PHOTOPRODUCTION OF MUON PAIRS: A TEST OF QUANTUM ELECTRODYNAMICS*

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(Received 11 August 1966)

An experiment on the photoproduction of muon pairs from carbon has been performed at the Cambridge Electron Accelerator using a 5.2-BeV bremsstrahlung beam. Earlier results of the experiment gave the branching ratio for the decay of rho mesons into muon pairs.[1] In the present note we compare a considerably expanded selection of the data with the predictions of quantum electrodynamics (QED).

In this experiment muons were identified by their ability to penetrate iron. Muon pairs were detected in coincidence, with one member on each side of the γ beam. Each muon had an energy between 1.8 and 2.4 BeV, measured in five equal intervals; a polar angle between 4.2° and 10.9°, measured in nine equal intervals; and was in an azimuthal interval of 42°, measured in 7 equal intervals, centered about the horizontal plane. The experimental equipment has been described previously.[1,2]

Experimental data were corrected for target-out rates, chance rates, geometrical and electronic efficiencies, dead-time losses, Coulomb-scattering losses, and background due to π-pair production. The backgrounds originating from π pairs were assumed to arise mainly from ρ^0 production as is indicated by previous measurements[3,4] and by data in this experiment.[1,5] The uncertainty in the correction for backgrounds due to the π pairs is a major source of our error.

The theoretical expressions used for the π-pair backgrounds and for the μ-pair yield from ρ^0 decay are those described in an earlier Letter.[1] The Bethe-Heitler theory used has also been previously discussed.[6]

The theoretical yields can be shown to have the form $Y_{\text{theory}} = Y_{\text{BH}} + BY_\rho$, under the assumption that charge conjugation is a good quantum number,[7] where B is the ρ^0 branching ratio for muon pairs compared with pion pairs, and Y_ρ is the ρ^0-to-two-muon yield for a branching ratio of 1. B is treated as an adjustable parameter because there is as yet no external measurement of B. Early simple models of breakdowns in QED[8] suggested deviations from theory linearly proportional to $q_\mu{}^2$, where q_μ is the four-momentum transferred to the virtual muon. Hence, we include a term $\beta|q_\mu|^2 Y_{\text{BH}}$ in the theory, where β is an adjustable parameter which, if different from zero, indicates a breakdown to QED. Also in order to allow for a difference in normalization between experiment and theory, we include an adjustable constant A. This puts the theory in the form $Y_{\text{theory}} = A(\{1+\beta|q_\mu{}^2|\}Y_{\text{BH}} + BY_\rho)$. Finally, it is convenient for comparison with other work to express the theory as a ratio, R_{theory}, to the Bethe-Heitler theoretical yields. Thus

$$R_{\text{theory}} = A(1+\beta|q_\mu{}^2| + B\{Y_\rho/Y_{\text{BH}}\}). \qquad (1)$$

In this expression A and β are constants which are directly meaningful only if a linear breakdown model is assumed. Since in a least-squares fit the resulting value of A is strongly model dependent, we will refer to A as the "extrapolated normalization constant" in order to distinguish it from a model-independent normalization measurement.

We compare the theory of Eq. (1) with the experimental yields, where the latter are also divided by $Y_{\text{BH, theory}}$. This latter ratio

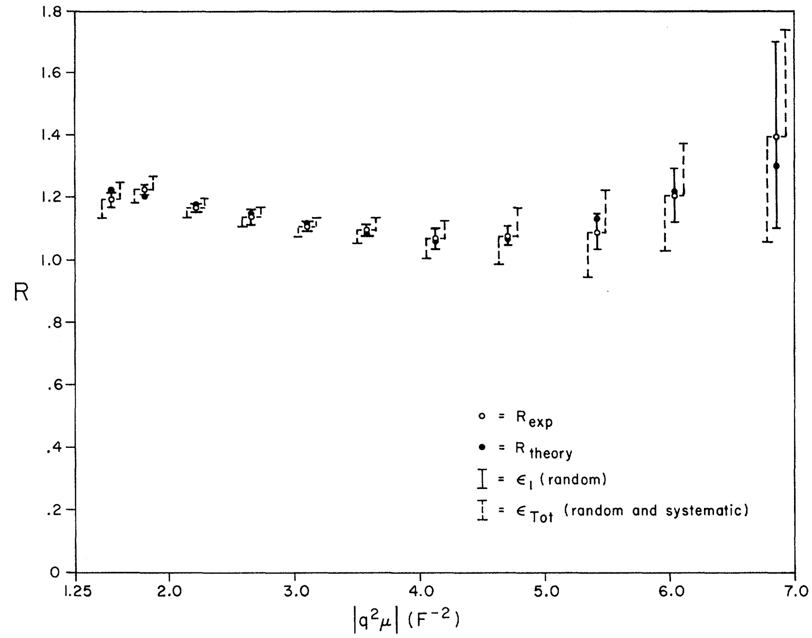

FIG. 1. Upper energy data and best theoretical fit. The best fit of Eq. (1) and the values of R_{expt} are shown for the case in which both muons are restricted to $2040 < E_\mu < 2400$ MeV. The errors shown as solid lines are statistical errors. The errors shown as dashed lines are total errors.

is denoted as R_{expt}. The adjustable parameters A, β, and B are determined by a best fit of experiment to theory.

All of our comparisons of R_{theory} to R_{expt} in this note are restricted to events such that $0.0 < |q_N| < 0.4$ F^{-1}, and $\Delta |q_\mu^2| / |q_\mu^2| < 0.4$. q_N is the four-momentum transferred to the nucleus. The restriction on q_N reduced the uncertainty in R_{expt} due to the inelastic and pion backgrounds. There are two possible values of q_μ^2 when the event is asymmetric, depending upon which muon has scattered from the nucleus. $\Delta |q_\mu^2|$ is the difference between these two ways of calculating q_μ^2. The restriction on $\Delta |q_\mu^2|$ insured that q_μ^2 was fairly well defined. These preselections utilize approximately 50% of our data.

The best fit of R_{theory} and the values of R_{expt} are shown in Fig. 1. This plot includes only events in which each charged particle stops in one of the last three of our five observed energy intervals. Hence, the energy of each muon is in the range $2040 < E < 2400$ MeV. We denote this range as upper energy, E_U, and the range in which each of the muons stops in one of the first two energy intervals ($1800 < E < 2040$ MeV) as lower energy, E_L. Data in which either muon stops in any of the five energy intervals is denoted by "sum E." Restriction of the data to E_U reduces the effect of backgrounds due to π pairs and their decay products. Results based on E_L and sum E will also be reported here, but these have significantly larger errors.

In Fig. 1 each value of R_{expt} is shown with two errors, one purely random, and the other the total error compounded from random and systematic errors. It is evident that the experimental error is dominated by systematic errors. The various contributions to the total error of Fig. 1 are given in Table I. ϵ_1 is the random error due to counting statistics, ϵ_2 is the error due to the uncertainty in the form factor[9] and in the inelastic yields,[6] ϵ_3 is the error in the pion background,[5] ϵ_4 is the error due to correcting the data for particles which multiple scatter in the iron and are not detected by trigger counters, and $\epsilon_{total}^2 = \epsilon_1^2 + \epsilon_2^2 + \epsilon_3^2 + \epsilon_4^2$.

In order to get a best fit, we weighted the experimental points according to the errors ϵ_{total} since these errors more accurately weighted the relative importance of each value of R_{expt} than did the random errors alone. In order to obtain the best fit, we assumed that the error in each value of R_{expt} was uncorrelated

Table I. Values of R_{theory}, R_{expt}, and associated experimental errors. Symbols are defined in the text.

$\lvert q_\mu^2\rvert$ (F^{-2})	R_{expt}	ϵ_1	ϵ_2	ϵ_3	ϵ_4	ϵ_{total}	R_{theory}
1.530	1.195	0.019	0.009	0.001	0.053	0.057	1.221
1.794	1.222	0.015	0.011	0.001	0.033	0.038	1.201
2.224	1.165	0.013	0.010	0.002	0.025	0.029	1.170
2.665	1.137	0.025	0.009	0.005	0.018	0.033	1.139
3.106	1.108	0.017	0.013	0.011	0.013	0.027	1.110
3.581	1.092	0.020	0.020	0.023	0.011	0.038	1.085
4.132	1.065	0.028	0.022	0.047	0.011	0.060	1.065
4.704	1.074	0.032	0.018	0.080	0.011	0.089	1.073
5.406	1.089	0.057	0.021	0.122	0.013	0.137	1.127
6.043	1.207	0.084	0.022	0.143	0.016	0.169	1.212
6.850	1.394	0.298	0.023	0.151	0.020	0.336	1.303

with the error in neighboring values of R_{expt}, and did a least-squares fit. The resulting best values of the parameters for the fit of Fig. 1 (E_U data) are $A = 1.34$, $\beta = -0.056$ F^2, and $B = 0.33 \times 10^{-4}$.

Because of the large effect of systematic errors, the final calculated errors in the parameters might have been underestimated if they had been determined by the usual methods for analyzing random errors. Instead we used a procedure which allowed what we considered to be reasonable systematic correlations in the errors. In order to get a measure of the effects of correlations in the errors, we made additional least-squares fits to values of R_{expt} which had been perturbed from their actual values in a correlated manner which represented the systematic effects. We took the resulting perturbed values of the parameters as a measure of the systematic errors in the parameters. We considered several possible error correlations in order to obtain a measure of the effect they had upon the data. We found that the errors in the fitted parameters were fairly insensitive to the exact model chosen. We chose, finally, error correlations which seemed most reasonable to us in the light of the physical sources of the errors and our technique of measurement. These will be described fully in a later publication. The allowances for correlations in the errors increased the total errors in the measured parameters by factors varying from 1.2 to 1.5 for various energy sorts of the data.

In addition to the systematic errors already mentioned, there was an over-all normalization error of 8.6% in the muon pair rate and an underestimation of β by 0.005 F^2 due to approximations made in calculations of Coulomb-scattering corrections to Y_{BH}.[5]

The final random errors in the parameters A, β, and B were determined by making an additional fit of R_{theory} to R_{expt} using only the random errors, ϵ_1, as the error in R_{expt}. The random errors in the parameters were then computed by the usual techniques for making a fit to data points which have uncorrelated errors.

Including all known sources of errors, the resulting values of A, β, and B[10] for the upper energy data are:

$$A_U = 1.34 \pm 0.14 \ (\pm 0.03),$$

$$\beta_U = -0.051^{+0.018}_{-0.017} \ (\pm 0.007) \ \text{F}^2,$$

$$B_U \times 10^4 = 0.33 \pm 0.15 \ (\pm 0.08), \tag{2}$$

where the systematic errors are given first, and the random statistical errors are enclosed in parentheses. The statistical errors correspond to one standard deviation, and the systematic errors correspond to our best estimate of one standard deviation. As discussed earlier, A is an extrapolated normalization, assuming a linear breakdown model. We have also calculated an "average experimental normalization" from those points of Fig. 1 which have less than a 2% ($\rho \to 2\mu$) contamination, as independent measures of the normalization. This average normalization is 1.14 ± 0.10.

If the negative value of β is to be assigned to an error in our backgrounds, we note that the only known background which is possibly large enough to produce such an effect is the background due to π pairs. A measure of the sensitivity of the slope to an error in this background is given by the result that a zero slope is produced when the fit is made with the assumption that there is no background due to pion pairs. In making the π-pair background correction in the actual fit, we have used the measured cross sections for pion pair production and have assumed a somewhat larger error then appears in the literature[3] (~25%). We feel that an error in this background cannot account alone for the negative slope.

Since inelastic effects are often a concern, we note that the inelastic cross sections used[6] are about 8% of σ_{BH} at the highest values of $|q_\mu^2|$ and are negligible at the lowest values. If we assume that there is no inelastic cross section, the experimental value of $|\beta|$ in Eq. (2) would be reduced by about $\frac{1}{4}$. If we have underestimated the inelastic cross sections, then the slope of β becomes more negative than that given by Eq. (2). We do not believe, therefore, that an error in the inelastic contributions can account for the value of β.

For the case in which each muon stops in one of the lower two of our five energy intervals, the results are

$$A_L = 1.10 \pm 0.15\ (\pm 0.03),$$

$$\beta_L = +0.029^{+0.070}_{-0.052}\ (\pm 0.016)\ \mathrm{F}^2.$$

The data in the lower energy interval include few events in the region of the rho mass, and hence B is not well determined. Fits to the data with B as an undetermined parameter produced negative values of B with, however, errors which leave these results in agreement with the results from our other data. In order to determine the values of A_L and β_L, we fixed B to be our best value of 0.33×10^{-4}. The errors in A_L and β_L quoted above include the effects of the uncertainty in B. The result for the lower energy data is consistent with a zero value for β, but has a large error compared with the upper energy result for β. The results for the upper and lower energy sorts are reasonably consistent internally as shown by the differences

$$A_U - A_L = 0.24 \pm 0.21,$$

$$\beta_L - \beta_U = (0.080 \pm 0.058)\ \mathrm{F}^2.$$

It should be noted that the average photon energies corresponding to the two batches of data differ by about 15%.

The results from the data which included events from all of our energy bins are

$$A_{\mathrm{sum}} = 1.15^{+0.14}_{-0.13}\ (\pm 0.01),$$

$$\beta_{\mathrm{sum}} = +0.002^{+0.036}_{-0.032}\ (\pm 0.005)\ \mathrm{F}^2,$$

$$B_{\mathrm{sum}} \times 10^4 = 0.06^{+0.23}_{-0.07}\ (\pm 0.06).$$

These data, which include E_U and E_L data, also include events in which one member of the pair falls into E_U and the other into E_L. This latter type of event accounts for an amount of data comparable with the E_L and E_U data put together. Events in which at least one member of the pair falls into E_L account for about $\frac{3}{4}$ of the data used in the sum-E fit. Note that the error in the E_U fit is smaller than in the fit utilizing all energies, despite the fact that the latter sort contains much more data. This is simply a reflection of the dominance of the systematic errors which are smallest in the E_U data.

We may compare the results of Eq. (2) with previous experiments. The comparison for experiments not very similar to this one is most easily done[8] in terms of the "breakdown distance," $1/\Lambda_\mu$. For the E_U data of this experiment, $|1/\Lambda_\mu^2| = (0.16^{+0.05}_{-0.08}\ \mathrm{F})^2$ for two standard deviations. The Frascati experiment[11] on muon pair production gave $|1/\Lambda_\mu^2| < (0.23\ \mathrm{F})^2$ for two standard deviations, which is consistent with the present work. The $g-2$ experiment[12] yields $|1/\Lambda_\mu^2| < (0.1\ \mathrm{F})^2$ for two standard deviations, if the deviation from theory is entirely attributed to the muon propagator.[13] This is consistent with our results. The previous experiment on the photoproduction of muon pairs in this energy range[6] gave a result of $|1/\Lambda_\mu^2| < (0.16\ \mathrm{F})^2$ for two standard deviations. It should be noted that the analysis of that experiment was based upon theoretical estimates[7,14] that the $\rho^0 \to 2\mu$ contribution would be unobservable. It was later demonstrated[2] that this ρ^0 decay requires a sizable correction. We cannot accurately reanalyze the earlier experiment with our present programs because of the different geometry of the two experiments. However, approximate corrections of that data for a ρ^0 branching ratio of 0.33×10^{-4} yield $\beta \approx (-0.03 \pm 0.02)\ \mathrm{F}^2$, or $|1/\Lambda_\mu^2| = (0.12^{+0.04}_{-0.05}\ \mathrm{F})^2$ for one standard deviation, in good agreement with the present work.

Following the earlier muon-pair-production experiment, an experiment was done by a Harvard group on electron pair production[15] in approximately the same angular and energy region as the present muon work. The results of a more recent experiment on e pairs done at Cornell[16] are in agreement with the Harvard experiment. Both of these experiments indicate, relative to the Bethe-Heitler theory, a rapid rise of $\sigma_{\rm expt}$ versus either q_e^2 or k^2, where k is the gamma-ray energy. If the deviations in the electron data relative to the Bethe-Heitler theory are interpreted as a slope versus q_e^2 ("q^2 fit"), the results are[15]

$$R_q = 0.67[(1.00 \pm 0.04) + 0.397^{+0.035}_{-0.031} |q_e^2|],$$

where q_e^2 is in units of F^{-2}; if interpreted as a slope versus k^2 ("k^2 fit"), the results are[15]

$$R_k = 0.62[(1.00 \pm 0.05) + k^2/(4.31 \pm 0.17)^2],$$

where k is in BeV.

Considering first the q_e^2 fit of the electron data, we note that the results are in marked disagreement with the present muon results.

Consider next the k^2 fit. We cannot presently compare the slope of the muon experiment with that of the electron experiment interpreted as a function of k^2, because of the limited range of k which we probe. Nevertheless, in a narrow energy region we can compare our data with the k^2 fit of the electron data. In order to be mutually consistent, R_k of the electron data and $R_{\rm expt}$ of the muon data must be equal at the same photon energy and in the same angular range, where $R_{\rm expt}$ is the ratio of muon experimental yields to the predictions of Bethe-Heitler. The published electron pair[15] data gives a ratio of 1.27 ± 0.08 in the energy and angular region of our experiment and a ratio of 0.62 ± 0.03 in the low-energy limit. However, the Cornell data[16] on electron pair production give a ratio consistent with 1.0 in the low-energy limit, and the Harvard data are reported to be consistent with this result[17] if systematic errors are assigned. On the assumption that this is the case, i.e., that the electron data agree with QED in the low-energy limit, the electron ratio R_k in our energy region is raised to $(1.0/0.62) \times 1.27 = 2.05$ with an error of about 10%. Our average experimental normalization of 1.14 ± 0.10, which is equivalent to the experimental value of R_k for muon pair production in this energy and angular range, is in striking disagreement with this. (Our extrapolated normalization is also in marked disagreement with this. However, this is less meaningful, since the extrapolated normalization is, in effect, an extrapolation to $\theta = 0$, while the average experimental normalization compares muon with electron data in approximately the same angular interval.)

We conclude from the present experiment that the behavior of muon pair production is not compatible with the electron-pair results interpreted as a function of q_e^2, nor is it compatible with the electron-pair results interpreted as a function of k^2, if the Harvard ratio, R_k, is set equal to 1.0 in the low-energy limit, in agreement with the Cornell data. The electron and the muon appear to be behaving differently in these experiments, and the present experiment indicates that the muon is better described by QED, or at least by the Bethe-Heitler theory,[18] in this kinematic range. There is, nevertheless, an indication in the muon results of a deviation from Bethe-Heitler theory which is about eight times smaller than the deviation in the electron experiments and of opposite sign.

We wish to express our appreciation to the staff of the Cambridge Electron Accelerator for its supporting effort and for the delightfully steady and reliable beams they provided.

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*Work supported in part through funds provided by the National Science Foundation under Grant No. GP-3389 and in part through funds provided by the Atomic Energy Commission under Contract No. AT(30-1)2098.

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§This material represents part of a thesis submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy to the Physics Department, Northeastern University at Boston, Boston, Massachusetts.

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$K\rightarrow 2\pi$ DECAY AND SU(3) SYMMETRY

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(Received 29 August 1966)

It is well known[1,2] that all two-pion decay modes of the K meson are forbidden in the limit of exact SU(3) symmetry if the decay Hamiltonian is of the current-current form, and if octet dominance is assumed. We shall show in this brief note that these decay modes remain forbidden if the requirement of octet dominance is relaxed, and the decay Hamiltonian is allowed to contain a part that transforms like the 27-dimensional representation. Thus both the $K_1^0 \rightarrow 2\pi$ and $K^{\pm}\rightarrow 2\pi$ decays occur through SU(3) breaking, and the small $K^{\pm}$ decay rate can perhaps be accounted for by a small 27 part. Since both decay modes are suppressed, an octet enhancement relative to the 27 on the order of a factor 25 is required to explain the ratio of the two decay rates rather than the much larger factor that would be required if the 27 were not also forbidden in the exact symmetry limit. We should like to note that the observed decay rates may be attributed principally to the large K-π mass difference, which can give rise to a large kinematical SU(3) breaking of otherwise symmetrical amplitudes.

We assume that the nonleptonic decay Hamiltonian has the current-current structure

$$\mathcal{H}_w = \tfrac{1}{2}\{J, J^\dagger\},$$

where the current, J, transforms like[3]

$$j = (\lambda^1 + i\lambda^2)\cos\theta + (\lambda^4 + i\lambda^5)\sin\theta,$$

and its adjoint, $J^\dagger$, transforms like $j^\dagger = j^{\mathrm{T}}$. Since J and $J^\dagger$ occur symmetrically in the decay Hamiltonian, the SU(3) invariants that represent its matrix elements must be invariant under the interchange of j and j^{T}. This restricts $\mathcal{H}_w$ to transform as a combination of a unitary