$\Delta R/R \sim +1\times10^{-4}$. V. A. Belyakov, Phys. Letters 16, 279 (1965) gives $\Delta R/R \sim -2.5\times10^{-4}$. Earlier estimates may be found in D. A. Shirley, Rev. Mod. Phys. 36, 339 (1964).

SIGN RATIO AND ABSOLUTE FLUX OF COSMIC-RAY ELECTRONS

C. J. Bland, G. Boella, G. Degli Antoni, C. Dilworth, L. Scarsi, and G. Sironi
Istituto di Scienze Fisiche dell'Università, Milano, Italy, and
Gruppo GIFCO* del Consiglio Nazionale delle Ricerche, Milano, Italy

and

B. Agrinier, Y. Koechlin, B. Parlier, and J. Vasseur
Service d'Electronique Physique, Centre d'Etudes Nucléaires de Saclay, Gif-sur-Yvette, Seine et Oise, France
(Received 6 July 1966)

Two balloon flights have been carried out over southern France (vertical geomagnetic cutoff given by Shea, Smart, and McCracken[1] as 5.36 BV) to measure with higher precision the value of the flux of cosmic-ray electrons already reported by the authors at this geomagnetic latitude[2] and to investigate the sign ratio of the electrons of energy about 5 BeV as revealed by the east-west asymmetry of the integral flux. The apparatus used consisted of a multiplate spark chamber containing 5 radiation lengths of lead and triggered for shower detection by a plastic-scintillator counter telescope (see Fig. 1).[2,3] The efficiency of detection of electron showers of 6 BeV was 94%, that of 6-BeV proton interactions 41%, and of noninteracting α particles ~40%. The axis of the chamber was inclined at a constant angle of 38° to the zenith and rotated continuously through azimuth. A flux-gate magnetometer monitored the orientation of the chamber with respect to the geomagnetic field.

Details of the two flights are given in Table I.

The analysis of the data consisted in the scanning of the stereo photos of events which triggered the chamber, of a binary coded series of lamps giving the number of single charged particles passing through the scintillator telescope without interacting in the chamber, and of the telemetered record of the orientation of the chamber.

A total of 238 electron showers in Flight I and 169 electron showers in Flight II were recorded during ceiling. These events were selected by visual criteria, in goemetrical conditions such that the axis of the shower lay within the visual area of the chamber. The geometrical factor corresponding to these conditions is 17 ± 1 cm^2 sr; the dead time imposed after each trigger being 1.02 sec, the relative val-

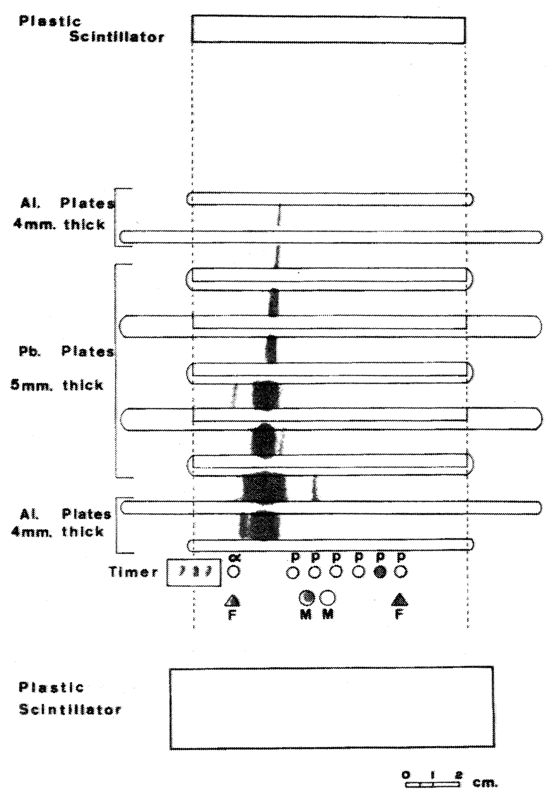

FIG. 1. Photomontage showing an electron shower recorded during the balloon flights and a superimposed sketch of the spark chamber and plastic scintillator telescope. F: fiducial marks for stereoreconstruction of the events; P: binary coded series of lamps giving the number of singly charged particles passing through the scintillator telescope without interacting in the chamber; α: lamp alight when I/I_0 is ≥ 2; M: coded lamps to indicate the geomagnetic quadrant containing the axis of the spark chamber at the moment of trigger.

Table I. Details of the balloon flights.

Flight	Date	Ceiling Altitude (mbar)	Ceiling Duration (h)	Total flux of electrons observed at ceiling $(m^2 \sec sr)^{-1}$	East-west asymmetry in the electron flux
I	11 May 1965	~3	9	7.3 ± 0.6	-0.28 ± 0.18
II	17 September 1965	4-5	6	7.2 ± 0.8	-0.31 ± 0.23

ues of flux in the two flights are those given in Table I. The validity of the visual criteria of discrimination between electron showers and nuclear interactions was confirmed by an analysis, on a sample of 965 events, of the distribution in successive plates of the origins of showers and interactions. The excess of origins in the first two radiation lengths led to an estimate of 128 ± 34 electrons in the sample, as compared with the 115 selected by the visual criteria.

The total flux of electrons observed has to be corrected for the presence of secondary electrons produced in the overlying atmosphere and of re-entrant albedo. The fraction of electrons of energy below the calculated geomagnetic cutoff at the top of the atmosphere was determined by measurements of shower size, based on calibration exposures at the proton synchrotrons of Saclay and CERN[4] and the Deutsches Elektronen Synchrotron (DESY), and was found to be $(8 \pm 2)\%$. The contribution of secondary electrons above the cutoff value was estimated from calculations based on experimental data on pion production[5] as 4%. The mean value of the flux of primary electrons, taking into account these corrections and the efficiency of detection, is then 6.8 ± 0.6 $(m^2 \sec sr)^{-1}$, a value consistent with that previously obtained on smaller statistics, $6.6^{+2.8}_{-1.7}$ $(m^2 \sec sr)^{-1}$.[2]

The azimuthal distribution of the direction of incidence of the electrons was determined by reconstruction from the stereophotos of the angle contained between the axis of the shower and the axis of the chamber and by reference to the magnetometer record of the azimuthal direction of the chamber axis at the moment of trigger. The relative counting rates in the four geomagnetic quadrants are given in Table II. The counting rate is seen to be higher in the eastern than in the western quadrant, indicating the existence of a negative excess in the electron flux. The value of the east-west asymmetry, defined classically as $A = 2[F(W)-F(E)]/[F(W)+F(E)]$, where $F(W)$ and $F(E)$ are the fluxes in the western and eastern quadrants, respectively, is consistent within statistical error in the two flights (Table I). The weighted mean value of $A = -0.29 \pm 0.14$ is two standard deviations from zero asymmetry.

The value of the negative excess $\epsilon = (F^- - F^+)/(F^- + F^+)$, where F^- and F^+ are, respectively, the negative and positive components of the electron flux, can be estimated from the east-west asymmetry A, given the relative values of cutoff rigidity as a function of azimuth, and the form of the rigidity spectrum of the electrons. Calculations of the cutoff rigidities based on the analysis of the geomagnetic field are available[1] and numerical computations for this geomagnetic location are in progress. In the meantime a direct estimate is obtained

Table II. East-west asymmetry; weighted means of the two flights.

	Counting rate in quadrant centered on North F(N)	West F(W)	South F(S)	East F(E)	Ratio of flux in western and eastern quadrants	East-west asymmetry
Electrons $(10^{-2}$ counts/sec$)^2$	1.27 ± 0.12	1.19 ± 0.13	1.24 ± 0.13	1.59 ± 0.14	0.75 ± 0.11	-0.29 ± 0.14
Protons (counts/sec)	1.24 ± 0.013	1.39 ± 0.014	1.20 ± 0.012	1.10 ± 0.012	1.27 ± 0.019	$+0.23 \pm 0.014$ (uncorrected)

Table III. Experimental values of the exponent in the integral spectrum of cosmic-ray electrons of energy ≥2 BeV.

Value derived from	Energy range (BeV)	Value of the exponent of the integral spectrum, γ
Differential flux measurement[a]	2-15	0.9 ± 0.4
Two integral flux values[b]	3.5-8	1.8 ± 0.36
Two integral flux values[c]	5-100	1.16 ± 0.52
Two integral flux values[d]	5-16	2.0 ± 0.25
Shower size distribution[e,f]	5-10	2.0 ± 0.5

[a]J. A. M. Bleeker, J. J. Burger, A. Scheepmaker, B. N. Swaneburg, and Y. Tanaka, in Proceedings of the Ninth International Conference on Cosmic Rays, London, 1965 (The Institute of Physics and The Physical Society, London, 1966), Vol. 1, p. 327.
[b]V. I. Rubtsov, in Proceedings of the Ninth International Conference on Cosmic Rays, London, 1965 (The Institute of Physics and The Physical Society, London, 1966), Vol. 1, p. 324.
[c]R. R. Daniel and S. A. Stephens, Phys. Rev. Letters 15, 769 (1965).
[d]Daniel and Stephens,[c] plus this experiment.
[e]Ref. 2.
[f]Ref. 3.

in this experiment by the azimuthal distribution of the proton flux (Table II), given by the number of singly charged, noninteracting particles passing through the counter telescope. The value obtained for the east-west asymmetry, $A = +0.23 \pm 0.014$, must be corrected for the wider angle of acceptance compared with that relative to the distribution of electron events and for the presence of splash and re-entrant albedo.[6] The final corrected value is $A = +0.28 \pm 0.02$ and represents the asymmetry for a 100% positive beam with an energy spectrum of the form $N(>E) = kE^{-\gamma}$, with $\gamma = 1.5$. The exponent of the integral rigidity spectrum of electrons has not yet been established with accuracy; values indicated experimentally, for energies above 2 BeV, are shown in Table III.

The negative excess ϵ resulting from the assumed value for γ and the asymmetry factor $A = -0.29 \pm 0.14$ is given in Table IV.

We can conclude that the primary electrons of energy around 5 BeV are predominantly negative; as has been pointed out by several authors, the existence of a negative excess in the cosmic-ray electron flux at high energy (some BeV) indicates a different origin than that resulting from nuclear collisions of cosmic rays in interstellar space. The flux value itself reported here exceeds that which can reasonably be expected from the collision mechanism.[7,8] Combining this information on the flux and the negative excess one is led, therefore, to consider that these electrons constitute an independent component of the cosmic radiation. The relative strength of this component, compared to that arising from collisions, is not necessarily constant over the whole energy spectrum. Data at present available in the lower energy region[9] are heavily contaminated by the contribution from secondary electrons produced in the atmosphere. At higher energies[9] no significant statistics have yet been reported. Accurate values both of the flux and of the sign ratio over a wide energy range are essential to a complete understanding of the origin and propagation of the cosmic-ray electrons.

The authors wish to express their gratitude to the Centre National Etudes Spatiales whose support made possible the balloon flights; to the staff of the Centre of Balloon Launching of Aire-sur-Adour for their expert and friendly collaboration; to Dr. H. O. Schulz and Dr. G. Weber of DESY who made possible the latest calibration of the spark chamber; to Madame J. Vasseur of the Service Calcule Electronique for assistance in data analysis; to the technical staff of both groups for their en-

Table IV. Negative excess ϵ as a function of γ, for $A = -0.29 + 0.14$.

γ	ϵ
2.5	0.62 ± 0.30
2.0	0.78 ± 0.37
1.5	1.0 ± 0.5

thusiastic support; and to Dr. J. Labeyrie and Professor G. Occhialini for help, criticism, and inspiration.

*Gruppi Italiani di Fisica Cosmica.

[1]M. A. Shea, D. F. Smart, and K. G. McCracken, J. Geophys. Res. 70, 4117 (1965).

[2]B. Agrinier, Y. Koechlin, B. Parlier, G. Boella, G. Degli Antoni, C. Dilworth, L. Scarsi, and G. Sironi, Phys. Rev. Letters 13, 377 (1964).

[3]B. Agrinier, Y. Koechlin, B. Parlier, J. Vasseur, C. F. Bland, G. Boella, G. Degli Antoni, C. Dilworth, L. Scarsi, and G. Sironi, in Proceedings of the Ninth International Conference on Cosmic Rays, London, 1965 (The Institute of Physics and The Physical Society, London, 1966), Vol. 1, p. 331.

[4]B. Agrinier, Y. Koechlin, B. Parlier, G. Boella, G. Degli Antoni, C. Dilworth, L. Scarsi, and G. Sironi, Nuovo Cimento 36, 1077 (1965).

[5]C. Perola and L. Scarsi, to be published.

[6]C. J. Bland, Space Research V (North-Holland Publishing Company, Amsterdam, 1965), p. 618.

[7]S. Hayakawa and H. Okuda, Progr. Theoret. Phys. (Kyoto) 28, 517 (1962).

[8]V. L. Ginzburg and S. I. Syrovatskii, The Origin of Cosmic Rays (Pergamon Press, New York, 1964).

[9]R. C. Hartman, R. H. Hildebrand, and P. Meyer, J. Geophys. Res. 70, 2713 (1965).

EQUILIBRIUM MODELS OF DIFFERENTIALLY ROTATING ZERO-TEMPERATURE STARS*

Jeremiah P. Ostriker and Peter Bodenheimer
Princeton University Observatory, Princeton, New Jersey

and

D. Lynden-Bell
Royal Greenwich Observatory, Herstmonceux Castle, Sussex, England

(Received 6 September 1966)

Recent work in stellar-evolution theory has been greatly influenced by the existence of rather strict limitations on the masses of models of white dwarfs, which represent the final evolutionary stages of cooling stars. Chandrasekhar[1] showed that no equilibrium models of nonrotating, completely degenerate white dwarfs exist for masses

$$M \geq M_3 = 5.75\mu_e^{-2}M_\odot = 1.44M_\odot \quad (\mu_e = 2), \qquad (1)$$

where μ_e is the mean molecular weight per electron. Subsequent work by Salpeter and Hamada,[2] with a more refined equation of state, reduced the critical mass M_3 to the range $(1.01-1.40)M_\odot$ depending upon chemical composition. Mestel[3] and Hartle and Sharp[4] have stressed that such a limit applies only to spherically symmetric configurations without rotation. However, work by Roxburgh,[5] Anand,[6] and James[7] shows that the mass limit M_3 is increased by only a few percent when uniform rotation is included in the models, although Hoyle[8] and Roxburgh[5] have indicated that considerably larger masses may be possible for disk-like configurations.

The mass limit for a nonrotating white dwarf is determined by the point at which the degenerate pressure can no longer support the material of the star against the forces of gravity (see Schwarzschild[9]). However, if rotation is present, the centrifugal force increases faster than the forces of gravity as the radius becomes smaller, and one would expect an equilibrium to be reached for any mass and angular momentum. In fact, however, for uniformly rotating configurations a different limit is reached which is determined by the maximum angular momentum compatible with solid-body rotation such that the effective gravity at the equator remains positive. This restriction on the angular momentum can be removed through the introduction of differential rotation or alterations in topology, as has been shown by Stoeckly[10] and Ostriker,[11] respectively, for the case of equilibrium models of rotating polytropes. In this Letter we demonstrate that white-dwarf models with masses considerably greater than M_3 are possible if differential rotation is allowed.

The calculated models are based on the physical assumption of an axially symmetric, completely degenerate, self-gravitating fluid, in which the effects of viscosity, magnetic fields, meridional circulation, and relativistic terms