DOUBLE SCATTERING IN HIGH-ENERGY ELASTIC COLLISIONS WITH DEUTERONS*

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(Received 31 August 1966)

We suggest that the broad shoulder observed in recent measurements of the pd elastic differential cross section at 2 GeV is a double-scattering effect and propose that further measurements be made to test whether double scattering persists at high energies.

In this note we suggest an explanation for the broad shoulder observed in recent measurements[1] of pd elastic scattering at 2 GeV and propose an experiment to establish clearly whether or not double-scattering effects in deuterons persist at high energies. Analyses[2,3] of a variety of particle-deuteron cross sections, based upon the Glauber high-energy approximation,[4] suggest that these effects tend to persist at least up to energies of 18 GeV. The influence of double scattering on the cross sections which have been considered up to now, however, has been considerably smaller in magnitude than that of single scattering. We suggest that high-energy pd elastic scattering at 0.5 $(\text{GeV}/c)^2 \lesssim -t \lesssim 1.5$ $(\text{GeV}/c)^2$, where t is the squared four-momentum transfer, consists mainly of double scattering. A test of this suggestion at high energies could have a bearing on the question of Regge-pole dominance since it has been argued[5] that if the nucleon-nucleon (NN) scattering amplitudes near the forward direction can be represented as a sum of Regge poles, double-scattering effects should vanish much more rapidly at high energies than is inferred from the Glauber approximation.

To determine the momentum transfers at which double scattering may dominate over single scattering, consider, for an arbitrary incident particle x, both xN and xd elastic scattering at momentum transfers q (we take $\hbar = 1$) which are sufficiently large so that the xN elastic scattering intensity $d\sigma_{xN}(q)/d\Omega$ is much smaller than its value near the forward direction. The contribution to the xd elastic-scattering intensity arising from single scattering is proportional to[6] $S^2(\frac{1}{2}q)d\sigma_{xN}(q)/d\Omega$, where $S(q) = \langle \exp(i\vec{q}\cdot\vec{r})\rangle$ is the form factor for the deuteron ground state. In the Glauber approximation the intensity for xd elastic double scattering is asymptotically proportional to[6] $[d\sigma_{xN}(\frac{1}{2}q)/d\Omega]^2$ in the limiting case of a deuteron whose radius very greatly exceeds the range of the xN strong interactions. But $S^2(\frac{1}{2}q)d\sigma_{xN}(q)/d\Omega$ typically decreases much more rapidly from its value at $q = 0$ than does $[d\sigma_{xN}(\frac{1}{2}q)/d\Omega]^2$. Consequently, although double scattering is much weaker than single scattering for small q, it becomes dominant at larger q. For some large values of q, in other words, it is more probable that the incident particle suffers two successive collisions, each with momentum transfer of the order of $\frac{1}{2}\vec{q}$, than that it is scattered with momentum transfer $\vec{q}$ by only a single collision with a target nucleon.

The formula we shall use for our calculation is independent of the deuteron radius. Its use removes the restrictions that the deuteron radius very greatly exceed the range of the xN strong interactions and that, consequently, in each of the two collisions comprising the double-scattering event a momentum of nearly $\frac{1}{2}\vec{q}$ be transferred by the incident particle. For a net momentum transfer $\vec{q}$, a momentum of $\frac{1}{2}\vec{q}+\vec{q}\,'$ may be transferred in a collision with one target nucleon and a momentum of $\frac{1}{2}\vec{q}-\vec{q}\,'$ may be transferred in a collision with the other, where the vector $\vec{q}\,'$ may take on a range of values. At large values of q the intensity for such elastic double scattering is greater than the intensity for scattering in which momentum $\vec{q}$ is transferred via a single collision with a target nucleon.

The differential cross section $d\sigma/d\Omega$ for xd elastic scattering may be written in the Glauber approximation in terms of the xp and xn elastic-scattering amplitudes f_{xp} and f_{xn} as

$$d\sigma/d\Omega = |S(\tfrac{1}{2}\vec{q})[f_{xn}(\vec{q}) + f_{xp}(\vec{q})] + \frac{i}{2\pi k}\int S(\vec{q}\,')\langle f_1(\tfrac{1}{2}\vec{q}+\vec{q}\,')f_2(\tfrac{1}{2}\vec{q}-\vec{q}\,')\rangle d^2\vec{q}\,'|^2, \quad (1)$$

where f_1 and f_2 are scattering matrices for collisions between particle x and target nucleon 1 and between particle x and target nucleon 2. The product $f_1 f_2$ is an operator in the composite isospin space of particle x and nucleons 1 and 2. The brackets $\langle\,\rangle$ denote the expectation value with respect to the deuteron ground state, which is an isotopic singlet ($I=0$) state, and with respect to the initial isospin state of the incident particle. This expression does not violate the charge independence of nuclear forces for particle interactions, and it includes elastic double-charge-exchange processes, i.e., processes in which the incident proton undergoes a charge exchange with the target neutron and the resulting scattered neutron then undergoes a charge exchange with the target proton, with the final state of the target being the deuteron ground state.[7] At momentum transfers larger than approximately 0.5 GeV/c this expression differs rather dramatically from that in which the integral, i.e., double-scattering term, is neglected.[8] We shall be concerned here with t in the range $0 \leqslant -t \lesssim 1.5$ (GeV/c)^2, where NN measurements are fairly well represented by elastic-scattering amplitudes of the form

$$f_{xN} = (i + \alpha_{xN})(k\sigma_{xN}/4\pi)\exp(\tfrac{1}{2}a_x t + \tfrac{1}{2}b_x t^2), \qquad (2)$$

$$N = n, p.$$

We have calculated $d\sigma/d\Omega$ for pd collisions at 2 GeV by means of Eqs. (1) and (2).[9] As input we have used the values $\alpha_{pp} = -0.12$, $a_p = 7.62$ (GeV/c)$^{-2}$, and $\sigma_{pp} = 45.1$ mb obtained directly from pp measurements,[10-12] and $\sigma_{pn} = 43.0$ mb and $\alpha_{pn} = 0.20$ obtained indirectly from pp and pd measurements.[12,13] Since no value of b_p for pN scattering at 2 GeV has, to our knowledge, been published, we have analyzed recent pp data[14] at 2.2 GeV and we obtain a value of 1.88 (GeV/c)$^{-4}$. We obtained $S(q)$ from the analytic expression for the deuteron wave function given as "Approximation III" by Moravcsik.[15] That expression is a sum of eight exponential functions multiplied by the inverse neutron-proton separation which is fitted to the Gartenhaus wave function.

The calculations, shown by the solid curve in Fig. 1 together with the measurements,[1,13] present evidence for the importance of double scattering at 2 GeV. The predicted minimum and secondary maximum, the existence of which is perhaps suggested by the measurements, result from the destructive interference between the single- and double-scattering amplitudes and from the relatively large amount of double scattering at $-t \gtrsim 0.5$ (GeV/c)^2. Our calculations indicate that the shoulderlike departure of the data for $-t$ between 0.4 and 1.5 (GeV/c)^2 from the exponential trend of the diffraction peak is not of the same nature as the secondary peaks observed in recent $\pi^{\pm}p$ and $K^{-}p$ elastic scattering.[16] We wish to note that this calculation con-

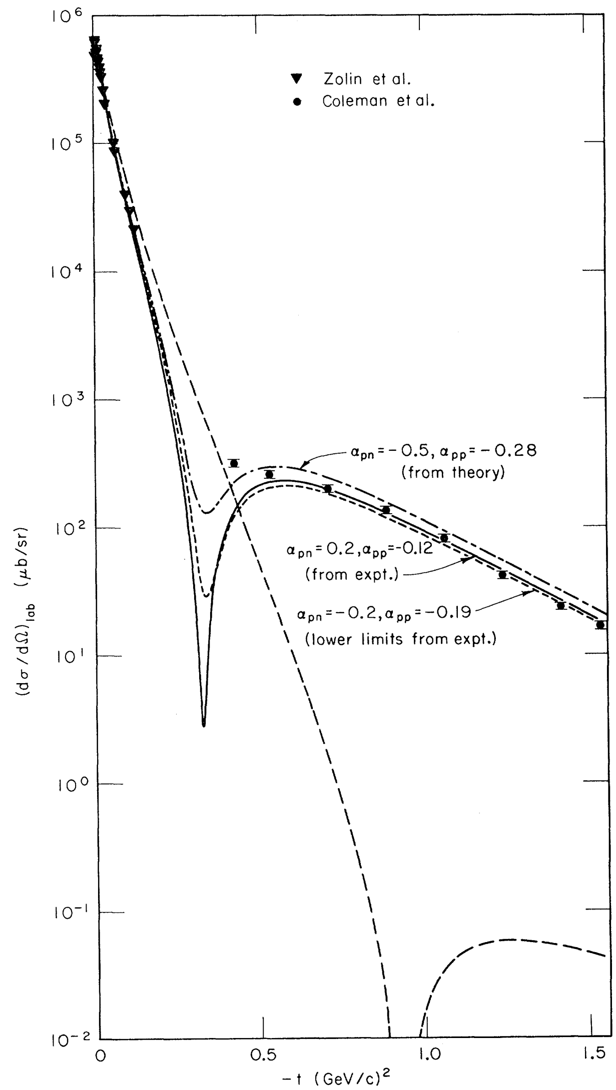

FIG. 1. Differential cross sections in the laboratory system for elastic pd scattering at 2.0 GeV. The solid (broken) curve is the theoretical prediction, using nucleon-nucleon data, when double scattering is treated (neglected). The dotted and dot-dashed curves are calculated with the experimental lower limits for α_{pn} and α_{pp} and with theoretical predictions for α_{pn} and α_{pp}, and include double scattering.

tains no adjustable parameters and that the input was determined from NN measurements. It should be clear that changes within the quoted statistical errors for the input could yield even better agreement with the pd data, but that no such changes would yield very large qualitative differences.

The depth of the minimum is intimately connected to the relative phase between the single- and double-scattering amplitudes, and hence, to the values of α_{pn} and α_{pp}. The measured values are[10,13] 0.20 ± 0.40 and -0.12 ± 0.07, respectively. Theoretical predictions[17] give $\alpha_{pn}=-0.50$ and $\alpha_{pp}=-0.28$. If we assume $\alpha_{pn}=-0.20$ and $\alpha_{pp}=-0.19$, for example, our calculations are represented by the dotted curve in Fig. 1. A further decrease in α_{pn} and α_{pp} to -0.50 and -0.28, respectively, yields a further reduction in the depth of the minimum as shown by the dot-dashed curve. These curves strongly suggest that α_{pn} at 2 GeV is negative.

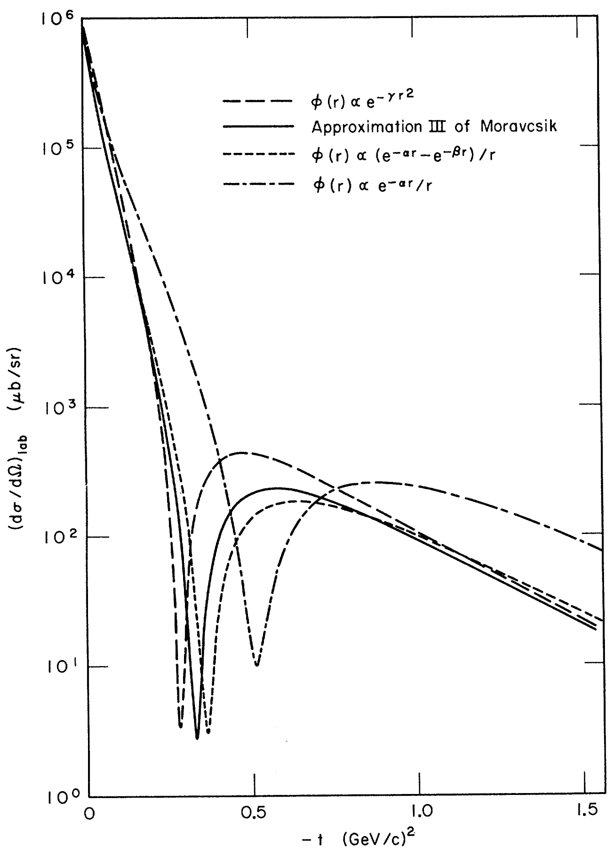

FIG. 2. Effect of different deuteron wave functions φ on the calculated differential cross sections. The wave function used for the solid curve is a sum of eight exponential functions multiplied by the inverse neutron-proton separation. The parameters α, β, and γ have the values 0.232 F^{-1}, 1.202 F^{-1}, and 0.0961 F^{-2}, respectively.

To compare the results of these calculations with those in which double-scattering effects are neglected, we have calculated $d\sigma/d\Omega$ with the integral in Eq. (1) set equal to zero. The values of σ_{pn} and α_{pn} used for this calculation were obtained by consistently neglecting double scattering and were derived from pp and pd measurements[10,12,13] to be 38.8 mb and 0.21, respectively. The results are shown by the broken curve in Fig. 1.

In Fig. 2 we show the effect of using different deuteron wave functions φ in our calculation. The wave function used for the solid curve is presumably the most accurate. We see that the results for the different wave functions are all qualitatively very similar.

To determine whether double-scattering effects persist at higher energies, we suggest further elastic pd measurements be made. On the basis of presently available NN data we would predict for pd elastic scattering at 19.3 GeV/c, for example, a broad shoulder centered near $-t\approx0.5$ (GeV/c)^2.

It is a pleasure to thank Professor R. J. Glauber for several useful discussions.

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*This work was performed under the auspices of the U. S. Atomic Energy Commission.

[1]E. Coleman, R. M. Heinz, O. E. Overseth, and D. E. Pellet, Phys. Rev. Letters 16, 761 (1966).

[2]V. Franco and R. J. Glauber, Phys. Rev. 142, 1195 (1966).

[3]V. Franco, Phys. Rev. Letters 16, 944 (1966).

[4]R. J. Glauber, in Lectures Delivered at the Summer Institute for Theoretical Physics, University of Colorado, 1958, edited by W. E. Britten and G. Dunham (Interscience Publishers, Inc., New York, 1959), Vol. I, p. 315.

[5]E. S. Abers, H. Burkhardt, V. L. Teplitz, and C. Wilkin, Nuovo Cimento 42, 365 (1966); Phys. Letters 21, 339 (1966).

[6]See Ref. 2. For simplicity we assume in the qualitative discussion that the xp and xn elastic scattering amplitudes are identical.

[7]C. Wilkin, Phys. Rev. Letters 17, 561 (1966). The charge-exchange correction noted by Wilkin is rather small in the present calculation. It contributes roughly 12% of the double scattering and affects the cross section near the forward direction by approximately 0.5%.

[8]Higher order multiple scattering is presumably unimportant because high-energy scattering is most in-

tense near the forward direction. The dominance of double scattering was predicted in Ref. 2 and in V. Franco, thesis, Harvard University, 1963 (unpublished), p. 67. It was calculated to occur in $\bar{p}d$ elastic scattering at 2 GeV near $-t \approx 0.4$ (GeV/c)^2, where a secondary maximum appeared in the calculated differential cross section, but no data had been available to test the prediction.

[9]Since single collisions in which $(\frac{1}{2}\vec{q} \pm \vec{q}')^2$ is greater than 2 (GeV/c)^2 yield negligibly small contributions to the integral in Eq. (1) for the momentum transfers of interest, we have restricted the integration over $\vec{q}'$ to $(\frac{1}{2}\vec{q} + \vec{q}')^2 \leq 2$ (GeV/c)^2.

[10]L. F. Kirillova et al., Zh. Eksperim. i Teor. Fiz. 50, 76 (1966).

[11]L. F. Kirillova et al., Yadern. Fiz. 1, 533 (1965) [translation: Soviet J. Nucl. Phys. 1, 379 (1965)]; L. F. Kirillova, private communication.

[12]D. V. Bugg et al., Phys. Rev. 146, 980 (1966).

[13]L. S. Zolin et al., Zh. Eksperim. i Teor. Fiz.—Pis'ma Redakt. 3, 15 (1966) [translation: JETP Letters 3, 8 (1966)].

[14]A. R. Clyde, University of California Radiation Laboratory Report No. UCRL-16275, 1966 (unpublished). We treat the angular distribution in the laboratory system.

[15]M. J. Moravcsik, Nucl. Phys. 7, 113 (1958).

[16]D. D. Allen et al., Phys. Letters 21, 468 (1966); C. T. Coffin et al., Phys. Rev. Letters 17, 458 (1966), and references cited therein; see also references cited in P. Sonderegger et al., Phys. Letters 20, 75 (1966).

[17]A. A. Carter and D. V. Bugg, Phys. Letters 20, 203 (1966); P. Söding, Phys. Letters 8, 285 (1964).

ROLE OF LEVINSON'S THEOREM FOR PARTIAL WAVES WITH $l > 0$

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(Received 6 June 1966)

In this note we consider the p-wave elastic scattering amplitude[1]

$$F = [\eta \exp(2i\delta) - 1]/2iq^3, \tag{1}$$

which satisfies in the physical region the relation

$$\mathrm{Re}\,F(s) = F_L(s) + \frac{P}{\pi}\int_{s_t}^{\infty} ds' \frac{\mathrm{Im}\,F(s')}{(s'-s)}, \tag{2}$$

where F_L is the left-hand-cut contribution and s_t is the physical threshold.[1] For simplicity we do not allow bound-state poles in the amplitude.

Our main purpose is to point out that such an amplitude satisfies Levinson's theorem [$\delta(\infty) = 0$] if[2] and only if the functions F_L and η satisfy a certain integral condition. Therefore, if we know F_L and η, we can predict whether or not Levinson's theorem is satisfied; conversely, if we assume Levinson's theorem, then a knowledge of F_L can be used to put a constraint on the inelasticity parameter η (or vice versa). This is in contrast to the "normal" (s-wave) situation, where F_L and η can be chosen independently, and where Levinson's theorem must be imposed separately if it is required to hold (and in general always may be imposed).

Essentially following Frye and Warnock,[3] we define an auxiliary amplitude

$$\bar{F} = F + [(1-\eta)/2iq^3] \equiv \eta[\exp(2i\delta) - 1]/2iq^3, \tag{3}$$

which satisfies [cf. Eq. (2)]

$$\mathrm{Re}\,\bar{F}(s) = \bar{F}_L(s) + \frac{P}{\pi}\int_{s_t}^{\infty} ds' \frac{\mathrm{Im}\,\bar{F}(s')}{(s'-s)} \tag{4}$$

with

$$\bar{F}_L(s) \equiv F_L(s) + \frac{P}{\pi}\int_{s_t}^{\infty} ds' \frac{\{[1-\eta(s')]/2q'^3\}}{(s'-s)}. \tag{5}$$

We then write $\bar{F} = \bar{N}/\bar{D}$, where

$$\bar{D}(s) \equiv \exp\left[-\frac{1}{\pi}\int_{s_t}^{\infty} ds' \frac{(s-s_1)}{(s'-s_1)}\frac{\delta(s')}{(s'-s)}\right], \tag{6}$$

where s_1 is an arbitrary subtraction point ($s_1 < s_t$). Assuming that[4] $\delta(\infty) \leq \frac{1}{2}\pi$, we may then set up the "N/D" equations

$$\bar{N}(s) = \bar{F}_L(s) + \frac{1}{\pi}\int_{s_t}^{\infty} ds' \frac{(s'-s_1)F_L(s') - (s-s_1)F_L(s)}{(s'-s_1)(s'-s)} \times \frac{q'^3}{\eta(s')}\bar{N}(s'), \tag{7}$$

$$\bar{D}(s) = 1 - \frac{1}{\pi}\int_{s_t}^{\infty} ds' \frac{s-s_1}{s'-s_1} q'^3 \eta^{-1}(s') \frac{\bar{N}(s')}{(s'-s)}. \tag{8}$$