vided the condition "$\delta(\infty)=0$" is replaced by "$s^{\epsilon}\delta(s)\to 0$, some $\epsilon>0$."

[2]Actually we should say "∘∘∘ satisfies $\delta(\infty)\leq 0$ if ···" [see Eq. (10) and the following remarks].

[3]G. Frye and R. L. Warnock, Phys. Rev. 130, 478 (1963).

[4]One has to assume also that the derivatives of F_L, δ, and η do not oscillate too violently at large physical energies. For full details, see D. H. Lyth, "Consequences of pure absorption at high energies, for the prediction of partial waves from unitarity" (to be published).

[5]This is a consequence[4] of the unitarity bound $F = O(q^{-3})$. See A. Donnachie and J. Hamilton, Phys. Rev. 138, B678 (1965) and also Ref. 4.

[6]Remember that $q\sim s^{1/2}$ for large s.

[7]The theorem is derived explicitly for the general case (Lyth, Ref. 4).

[8]This fact has been emphasized by A. Donnachie, J. Hamilton, and A. T. Lea, Phys. Rev. 135, B515 (1964).

[9]Donnachie, Hamilton, and Lea, Ref. 8.

[10]D. H. Lyth, Phys. Letters 21, 338 (1966).

[11]E.g., A. Donnachie, A. T. Lea, and C. Lovelace, Phys. Letters 19, 146 (1965).

[12]Assuming purely imaginary, spin-independent, forward-peak-dominated high-energy πN amplitudes, we have $1-\eta=\sigma_{\rm EL}/\sigma_{\rm TOT}\simeq 0.4$ at high energies.

[13]As s_I is increased from 100 to 200, s_Y increases from about 55 to about 75.

NEW METHOD FOR ASSIGNING BARYON RESONANCES TO SU(3) MULTIPLETS*

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SU(3) assignments for baryon resonances can be inferred from the relative phases of the resonant amplitudes in two-body inelastic channels. We give an application to Y_1^* resonances in the channel $K^-+n\to\Lambda+\pi^-$.

In a partial-wave analysis of the reaction $K^-+n\to\Lambda+\pi^-$ in the c.m. energy interval 1660 to 1900 MeV, we examined the relative phases of the resonant amplitudes $Y_1^{*-}(1660)$, $Y_1^{*-}(1765)$, $Y_1^{*-}(1915)$, and $Y_1^{*-}(2030)$.[1] In the elastic channel $K^-+n\to K^-+n$ the relative phase of two resonant amplitudes, taken at the resonant energy E_R, is always zero because the resonant amplitude T_R is proportional to $g_{N\bar{K}Y^*}{}^2/(E_R-E-i\Gamma/2)$. In the inelastic channel $\Lambda\pi$ the amplitude varies as $g_{N\bar{K}Y^*}g_{\Lambda\pi Y^*}/(E_R-E-i\Gamma/2)$, and the relative phase of two resonant amplitudes may be 0 or 180 deg depending on the relative sign of $g_{N\bar{K}Y^*}g_{\Lambda\pi Y^*}$ for the two Y^* states.

In this paper we show that knowledge of the relative sign of the coupling constants is a powerful aid in assigning particles to SU(3) multiplets.

The analysis in Ref. 1 showed that $Y_1^*(1765)$ and $Y_1^*(2030)$ are 180° out of phase at energy E_R. The experiment gave some evidence that $Y_1^*(1660)$ is in phase with $Y_1^*(2030)$, and that $Y_1^*(1915)$ is in phase with $Y_1^*(1765)$ at the resonant energy. The $Y_1^*(1660)$ and $Y_1^*(1915)$ amplitudes are relatively weak in the $\Lambda\pi$ channel, and a conclusive measurement of their phases was not possible. However, in discussing the relevance of the phase angle to SU(3) assignments, we shall use for $Y_1^*(1660)$ and $Y_1^*(1915)$ the tentative phase values indicated in Ref. 1.[2]

The experimental observations imply that $g_{N\bar{K}Y^*}g_{\Lambda\pi Y^*}$ is of one sign (±) for $Y_1^*(1765)$ and $Y_1^*(1915)$ and of opposite sign (∓) for $Y_1^*(1660)$ and $Y_1^*(2030)$. The ambiguity in sign arises because the experiment does not measure the phase relationship of the resonant amplitude in the $\Lambda\pi$ and $N\bar{K}$ channels.

Y_1^* resonant states have hypercharge $Y=0$ and isospin $I=1$. Table I shows the Clebsch-Gordan coefficients of SU(3) for the decay of a Y_1^{*-} into a baryon B and a meson M, both members of octets.[3] We consider the case that the baryon is a member of the $J^P=\frac{1}{2}^+$ $(N\Lambda\Sigma\Xi)$ octet, and the meson is a member of the pseudoscalar $(K\eta\pi\bar{K})$ octet.

In the limit of unitary symmetry the coupling of a member of a multiplet $\{\mu\}$ to $\{8\}\otimes\{8\}$ is described by a single invariant coupling constant g_μ. Then the coupling constant g_{BMY^*} is g_μ times the Clebsch-Gordan coefficient for the transition $Y^*\to B+M$ for Y^* a member of a $\{27\}$, $\{10\}$, or $\{10^*\}$. The situation is more complicated when Y^* is a member of an octet. Because $\{8\}\otimes\{8\}$ contains the $\{8_1\}$ and $\{8_2\}$ representations, there are two coupling constants for this process, g_1 and g_2. The coupling of an octet to $\{8\}\otimes\{8\}$ is described in the notation of de Swart[4] by two parameters g_8 and α, where $g_8=(30^{1/2}/40)g_1+(6^{1/2}/24)g_2$ and $\alpha=(6^{1/2}/24)g_2/$

Table I. SU(3) Clebsch-Gordan coefficients for the decomposition of $|\mu, Y, I, I_3\rangle = |\mu, 0, 1, -1\rangle$ into $|8, y, i, i_3\rangle \otimes |8, y', i', i_3'\rangle$.

	$y\ i\ i_3;\ y'\ i'\ i_3'$ \ $\{\mu\}$ / Y / I / I_3	Y_1^{*-} {27} 0 1 −1	{10} 0 1 −1	{10*} 0 1 −1	$\{8_1\}$ 0 1 −1	$\{8_2\}$ 0 1 −1	{8} 0 1 −1
nK^-	$1\ \frac{1}{2}\ -\frac{1}{2}\quad -1\ \frac{1}{2}\ -\frac{1}{2}$	$1/\sqrt{5}$	$-1/\sqrt{6}$	$1/\sqrt{6}$	$-\sqrt{3}/\sqrt{10}$	$1/\sqrt{6}$	$-(1-2\alpha)\sqrt{16}$
$\Sigma^0\pi^-$	$0\ 1\ 0\quad 0\ 1\ -1$	0	$1/\sqrt{12}$	$-1/\sqrt{12}$	0	$1/\sqrt{3}$	$\alpha\sqrt{32}$
$\Sigma^-\pi^0$	$0\ 1\ -1\quad 0\ 1\ 0$	0	$-1/\sqrt{12}$	$1/\sqrt{12}$	0	$-1/\sqrt{3}$	$-\alpha\sqrt{32}$
$\Sigma^-\eta$	$0\ 1\ -1\quad 0\ 0\ 0$	$3/\sqrt{10}$	$1/\sqrt{4}$	$1/\sqrt{4}$	$1/\sqrt{5}$	0	$[(1-\alpha)\sqrt{32}]/\sqrt{3}$
$\Lambda\pi^-$	$0\ 0\ 0\quad 0\ 1\ -1$	$3/\sqrt{10}$	$-1/\sqrt{4}$	$-1/\sqrt{4}$	$1/\sqrt{5}$	0	$-(1-\alpha)\sqrt{32}/\sqrt{3}$
Ξ^-K^0	$-1\ \frac{1}{2}\ -\frac{1}{2}\quad 1\ \frac{1}{2}\ -\frac{1}{2}$	$1/\sqrt{5}$	$1/\sqrt{6}$	$-1/\sqrt{6}$	$-\sqrt{3}/\sqrt{10}$	$-1/\sqrt{6}$	$-\sqrt{16}$

Table II. Quantity $g_{N\bar{K}Y^*}g_{BMY^*}$ for Y_1^* a member of a {27}, {10}, {10*}, or {8} multiplet; B and M denote members of the $J^P = \frac{1}{2}^+$ baryon octet and of the pseudoscalar meson octet, respectively.

B,M	{27}	{10}	{10*}	{8}
$\Sigma^0\pi^-$	0	$-g_{10}^2/\sqrt{72}$	$-g_{10^*}^2/\sqrt{72}$	$-(g_8^2\sqrt{512})\alpha(1-2\alpha)$
$\Sigma^-\pi^0$	0	$g_{10}^2/\sqrt{72}$	$g_{10^*}^2/\sqrt{72}$	$(g_8^2\sqrt{512})\alpha(1-1\alpha)$
$\Sigma^-\eta$	$g_{27}^2\sqrt{3}/\sqrt{50}$	$-g_{10}^2/\sqrt{24}$	$g_{10^*}^2/\sqrt{24}$	$-[(g_8^2\sqrt{512})/\sqrt{3}](1-\alpha)(1-2\alpha)$
$\Lambda\pi^-$	$g_{27}^2\sqrt{3}/\sqrt{50}$	$g_{10}^2/\sqrt{24}$	$-g_{10^*}^2/\sqrt{24}$	$-[(g_8^2\sqrt{512})/\sqrt{3}](1-\alpha)(1-2\alpha)$
Ξ^-K^0	$g_{27}^2/\sqrt{25}$	$-g_{10}^2/\sqrt{36}$	$-g_{10^*}^2/\sqrt{36}$	$(g_8^2\sqrt{256})(1-2\alpha)$

g_8. The coefficient which when multiplied into g_8 gives the coupling constant g_{BMY^*} for $\{8\}\otimes\{8\}$ is shown in the last column of Table I. The quantity $g_{N\bar{K}Y^*}g_{\Lambda\pi Y^*}$ is simply the product of the first and fifth rows in Table I times g_μ, and is shown in Table II. Using Table II we make some observations on SU(3) assignments for $Y_1^*(1660)$, $Y_1^*(1765)$, $Y_1^*(1915)$, and $Y_1^*(2030)$.

The $Y_1^*(2030)$ has $J^P = \frac{7}{2}^+$, and it has been suggested that this particle along with $N_{3/2}^*(1920)$ belongs to a $\frac{7}{2}^+$ {10} multiplet, which is the Regge recurrence of the $\frac{3}{2}^+$ δ decuplet.[5,6]

In order to make an SU(3) assignment for $Y_1^*(1765)$, we assume that $Y_1^*(2030)$ is a member of a {10} representation. Table II shows that $g_{N\bar{K}Y^*}g_{\Lambda\pi Y^*}$ is positive for a {10}. Since $g_{N\bar{K}Y^*}g_{\Lambda\pi Y^*}$ has opposite sign for $Y_1^*(2030)$ and $Y_1^*(1765)$ by Ref. 1, the {27} and {10} assignments are ruled out for $Y_1^*(1765)$, as is {8} with $\frac{1}{2} < \alpha < 1$. The only possible assignment for $Y_1^*(1765)$ is {10*} or {8} with $\alpha < \frac{1}{2}$ or $\alpha > 1$, if $Y_1^*(2030)$ is a member of a {10}. A measurement of the relative phase of $Y_1^*(1765)$ and $Y_1^*(2030)$ in the $\Sigma\eta$ channel would resolve this ambiguity. Unfortunately the branching ratio for $Y_1^*(1765)$ decaying to $\Sigma\eta$ is probably very small because of limited phase space, since the Q value for the decay is only 30 MeV. A measurement of the relative sign of $g_{N\bar{K}Y^*}g_{\Sigma\pi Y^*}$ for $Y_1^*(1765)$ and $Y_1^*(2030)$ could restrict further the value of α, and possibly rule out {10*}. A {10*} multiplet would contain a $Y = 2$, $I = 0$ resonance; evidence for this state has recently been reported.[7] A recent study of the branching ratios of $Y_1^*(1765)$ favors the octet assignment with $\alpha = -1.5_{-1.1}^{+0.7}$ or $-0.5_{-0.3}^{+0.2}$, depending on the energy-dependent form assumed for the resonant width Γ.[8] This is consistent with our limits on α.

Since $g_{N\bar{K}Y^*}g_{\Lambda\pi Y^*}$ has the same sign for $Y_1^*(1765)$ and $Y_1^*(1915)$, the same possible assignments are indicated for $Y_1^*(1915)$.

It has been suggested that $Y_1^*(1915)$ belongs to a $\frac{5}{2}^+$ octet along with $N_{1/2}^*(1688)$, $Y_0^*(1815)$, and $\Xi^*(1933)$,[6] and this is consistent with the conclusions drawn above. If $Y_1^*(2030)$ is a member of a {10}, then α is less than $\frac{1}{2}$ or greater than 1 for the $\frac{5}{2}^+$ baryon octet. The $\frac{1}{2}^+$ baryon octet has $\alpha \approx \frac{1}{4}$,[9,10] so the results are consistent with α being the same for the $\frac{1}{2}^+$ and $\frac{5}{2}^+$ baryon octets.

Y_1*(1660) is usually assigned to the $\frac{3}{2}^-$ γ octet of baryons. Then from Table II the $\frac{3}{2}^-$ γ octet has $\frac{1}{2}<\alpha<1$ if Y_1*(2030) is assigned to $\{10\}$.

Regardless of the Y_1*(2030) assignment, one can still state that α is different for the $\frac{3}{2}^-$ γ baryon octet and the proposed $\frac{5}{2}^+$ baryon octet. For one octet α lies in the range $\frac{1}{2}<\alpha<1$, and for the other $\alpha<\frac{1}{2}$ or >1. Cutkosky has discussed the conditions under which α might be the same for different baryon octets.[10]

We emphasize that most of the above conclusions are based on the assumption that Y_1*(2030) belongs to a $\{10\}$ representation. Ideally one should measure the Y_1* phases relative to Y_1*(1385), which is firmly established as a number of the $\frac{3}{2}^+$ baryon δ $\{10\}$.

We have used the experimental data from Ref. 1 primarily to illustrate that a measurement of the relative phase of resonant amplitudes in a two-body inelastic reaction can be used to make SU(3) assignments. This method is applicable to the higher spin resonance formed in π-N and K-N scattering, and may prove to be more reliable than assignments made on the basis of measured partial decay widths. The SU(3) predictions of the relative signs of coupling constants involve only one parameter, α, and this only for octets. On the other hand, SU(3) calculations of partial widths depend upon g_μ and kinematical factors, as well as on α. Inexactness of SU(3) symmetry may cause a splitting in g_μ, giving rise to discrepancies between calculated and experimental partial decay rates. For example, a calculation of $\Gamma_{\Xi\pi}$ for $\Xi_{1/2}$*(1530) (a member of the δ decuplet), using as input the current values of $\Gamma_{\Lambda\pi}$ and $\Gamma_{\Sigma\pi}$ for Y_1*(1385) and $\Gamma_{N\pi}$ for $N_{3/2}$*(1236),[11] predicted $\Gamma_{\Xi\pi}=16$ MeV, compared to the measured value of 7.5 ± 1.7 MeV.

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REGGE-POLE ANALYSIS OF πp CHARGE-EXCHANGE POLARIZATION*

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The π^-p charge-exchange reaction, $\pi^-+p\rightarrow\pi^0+n$, at high energy and low momentum transfer provides an excellent test of the Regge-pole hypothesis. Only the ρ meson may be exchanged in the crossed channel, t, because only the ρ has $I=1$, $G=+1$, and $P=(-1)^J$. The differential cross section $d\sigma/dt$ has been measured[1-3] in the energy range 6-18 GeV. Several analyses[4-6] have shown the consistency of these data with a single ρ Regge-pole exchange. The single-Regge-pole model predicts zero polarization because the spin-nonflip and the spin-flip amplitudes have the same phase. A recent measurement[7] at 6 GeV and low momentum transfer shows a nonzero polarization in apparent contradiction to the Regge-pole hypothesis.

We wish to demonstrate in this Letter that this polarization may be explained in terms of the