$Y_1^*(1660)$ is usually assigned to the $\frac{3}{2}^-$ $\gamma$ octet of baryons. Then from Table II the $\frac{3}{2}^-$ $\gamma$ octet has $\frac{1}{2}<\alpha<1$ if $Y_1^*(2030)$ is assigned to $\{10\}$.

Regardless of the $Y_1^*(2030)$ assignment, one can still state that $\alpha$ is different for the $\frac{3}{2}^-$ $\gamma$ baryon octet and the proposed $\frac{5}{2}^+$ baryon octet. For one octet $\alpha$ lies in the range $\frac{1}{2}<\alpha<1$, and for the other $\alpha<\frac{1}{2}$ or $>1$. Cutkosky has discussed the conditions under which $\alpha$ might be the same for different baryon octets.[10]

We emphasize that most of the above conclusions are based on the assumption that $Y_1^*(2030)$ belongs to a $\{10\}$ representation. Ideally one should measure the $Y_1^*$ phases relative to $Y_1^*(1385)$, which is firmly established as a number of the $\frac{3}{2}^+$ baryon $\delta$ $\{10\}$.

We have used the experimental data from Ref. 1 primarily to illustrate that a measurement of the relative phase of resonant amplitudes in a two-body inelastic reaction can be used to make SU(3) assignments. This method is applicable to the higher spin resonance formed in $\pi$-$N$ and $K$-$N$ scattering, and may prove to be more reliable than assignments made on the basis of measured partial decay widths. The SU(3) predictions of the relative signs of coupling constants involve only one parameter, $\alpha$, and this only for octets. On the other hand, SU(3) calculations of partial widths depend upon $g_\mu$ and kinematical factors, as well as on $\alpha$. Inexactness of SU(3) symmetry may cause a splitting in $g_\mu$, giving rise to discrepancies between calculated and experimental partial decay rates. For example, a calculation of $\Gamma_{\Xi\pi}$ for $\Xi_{1/2}^*(1530)$ (a member of the $\delta$ decuplet), using as input the current values of $\Gamma_{\Lambda\pi}$ and $\Gamma_{\Sigma\pi}$ for $Y_1^*(1385)$ and $\Gamma_{N\pi}$ for $N_{3/2}^*(1236)$,[11] predicted $\Gamma_{\Xi\pi}=16$ MeV, compared to the measured value of $7.5\pm1.7$ MeV.

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# REGGE-POLE ANALYSIS OF $\pi p$ CHARGE-EXCHANGE POLARIZATION*

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The $\pi^-p$ charge-exchange reaction, $\pi^-+p\to\pi^0+n$, at high energy and low momentum transfer provides an excellent test of the Regge-pole hypothesis. Only the $\rho$ meson may be exchanged in the crossed channel, $t$, because only the $\rho$ has $I=1$, $G=+1$, and $P=(-1)^J$. The differential cross section $d\sigma/dt$ has been measured[1-3] in the energy range 6-18 GeV. Several analyses[4-6] have shown the consistency of these data with a single $\rho$ Regge-pole exchange. The single-Regge-pole model predicts zero polarization because the spin-nonflip and the spin-flip amplitudes have the same phase. A recent measurement[7] at 6 GeV and low momentum transfer shows a nonzero polarization in apparent contradiction to the Regge-pole hypothesis.

We wish to demonstrate in this Letter that this polarization may be explained in terms of the interference of the Regge-pole amplitude with the lower lying resonance in the direct channel, $s$. Predictions of $P$ for various values of $s$ and $t$ are made.

The charge-exchange scattering amplitude is given by

$$m = f + i\frac{\sigma\cdot\vec{q}'\times\vec{q}}{q^2}\tilde{f}, \tag{1}$$

where $f$ and $\tilde{f}$ are the spin-nonflip and spin-flip amplitudes and $\vec{q}$ and $\vec{q}'$ are the center-of-mass momenta of the initial and final states. The differential cross section $d\sigma/dt$ and the polarization are related to $f$ and $\tilde{f}$ by

$$\frac{d\sigma}{dt} = \frac{\pi}{q^2}\{|f|^2 + \sin^2\theta\,|\tilde{f}|^2\}, \tag{2}$$

and

$$P = -\frac{2\,\mathrm{Im}(f\tilde{f}^*)\sin\theta}{|f|^2 + \sin^2\theta\,|\tilde{f}|}, \tag{3}$$

where $\theta$ is the scattering angle in the $s$ channel.

We shall assume that

$$f = f_{\mathrm{Reg}} + f_{\mathrm{res}} \tag{4}$$

and

$$\tilde{f} = \tilde{f}_{\mathrm{Reg}} + \tilde{f}_{\mathrm{res}}, \tag{5}$$

where $f_{\mathrm{Reg}}$ and $\tilde{f}_{\mathrm{Reg}}$ are the contributions to the scattering amplitudes arising from the exchange of a single $\rho$ Regge pole in the crossed channel and $f_{\mathrm{res}}$ and $\tilde{f}_{\mathrm{res}}$ are the contributions from the exchange of $\pi N$ resonances in the direct channel.

We shall consider contributions to $f_{\mathrm{res}}$ ($\tilde{f}_{\mathrm{res}}$) from all the known $\pi N$ resonances. The spin and parity assignments for the lower energy resonances are well known, whereas assignments for the higher energy resonance only recently observed[8] have not yet been established. Theoretical assignments have been made, however, on the basis of a Regge recurrence scheme.[9,10] These assignments were tested in two separate analyses of $\pi^-p$ elastic scattering at 180° and $\pi^-p$ charge-exchange scattering at 0°, in which the resonances in the direct channel interfere with the Regge-pole ($\Delta_{33}$ for backward elastic and $\rho$ for forward charge exchange) exchange in the crossed channel. In view of the success of these two analyses, we shall make use of these assignments. The list of resonances, their spin, parity, width, position, and elasticity are all given in Table I. This list is identical to the one in Ref. 9, as we have made use of their improved determination of the widths and elasticities of the higher energy states. We have not included $N_\gamma(3350)$, however, because its status is still uncertain. It does not really matter very much whether or not this state is included because it makes such a small contribution to the polarization because of its exceedingly small elasticity. The largest contributions to $P$ actually do not come from the very highest lying states but from the resonances in the middle range. This is because these latter resonances have much larger elasticities than their higher energy counterparts.

The charge-exchange amplitude is related to the $I=\frac{3}{2}$ and $I=\frac{1}{2}$ $\pi p$ scattering amplitudes by $m_{\mathrm{cex}} = \sqrt{2}/3(m_{3/2} - m_{1/2})$ so that

$$f_{\mathrm{res}} = \frac{\sqrt{2}}{3q}\sum_{\mathrm{resonances}}(-1)^{I+\frac{1}{2}}\frac{(J+\frac{1}{2})\eta_l P_l(1+t/2q^2)}{W_l - W - i\Gamma_l/2}, \tag{6}$$

where $W=\sqrt{s}$, and $J$, $l$, $W_l$, $\Gamma_l$, and $\eta_l$ are the total spin, orbital angular momentum, energy, width, and elasticity of the resonance, respectively. The spin-flip amplitude, on the other hand, is given by

$$\tilde{f}_{\mathrm{res}} = \frac{\sqrt{2}}{3q}\sum_{\mathrm{resonances}}(-1)^{I+\frac{1}{2}}(-1)^{J-l-\frac{1}{2}}\frac{\eta_l P_l'(1+t/2q^2)}{W_l - W - i\Gamma_l/2},$$

where $P_l'$ is the first derivative of the Legendre polynomial. Notice that our two expressions for $f_{\mathrm{res}}$ and $\tilde{f}_{\mathrm{res}}$ contain no free parameters.

The Regge amplitudes are dominated by a single $\rho$ trajectory; hence

$$f_{\mathrm{Reg}} = \frac{-M\mu b_1(t)}{4\pi W}\left[\frac{S-M^2-\mu^2}{S_0}\right]^{\alpha_\rho(t)}\left[i + \tan\frac{\pi}{2}\alpha_\rho(t)\right],$$

$$\tilde{f}_{\text{Reg}} = \frac{\mu}{16\pi}[b_1(t) - \alpha_\rho(t) b_2(t)]\left[\frac{S-M^2-\mu^2}{S_0}\right]^{\alpha_\rho(t)}\left[i + \tan\frac{\pi}{2}\alpha_\rho(t)\right],$$

where $M$ and $\mu$ are the nucleon and pion mass, $\alpha_\rho(t)$ is the trajectory of the $\rho$, and $b_1(t)$ and $b_2(t)$ are the residues of the Regge pole. The total center-of-mass energy squared, $S$, is related to the pion lab energy $E$ by $S = 2ME - M^2 - \mu^2$, so that on choosing $S_0 = 2M\mu$, $(S-M^2-\mu^2)/S_0$ reduces simply to $E/\mu$. With this choice of $S_0$ it was shown[4] that the residue functions $b_1(t)$ and $b_2(t)$ do not have very strong $t$ dependences. We have in fact chosen the residues to be constants. One can obtain a better fit to $d\sigma/dt$, naturally, by giving the residues a more complicated $t$ dependence. Since our purpose here is to explain the polarization and since $P$ does not depend sensitively on small variations of $b_1(t)$ and $b_2(t)$, we will not complicate our model with $t$-dependent residues. In the same spirit we take a simple linear relationship for the $t$ dependence of $\alpha_\rho(t)$, viz. $\alpha_\rho(t) = 0.58 + 0.90t$. Since the polarization does not depend very sensitively on $\alpha_\rho(t)$ we shall not vary the parameters $\alpha_\rho(0)$ and $\alpha_\rho'(0)$. This leaves us with only two free parameters, $b_1$ and $b_2$.

The parameters $b_1$ and $b_2$ are constrained to fit $d\sigma/dt$. Only the sign of $b_2$ is really free to determine the polarization. (The sign of $b_1$ is fixed by the optical theorem.) The sign of $b_2$ is fixed in a certain sense also because only the negative sign will give a positive polarization. So there are no free parameters to determine $P$. The best fit to the data is obtained with $b_1 = 12.9$ mb$^{1/2}$/GeV and $b_2 = 176$ mb$^{1/2}$/GeV and is shown in Figs. 1 and 2. Notice that the theoretical values of $P$ are on the whole less than the experimental values but all within the error bars except for one point.

The extrapolation of $P(t)$ over a larger range

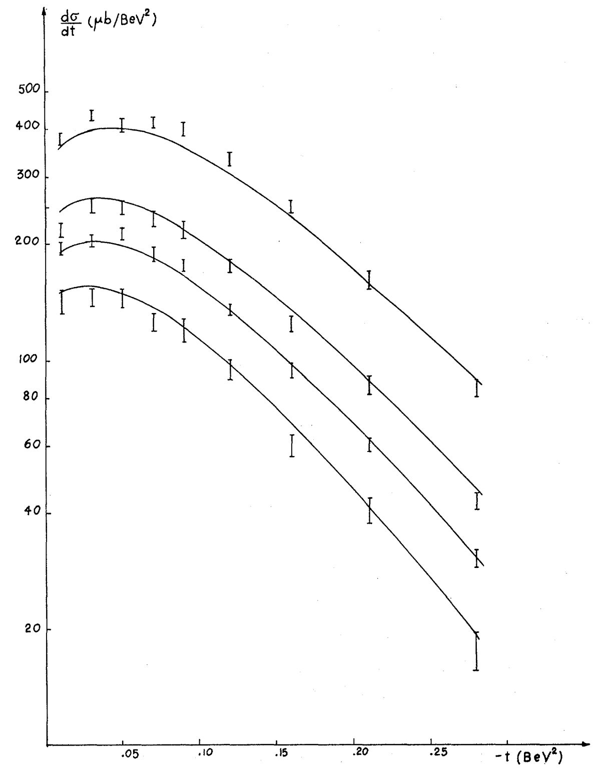


FIG. 1. Fit of differential cross sections at 5.9, 9.8, 13.3, and 18.2 GeV.

Table I. Resonance parameters.

| Resonance mass (MeV) | Width (MeV) | Total spin $J$ | Orbital angular momentum $l$ | Parity | Elasticity |
|---|---|---|---|---|---|
| $N_\alpha(1683)$ | 105 | 5/2 | 3 | + | 1.0 |
| $N_\gamma(1518)$ | 125 | 3/2 | 2 | − | 0.77 |
| $N_\gamma(2216)$ | 240 | 7/2 | 4 | − | 0.25 |
| $N_\gamma(2633)$ | 425 | 11/2 | 6 | − | 0.076 |
| $N_\gamma(3030)$ | 400 | 15/2 | 8 | − | 0.011 |
| $\Delta_\delta(1236)$ | 120 | 3/2 | 1 | + | 1.0 |
| $\Delta_\delta(1929)$ | 170 | 7/2 | 3 | + | 0.49 |
| $\Delta_\delta(2452)$ | 275 | 11/2 | 5 | + | 0.117 |
| $\Delta_\delta(2850)$ | 400 | 15/2 | 7 | + | 0.028 |
| $\Delta_\delta(3230)$ | 440 | 19/2 | 9 | + | 0.003 |

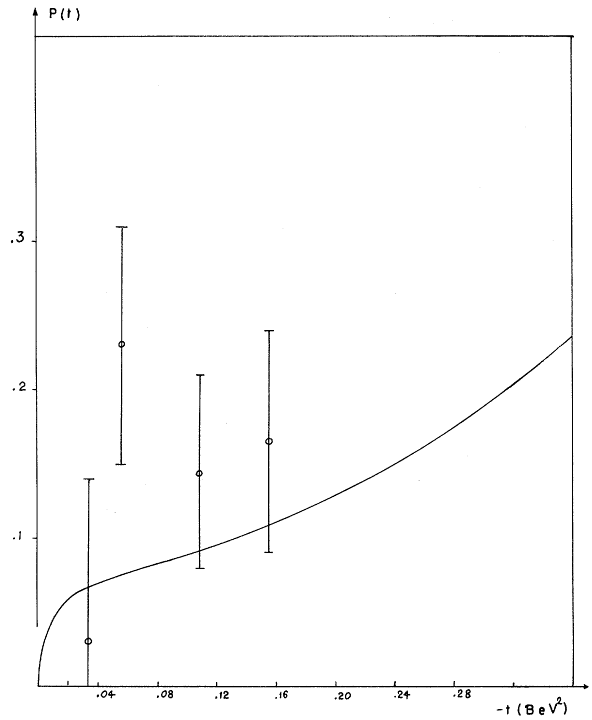


FIG. 2. Fit of polarization at 6.0 GeV.

of $t$ and for different values of the energy is given in Fig. 3. $P(t)$ for $-t>0.5$ (GeV)$^2$ is not given, however, because our simple model with constant residues is no longer able to fit $d\sigma/dt$ with sufficient accuracy. The decrease of the polarization with increasing energy is expected and may be understood in the following manner. The polarization arises only through the interference of $f_{\rm res}$ with $f_{\rm Reg}$ so that in a crude sense $P$ is proportional to $f_{\rm res}/f_{\rm Reg}$. The decrease of $P$ with energy arises from the fact that $f_{\rm res}$ decreases faster with energy than $f_{\rm Reg}$. The general increase of $P$ with $-t$ is due to its explicit $\sin\theta$ dependence. The sharper rise of $P$ near $t=-0.4$ GeV$^2$, however, is accounted for by considering the ratio $f_{\rm res}/f_{\rm Reg}$. The amplitude $f_{\rm Reg}$ decreases exponentially with $-t$ whereas $f_{\rm res}$ decreases less rapidly, hence both the ratio and $P(t)$ increase as $-t$ increases.

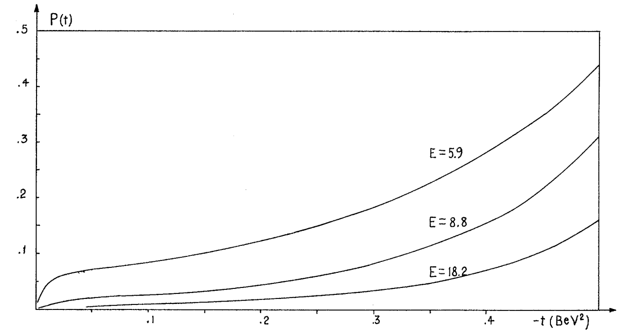


FIG. 3. The polarization, $P(t)$, at 5.9, 9.8, and 18.2 GeV.

Our model for $P(t)$ can easily be tested through measurements of $P(t)$ at larger values of $-t$.

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