\delta_\lambda + \varphi_{\lambda b}) = -0.23 \pm 0.02, \quad (10a)$$

$$(c/\lambda) \cos(\delta_\lambda + \varphi_{\lambda c}) = 0.02 \pm 0.02, \quad (10b)$$

$$\Delta a(\tau) = 0.13 \pm 0.18. \quad (10c)$$

All other data on $K \rightarrow 3\pi$ are consistent with the $\Delta I = \frac{1}{2}$ rule to within about 10%.

We consider the various possibilities of testing CP noninvariance in the following cases:

(1) The $\Delta I = \frac{1}{2}$ rule is exact ($\bar{\lambda} = \lambda$, $\bar{\varphi}_\lambda \approx \varphi_\lambda$, $\bar{b} = b$, $\bar{\varphi}_b = \varphi_b$, $c = \bar{c} = d = \bar{d} = 0$).—In this case there will be no differences between the K^+ and K^- partial decay rates,³ but differences in the slopes

of the odd-pion spectra may appear if $\varphi_b \neq \varphi_\lambda$. We have

$$\Delta a(\tau) = \Delta a(\tau') = \frac{-2 \sin \varphi_{\lambda b} \sin \delta_\lambda}{\cos(\delta_\lambda + \varphi_{\lambda b})}, \quad (11)$$

and the experimental limit (10c) requires $\tan(\varphi_\lambda - \varphi_b) \approx 0.13 \pm 0.18$; then $\Delta N(\tau') = 0.03 \pm 0.04$. The ratio $\gamma(K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0)/2\gamma(\tau'^+)$ $= 0.88 \pm 0.07$ sets a separate limit on φ_λ alone¹⁹:

$$|\varphi_\lambda| \lesssim 0.4.$$

(2) The $\Delta I = \frac{1}{2}$ rule is not exact.—The best evidence for this is the decay of $K^+ \rightarrow \pi^+ + \pi^0$, and small admixtures of $I=2$ and $I=3$ states in $K^+ \rightarrow 3\pi$ (say $c/\lambda \approx d/\lambda \approx 2.5\%$) with maximal CP -nonconserving phases $\varphi_{\lambda c} \approx \varphi_{\lambda d} \approx \pi/2$, are not incompatible with the present data. If for simplicity we here take $\varphi_b = \varphi_\lambda = 0$, i.e., ascribe all the CP nonconservation to the breaking of the $\Delta I = \frac{1}{2}$ rule, we have from (8a) and (8b)

$$\Delta a(\tau) = -\Delta a(\tau') \approx -5(c/\lambda) \sin \varphi_c, \quad (12)$$

so that a difference in slopes of up to 10% would not be unreasonable. Then the integrated difference,

$$\Delta N(\tau) = \frac{1}{2} \Delta N(\tau') \approx -\frac{4}{3\pi} \frac{c}{\lambda} \sin \varphi_c,$$

could be about 1%. A corresponding estimate²⁰ for the parameters $\tilde{c}/\tilde{\lambda}$ and φ_c in the decay $K_2^0 \rightarrow \pi^+ + \pi^- + \pi^0$ would give the $\pi^+\pi^-$ asymmetry $\Delta N(K_2^0) \sim 1\%$. We note from Eqs. (11) and (12) that CP nonconservation in the state with $I=2$ can be distinguished from that in $I=1$ (case 1 above) by a comparison of the asymmetries between the slopes τ^\pm and τ'^\pm .

Differences in the K^+ and K^- partial rates may also be detectable if some $I=3$ amplitude is present; we have

$$\Delta \Gamma(\tau) \approx (d/\lambda) \sin \varphi_d \sim \pm 2.5\%,$$

$$\Delta \Gamma(\tau') \approx -4 \Delta \Gamma(\tau) \sim \mp 10\%,$$

and the difference in the $\tau/4\tau'$ branching ratio, $\Delta R \sim 12.5\%$.

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⁸For an analysis of isospin in $K \rightarrow 3\pi$, see G. Barton, C. Kacser, and S. P. Rosen, Phys. Rev. **130**, 793 (1963).

⁹This result is clearly independent of the particular form for the matrix elements that we have assumed; in general, we would write $\langle 3\pi | H_w | K^+ \rangle = \sum_I A_I \exp(i\varphi_I) \times \exp(i\delta_I)$ where the A_I , δ_I , and φ_I can all be functions of the s_i ; then CPT invariance requires $\langle 3\pi | H_w | K^- \rangle = \sum_I A_I \exp(-i\varphi_I) \exp(i\delta_I)$. This result was noted earlier in Ref. 4.

¹⁰For all data on K^+ decay we refer to the compilation by G. H. Trilling, Argonne National Laboratory Report No. ANL-7130, 1965 (unpublished); or University of California Radiation Laboratory Report No. UCRL-16473, 1965 (unpublished). The value for $a(\tau^+)$ is from the paper by Fung *et al.* presented at the Thirteenth International Conference on High Energy Physics, 1966 (unpublished).

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¹³This formula can be derived by defining $G(s) = \tilde{A}(s) + \frac{1}{3}\tilde{B}(s) + \frac{1}{3}\tilde{C}(s)$ (using the notation of the first paper in Ref. 12). It is found that $G(s)$ satisfies an Omnes-type integral equation, when the S -wave $\pi\pi$ scattering in $I=2$ and the multiple scattering are neglected.

¹⁴Much smaller $\pi\pi$ scattering lengths ($a_0 \approx 0.2\mu^{-1}$) have been suggested recently by S. Weinberg, to be published. However, larger estimates are deduced from experiments by L. W. Jones *et al.*, Phys. Letters **21**, 590 (1966).

¹⁵We write $\varphi_{\lambda b} = \varphi_\lambda - \varphi_b$, etc., ...

¹⁶In terms of the nonrelativistic Dalitz-plot variables $y = (2T_3/T_{\max} - 1)$, $x = 2(T_1 - T_2)/\sqrt{3}T_{\max}$, we have $(s_3 - s_0)/\mu^2 = m(T_{\max} - 2T_3)/\mu^2 = -\alpha y$ with $\alpha = mT_{\max}/\mu^2 = 1.22$ for τ , 1.35 for τ' , and 1.39 for K_2^0 decays.

¹⁷The absolute rates such as $\Gamma(\tau^+)$ can be replaced by branching ratios $\Gamma(\tau^+)/\Gamma(K^+\rightarrow\text{all})$ or $\Gamma(\tau^+)/\Gamma(K^+\rightarrow\mu+\nu)$, etc., since *CPT* invariance ensures $\Gamma(K^+\rightarrow\text{all}) = \Gamma(K^-\rightarrow\text{all})$, etc. We note that the equality $\Gamma(\tau^+) + \Gamma(\tau'^+) = \Gamma(\tau^-) + \Gamma(\tau'^-)$ is also guaranteed by *CPT* invariance.

¹⁸L. T. Smith, D. J. Prowse, and D. H. Stork, Phys. Letters 2, 204 (1962).

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²⁰We use the same estimate for $\bar{c}/\bar{\lambda}$ as in Ref. 7. The asymmetry is somewhat smaller than estimated in that paper because a smaller scattering length is used.

ISOBAR PRODUCTION IN $p+p \rightarrow p+p+\pi^++\pi^-$ AT 6.6 GeV/c*

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At 6.6 GeV/c, the reaction $p+p \rightarrow p+p+\pi^++\pi^-$ proceeds dominantly through the $N^{*++}p\pi^-$ channel. When peripheral N^{*++} 's are selected, the $p\pi^-$ angular distribution reproduces the angular distributions of free π^-p scattering in the c.m. energy range from threshold to 2.0 GeV. The diffraction scattering at the upper end of this energy band can account for the 1.4-GeV N^* deduced in recoil-proton spectrum studies.

The spectrum of masses recoiling against a final-state proton in proton-proton collisions has been measured in several recent counter and spark-chamber experiments¹⁻⁵ for incident momenta from 2 to 26 GeV/c. The "missing mass" spectra have demonstrated the formation of various baryon isobars. Of particular interest is the enhancement near 1.43 GeV which appears with a full width of ~ 0.2 GeV. These experiments,¹⁻⁵ although performed at a variety of incident momenta and proton recoil angles, all essentially agree on the existence of this enhancement, particularly for low momentum transfers (≤ 0.1 GeV²) to the "isobar." It has been tempting to associate this enhancement with a P_{11} resonance at approximately the same mass which has been inferred^{6,7} from phase-shift analyses of pion-proton elastic scattering. However, the study of the recoil proton spectrum alone in the above experiments cannot rule out the possibility that an observed enhancement is, in fact, due to a kinematic effect.

We have investigated this question in an analysis of 2097 events of the type

$$p+p \rightarrow p+p+\pi^++\pi^-, \quad (1)$$

produced by 6.6-GeV/c incident protons in the Lawrence Radiation Laboratory 72-inch hydrogen bubble chamber. The inelasticity of the P_{11} resonance is apparently large⁸; therefore a detailed analysis of Reaction (1) appears to

be a suitable way to investigate the relation of the P_{11} resonance⁹ to the 1.43-GeV enhancement reported in Refs. 1-5. Our results lead to the following conclusions: (a) Reaction (1) is dominated by one-pion exchange with pion-nucleon elastic scattering at each vertex. (b) A large enhancement of the $p\pi^+\pi^-$ mass spectrum in the 1.38- to 1.58-GeV range is observed; this enhancement can be understood as a kinematic reflection of the $p\pi^-$ elastic scattering at one vertex.

Thus we find no evidence in our data for the formation of a resonant state in the 1.38- to 1.58-GeV mass range which decays into the $p\pi^+\pi^-$ channel. Furthermore, the enhancement which we do observe can account quantitatively for the low-mass enhancement observed in counter experiments at nearby momenta.^{4,5}

The dominance of peripheral $N^{*++}(1238)$ production in Reaction (1) is illustrated in the following manner. Two $p\pi^+$ effective-mass combinations can be formed from the final-state particles of Reaction (1); furthermore, for each mass combination the invariant momentum transfer from either projectile or target proton may be considered. However, for each $p\pi^+$ combination only the smaller of the two possible momentum transfers is plotted on a Chew-Low graph. The clustering of events in the N^{*++} mass range and at low momentum transfer is strikingly apparent. A $p\pi^+$ mass