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$K \rightarrow 2\pi$ DECAY AND SU(3) SYMMETRY

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It is well known^{1,2} that all two-pion decay modes of the K meson are forbidden in the limit of exact SU(3) symmetry if the decay Hamiltonian is of the current-current form, and if octet dominance is assumed. We shall show in this brief note that these decay modes remain forbidden if the requirement of octet dominance is relaxed, and the decay Hamiltonian is allowed to contain a part that transforms like the 27-dimensional representation. Thus both the $K_1^0 \rightarrow 2\pi$ and $K^\pm \rightarrow 2\pi$ decays occur through SU(3) breaking, and the small K^\pm decay rate can perhaps be accounted for by a small 27 part. Since both decay modes are suppressed, an octet enhancement relative to the 27 on the order of a factor 25 is required to explain the ratio of the two decay rates rather than the much larger factor that would be required if the 27 were not also forbidden in the exact symmetry

limit. We should like to note that the observed decay rates may be attributed principally to the large $K-\pi$ mass difference, which can give rise to a large kinematical SU(3) breaking of otherwise symmetrical amplitudes.

We assume that the nonleptonic decay Hamiltonian has the current-current structure

$$\mathcal{H}_w = \frac{1}{2} \{J, J^\dagger\},$$

where the current, J , transforms like³

$$j = (\lambda^1 + i\lambda^2) \cos\theta + (\lambda^4 + i\lambda^5) \sin\theta,$$

and its adjoint, J^\dagger , transforms like $j^\dagger = j^T$. Since J and J^\dagger occur symmetrically in the decay Hamiltonian, the SU(3) invariants that represent its matrix elements must be invariant under the interchange of j and j^T . This restricts \mathcal{H}_w to transform as a combination of a unitary

singlet, an octet, and a 27-dimensional representation. Since the parity-nonconserving part of the Hamiltonian is responsible for $K \rightarrow 2\pi$ decay, CP conservation requires that the decay amplitude $A^{abc}(p_a^2, p_b^2, p_c^2)$ be odd under charge conjugation. Bose statistics requires that this amplitude remain invariant if any of the SU(3) indices a , b , or c are interchanged together with the interchange of the corresponding momenta p_a , p_b , or p_c . All the meson masses are equal in the exact-symmetry limit, $p_a^2 = p_b^2 = p_c^2 = m^2$, and the decay amplitude A^{abc} is completely symmetrical in the indices a , b , c . In this limit it can be expanded in terms of the following complete set⁴ of SU(3) invariants:

$$\text{tr} \lambda^a \lambda^b \lambda^c \{j, j^T\} + \text{perm.}, \quad (1)$$

$$\text{tr} \lambda^a \lambda^b \text{tr} \lambda^c \{j, j^T\} + \text{perm.}, \quad (2)$$

$$\text{tr} \lambda^a \lambda^b j \text{tr} \lambda^c j^T + \text{tr} \lambda^a \lambda^b j^T \text{tr} \lambda^c j + \text{perm.} \quad (3)$$

We have indicated by +perm. the additional terms necessary to give complete symmetry in the indices a, b, c . All of these SU(3) invariants are even under the charge-conjugation operation which takes each of the λ matrices into its transpose λ^T . Since the decay amplitude must be odd under this operation, it must, in fact, vanish.

The SU(3)-invariant decay amplitude $A^{abc}(p_a^2, p_b^2, p_c^2)$ does not vanish if the momenta take on their physical values, say $p_a^2 = p_b^2 = m_\pi^2$, $p_c^2 = m_K^2$. Bose statistics now requires only that the amplitude be symmetrical in the two pion indices a and b , and charge-conjugation invariance is maintained by two independent SU(3) invariants which we take to be

$$\text{tr} \{[\lambda^a, \lambda^b], \lambda^c\} \{j, j^T\}, \quad (4)$$

and

$$\text{tr}([\lambda^a, \lambda^c] j \lambda^b j^T + [\lambda^a, \lambda^c] j^T \lambda^b j), \quad (5)$$

where the parentheses in the latter indicate symmetrization of a and b . Invariant (4) corresponds to the pure octet part of the decay Hamiltonian, while (5) contains a $\underline{27}$ part as well as an octet contribution. This kinematical breaking of SU(3) invariance might well be large, perhaps on the order of the mass difference $m_K^2 - m_\pi^2$ divided by a characteristic mass of $(1 \text{ BeV})^2$, or a factor of $\frac{1}{4}$. There is, of course, in addition, a symmetry breaking of a more dynamical nature that involves SU(3) structures that are not invariants, but that transform like λ^8 . This additional symmetry breaking, however, does not give rise to any additional structure for the 2π decay modes of the K , since the kinematical breaking already yields both $\Delta T = \frac{1}{2}$ and $\Delta T = \frac{3}{2}$ terms.

We have enjoyed conversations with M. A. B. Bég and N. Cabibbo.

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