fects are important and the attenuation is predicted to be modified, eventually at very low T varying as ω^2 and independent of T. The temperature region at which the change occurs is given by $T=\hbar\omega/2\pi k$, which is approximately 0.34 mdeg K for $\omega/2\pi = 45$ MHz.

Finally, it would be very interesting to observe zero sound in a weakly interacting Fermi liquid where distortion of the Fermi surface could be relatively large compared with pure He^3, where the effective interactions are strong and distortions of the Fermi surface are small. At first sight the dilute solutions of He^3 in He^4 would be ideal for this purpose, second sound in the solution being analogous to first sound in the He^3. However, for zero sound to propagate [see Eq. (3)], one must have approximately $F_0+F_1>0$. Calculation of F_0+F_1 for a 5.0% solution of He^3 in He^4 using the potential suggested by Bardeen, Baym, and Pines[10] gives $F_0+F_1=-0.25$. Hence it appears unlikely that zero sound can be propagated in the weakly interacting Fermi gas in a dilute solution.

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LINEAR INSTABILITY THEORY OF LASER PROPAGATION IN FLUIDS*

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The phenomenon of anomalous Stokes gain,[1] observed when a laser beam travels through a Raman cell longer than a certain critical length, has recently been the subject of intensive discussions.[2-4] In the present note, it is proposed that this anomalous gain is due to rapidly growing spatial and temporal instabilities which arise when intense laser beams interact with the medium. By incorporating the light-medium coupling terms in the linearized hydrodynamical equations of momentum and energy, we will show that for a fluid such instabilities are indeed predicted by the solutions of the appropriate dispersion relation. The process considered here is essentially that of stimulated Brillouin scattering, but with the reverse effect of the sound wave on the laser wave—by the varying induced dipole moment density—taken into account.

The light wave is coupled to the fluid—assumed to be optically transparent, nonmagnetic, and electrically nonconducting—by the mechanisms of electrostriction, thermal energy deposition, and the high-frequency Kerr effect.[5] For a linearly polarized light wave E, the electrostrictive force density $\vec{f}_{es}$ acting on a volume element of uniform composition with dielectric constant ϵ and density ρ is given by

$$\vec{f}_{es}=\frac{\rho}{8\pi}\nabla\left\{E^2\left(\frac{\partial\epsilon}{\partial\rho}\right)_T\right\}-\frac{E^2}{8\pi}\left(\frac{\partial\epsilon}{\partial T}\right)_\rho\nabla T, \qquad (1)$$

where ∇T is the gradient of the temperature. The modified Navier-Stokes equation for a volume element of velocity $\vec{v}$ becomes

$$\rho\frac{d\vec{v}}{dt}=-\nabla p+\vec{f}_{visc}+\vec{f}_{es}. \qquad (2)$$

Here, p is the pressure without the field, and the viscous force density is $\vec{f}_{visc}=(\eta'+\eta)\nabla(\nabla\cdot\vec{v})$

$+\eta\nabla^2\vec{v}$ with the coefficients of shear viscosity η and of the "second" or compressional viscosity η'. Combination of the second law of thermodynamics with the equation of state $S=S(T, V, E)$ – relating the entropy S, the absolute temperature T, the volume V, and the electric field E – leads to the following power equation, where an absorption term for the electromagnetic (em) energy flux $F=(nc/4\pi)E^2$ has been included:

$$\rho C_v \frac{dT}{dt} - \frac{C_v(\gamma-1)}{\beta}\frac{d\rho}{dt} + \frac{\rho T}{4\pi}\left(\frac{\partial\epsilon}{\partial T}\right)_\rho E\frac{dE}{dt} = \kappa\nabla^2 T + K_{\text{abs}}F + \varphi_\eta, \quad (3)$$

where C_v (C_p) is the specific heat at constant volume (pressure), $\gamma=C_p/C_v$, β is the coefficient of thermal expansion, κ is the thermal conductivity, K_{abs} is the linear absorption coefficient in cm^{-1} for the em energy flux F, and φ_η is the viscous dissipation function defined by Hunt.[6]

The refractive index n is, in the present case, affected by the changes in density of the medium and by the high-frequency Kerr effect. Moreover, the coupling between the em and induced vibrational waves leads to a nonlinear polarization term in the wave equation for the electric field.[7] The real part of the resulting susceptibility, χ'', leads to positive or negative gain for the em wave, depending on whether χ'' is negative or positive. For a Raman-active medium, the equation for the Stokes refractive index n_S may thus be expressed as

$$n_S = n_S^{(0)}\rho/\rho^{(0)} + (\mu/K_S)E_S^2 + n_2E^2, \quad (4)$$

where $\mu/K_S=2\pi\chi_R'/n_0$, K_S is the Stokes wave number, E^2 is the sum of the squares of the laser and Stokes fields E_L and E_S, respectively, n_2 is the Kerr constant in esu, and $n_S^{(0)}$ and $\rho^{(0)}$ are the Stokes index and the density of the field-free medium. There is, of course, a similar equation for the laser index n_L. The transport equation for the em flux F in a medium with an inhomogeneous refractive index n follows from the second eikonal equation of geometric optics.[8] Neglecting diffraction effects, we obtain to first order that for the flux traveling in the z direction $\partial^2F/\partial z^2=-(F/n_0)\times\nabla_\perp^2 n$, where $\nabla_\perp^2$ is the transverse Laplacian.

The above equations are linearized by writing the dependent variables as $\rho=\rho^{(0)}+\rho^{(1)}$, $T=T^{(0)}+\theta^{(1)}$, $n=n^{(0)}+n^{(1)}$, $E=E^{(0)}+E^{(1)}$. The first terms on the right of these expressions denote constant reference values, while the terms with superscript (1) signify small deviations from these levels. Since the perturbations are small, the "acoustic" velocity $\vec{v}$ may also be regarded as a first-order quantity, i.e., small compared to the speed of sound in the undisturbed medium. Further, we assume an infinite medium, and represent the perturbations by two-dimensional infinite plane waves of the form $\exp\{i(K_x x+K_z z-\omega t)\}$. The tacit assumption made here is that the phase discrepancy between $E_L^{(1)}$ and $E_S^{(1)}$ caused by instability growth can be compensated by the phase uncertainty of $E_S^{(1)}$ due to the finite Raman linewidth. (A detailed justification for this assumption will be given in a subsequent paper.) The slab-shaped laser beam considered here is linearly polarized and incident along the z axis. For small-angle forward scattering, the vibrational waves will be practically transverse, and consequently $|K_z| \ll |K_x|$. At $z=0$, we postulate a plane boundary so that K_x is to be real. This boundary condition implies that K_x must exceed the acoustic damping coefficient, α_x. For viscous losses, $\alpha_x=\alpha_0\omega^2\approx\alpha_0(c_0K_x)^2$, where c_0 is the speed of sound in the undisturbed medium and $\alpha_0=\frac{1}{2}(2\eta+\eta')/\rho^{(0)}c_0^3$. Typically, $\alpha_0\sim10^{-15}$ sec^2 cm^{-1} and hence the corresponding restriction on K_x is $(K_x)_{\max}\approx10^4$ cm^{-1}.

By way of (2), (3), (4), and the continuity equation $d\rho/dt=-\rho\nabla\cdot\vec{v}$, the following momentum and energy equations are now easily derived:

$$\left\{\omega^2-\frac{c_0^2}{\gamma}K^2-\frac{(n_0^2-1)}{8\pi\rho}K^2\alpha^2\chi+\frac{\eta'+2\eta}{\rho}K^2i\omega\right\}\rho^{(1)}-\frac{c_0^2}{\gamma}\beta\rho K^2\left\{1+\frac{E_0^2}{8\pi}\left(\frac{\partial\epsilon}{\partial T}\right)_\rho\frac{\gamma}{c_0^2\beta\rho}\right\}\theta^{(1)}=0, \quad (5)$$

$$i\omega\left[\frac{C_v(\gamma-1)}{\beta}+\left\{\frac{cn_0}{i\omega}K_{\text{abs}}+\tfrac{1}{2}T_0\left(\frac{\partial\epsilon}{\partial T}\right)_\rho\right\}\frac{\alpha^2\chi}{4\pi\rho}\right]\rho^{(1)}+(K^2\kappa-i\omega\rho C_v)\theta^{(1)}=0, \quad (6)$$

with the notations

$$\chi=\left[E_0{}^2-\frac{\mu}{n_0K_0\alpha^2}E_L{}^{(0)2}E_S{}^{(0)2}\right]\left[\alpha^4+\frac{n_2}{2n_0}\alpha^2E_0{}^2-\frac{\mu^2\xi}{4n_0{}^2K_0{}^2}E_L{}^{(0)2}E_S{}^{(0)2}\right]^{-1}, \tag{7}$$

and the approximations $n_L{}^{(0)}=n_S{}^{(0)}=n_0$, $K_L=K_S=K_0$. The dispersion relation now follows from the consistency condition for nonzero solutions of $\rho^{(1)}$ and $\theta^{(1)}$. The detailed analysis of this relation is laborious, and to curb the length of this note we will ignore the dissipative terms and only present semiquantitative results for appreciable (~1%) and for negligible Stokes conversion, respectively. A more complete treatment will be presented in a forthcoming paper, now in preparation.

If there is appreciable Stokes conversion, consider the special case of an incompressible liquid (corresponding formally to the limit $c_0{}^2\to\infty$, $\gamma=1$, and $\beta=0$). Invoking the condition $|K_x|\gg|K_z|$, we deduce the approximate equation

$$K_z{}^4+\delta K_z{}^2-A=0, \tag{8}$$

where $\delta=(n_2K_x{}^2/2n_0)E_0{}^2$, $A=\xi\mu^2E_S{}^{(0)2}E_L{}^{(0)2}K_x{}^4/4n_0{}^2K_0{}^2$. The solution for maximum growth is seen to be $\mathrm{Im}K_z=-\frac{1}{2}[\delta+(\delta^2+A)^{1/2}]^{1/2}$. An estimate for A is obtained by making recourse to the Stokes and laser gain equations[7] $g_{S,L}=2\pi\omega_{S,L}\chi_R{}''E_{L,S}{}^{(0)2}/nc$. Under the assumption that $\omega_L-\omega_S$ differs from the Raman frequency ω_R by half the Raman linewidth, $\chi_R{}'=\chi_R{}''$, and hence $A\approx 2\xi K_x{}^4g_Sg_L/n_0{}^2K_0{}^2$.

For benzene $\chi_R{}''\sim 2\times10^{-12}$ esu and $n_2=49\times10^{-13}$ esu. With $\omega_L\approx\omega_S\sim 2\times10^{15}$ sec^{-1}, it therefore follows that for a laser beam of power P_L MW/cm^2, the expected e-folding length is approximately $(2/P_L)^{1/2}$ cm when the Stokes conversion has reached about 1%.

If the Stokes intensity may be neglected, the dispersion relation is greatly simplified, and a more revealing analysis becomes possible. The qualitative point should be made here that electrostriction and thermal energy deposition can lead to instability growth, because for these effects there is a time lag in the medium's response, due, for instance, to the finite hydrodynamical relaxation time. The Kerr effect, being of electronic origin, has no such time lag associated with it, and leads to stable oscillations, as is apparent from an analysis of the dispersion relation. It cannot, therefore, by itself cause instability growth. However, as will be evident from the subsequent results, the Kerr effect can enhance such growth, once it has been established.

Consider the cases of the Kerr effect combined with either electrostriction or thermal energy deposition. If the departure from the relation $\omega=c_0K$ due to the perturbing terms is not too large, the approximate dispersion relation for the former case becomes

$$\omega=c_0K\left\{1+\frac{1}{2}\frac{E_0{}^2(n_0{}^2-1)}{8\pi\rho c_0{}^2}\frac{(|\alpha|^2\cos2\varphi+\sigma)-i|\alpha|^2\sin2\varphi}{(|\alpha|^2\cos2\varphi+\sigma)^2+|\alpha|^4\sin^22\varphi}\right\}, \tag{9}$$

where we have set $K_z=|K_z|e^{i\varphi}$, and $\sigma=n_2E_0{}^2/2n_0$. Temporal growth is governed by the imaginary part of ω, say $\mathrm{Im}\omega=\mathcal{E}$, and for optimal choice of φ,

$$\mathcal{E}=\frac{E_0{}^2(n_0{}^2-1)}{16\pi\rho c_0K_x}\frac{|K_z|^2}{||\alpha|^4-\sigma^2|}.$$

At $|\alpha|^2=\sigma$, the growth rate becomes infinite, indicating the importance of the Kerr effect in instability growth. The visco-thermal damping terms have been neglected here, and this phenomenon is closely analogous to the classical resonance infinity of an undamped forced harmonic oscillator. The corresponding spatial growth is given by $\mathrm{Im}K_z=-|K_z|(|\alpha|^2+\sigma)/\{2(|\alpha|^4+\sigma^2)\}^{1/2}$. With appropriate data for benzene, and $|K_z|=1$, $K_x=10^4$ cm^{-1}, these results predict an e-folding in time of about 10^{-9} sec, and in space of about 1 cm, for laser power 1 MW/cm^2.

Similarly, for thermal energy deposition and the Kerr effect,

$$\mathcal{E}=\left(\frac{cn_0{}^{\beta K}\mathrm{abs}}{8\pi\rho C_p}\right)\Big/||\alpha|^2-\sigma|,\quad \mathrm{Im}K_z=-|K_z|.$$

Adopting the above values for $|K_z|$, K_x, and P_L, and setting $K_{\mathrm{abs}}=10^{-3}$ cm^{-1}, we obtain

a growth rate for benzene of about 3×10^{7} sec^{-1}.

In summary, it has been demonstrated that the light-medium coupling can lead to rapidly growing instabilities, and hence to large density fluctuations. Any small-scale structure in the laser beam, either present initially or caused by inhomogeneities in the medium, will thereby be enhanced, resulting in large intensity changes and so in anomalous gain.

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EFFECT OF SPIN-ORBIT SCATTERING ON THE UPPER CRITICAL FIELD OF HIGH-FIELD SUPERCONDUCTORS

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We present experimental evidence for the effect of spin-orbit scattering on the bulk upper critical field, H_{c2}, of high-field superconductors. We have measured the temperature variation of H_{c2} for three concentrated titanium alloys, viz. Ti-58 at.% V, Ti-44 at.% Nb, and Ti-52 at.% Ta. For each of these alloys the Pauli spin paramagnetism (PSP) should limit H_{c2} to a value lower than that predicted by the Ginzburg-Landau-Abrikosov-Gor'kov (GLAG) theory. It is found, however, that the measured values of H_{c2} are substantially higher than those predicted by Maki's theory[1] for the effects of the PSP on H_{c2}. More significantly, the observed deviation of the data from Maki's predictions shows a systematic trend, namely, the higher the atomic number (Z) of the column-V constituent [V(23) → Nb(41) → Ta(73)], the greater is the deviation. This trend is the first evidence to support the recent theoretical conjecture[2,3] that spin-orbit scattering counteracts the effect of PSP on H_{c2}. Values of the spin-orbit scattering times, deduced from the data using the theory of Werthamer, Helfand, and Hohenberg[2] (WHH), are found to decrease rapidly with increasing Z, as expected,[4] and are also in order-of-magnitude agreement with theoretical estimates.

It has been pointed out independently by Clogston[5] and by Chandrasekhar[6] that the PSP places an upper limit on H_{c2}. Subsequently Maki[1] showed, by a detailed calculation, that the PSP lowers the upper critical field at temperature T from the GLAG value $H_{c2}^{*}(T)$ to a value $H_{c2}(T)$, and that it also modifies the temperature variation of H_{c2}. Kim, Hempstead, and Strnad[7] and the present authors[8,9] have shown experimentally that while $H_{c2}(0)$ is lower than $H_{c2}^{*}(0)$, it is higher than predicted by Maki's theory. Moreover, it was found[8,9] that the effect of the PSP on the temperature variation of H_{c2} is smaller than predicted by Maki. To explain these findings WHH[2] and Maki[3] have suggested that spin-orbit scattering, which was not included in Maki's original treatment,[1] must be considered. In particular, they have shown theoretically that spin-orbit scattering counteracts the effects of PSP on H_{c2}. The present work was undertaken for the purpose of testing the validity of this explanation. In the theories of WHH and Maki, the spin-orbit scattering is characterized by a parameter which involves the spin-orbit scattering time τ_{s}. Since the magnitude of τ_{s} is not known accu-