NEW DEUTERON-STRIPPING THEORY

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A new theory for deuteron stripping has been recently proposed by Butler[1] and Tanifuji[2] and has been used to fit experimental data by Butler, Hewitt, and May.[3] The purpose of this note is to show that the basic "sudden" approximation of this theory is invalid and should result in a zero direct-reaction matrix element. The previous evaluations[1-3] of the matrix element contain serious and instructive errors which are pointed out.

The theory is described in a many-body framework, although it can be shown that the matrix element vanishes identically in a three-body problem. For the special case of an energy-independent neutron potential this was pointed out by May.[4] However, the theory is meant to describe the direct-reaction component of a many-body reaction and for this we need to define the latter in a many-body theory. Firstly, the previous derivations of the matrix element are critically examined. Then it is shown that two approximations are necessary to obtain the form of the scattering wave function Ψ used in the theory. The first of these is shown to lead to a Born-approximation expression (which contains the original Butler amplitude[5]). The second is the "sudden" approximation and leads to a zero direct-reaction matrix element.

The Schrödinger equation for the system is

$$[H_0+T_n+T_p+V_n+V_p+V_{np}]\Psi=E\Psi, \qquad (1)$$

where H_0 is the Hamiltonian for the target and $V_n(r_n\xi)$ is the full interaction between the neutron and the nucleons in the target. Since we are not interested in the proton's detailed interaction with the target, V_p is taken to be an optical-model potential as in Refs. 1 and 3.

The exact stripping matrix element is then

$$M=C\langle\Psi_p^{(-)}(k_p,r_p)\varphi_f(r_n\xi),V_{np}\Psi\rangle, \qquad (2)$$

where φ_f is the final-state nuclear wave function, $\Psi_p^{(-)}$ is an optical-model wave function with incoming waves at infinity, and C is a constant. As in Refs. 1 and 2, Eq. (1) can be used to rewrite M:

$$M=C\langle\Psi_p^{(-)}\varphi_f,[(E_f-T_n-V_n-H_0)+(E_p-T_p-V_p)]\Psi\rangle=M_1+M_2, \qquad (3)$$

where M_1, M_2 refer to the matrix elements of the two bracketed operators in which E_f and E_p are the energies corresponding to φ_f and $\Psi_p^{(-)}$, respectively. With a given approximate form for Ψ, the two matrix elements (2) and (3) are not identical, since the above transformation can only be made with the exact wave function.

In the theory of Refs. 1-3 the procedure adopted is to approximate Ψ in Eq. (3) and then evaluate M_1 and M_2. The results obtained were that $M_2=0$ and that M_1 gave the final expression to be compared with experiment. Both these results are incorrect, as will now be shown. With the "sudden" approximation form for Ψ the results should be that $M_1=0$ and M_2 is nonzero but small, and does not correspond to the direct-reaction matrix element. We shall find that in the "sudden" approximation the direct portion of M_2 does equal zero.

We first consider M_1 and <u>any</u> approximation for Ψ. Since V_n is the sum of two-body potentials it is Hermitian, and so are T_n and H_0 because φ_f is a bound-state wave function. Operation on the final state clearly gives zero for M_1, independent of the form of Ψ. The reason why a nonzero result is obtained in Refs. 1-3 is that V_n is approximated by an optical-model potential, U_n, defining the neutron wave function in Ψ. Action of the operator $(E_f-T_n-U_n-H_0)$ on the approximate Ψ then gives a finite result. However, it is clear that the contribution of the correction term, U_n-V_n, will cancel this result. This canceling term cannot be neglected as not corresponding to the "direct reaction" matrix element, since it is the choice of the form for Ψ which specifies the "direct reaction" contribution. The matrix element M_1 vanishes separately for <u>all</u> forms of Ψ.

Since M_1 is zero, M_2 contains all contributions to the matrix element and cannot vanish. When the operator $(E_p-T_p-V_p)$ acts on the final state $\Psi_p^{(-)}$ the result is zero. This implies that T_p is not Hermitian between the final state and Ψ. We now follow the evaluation of M_2 by Butler[1,3] to show where the error lies in their evaluation.

A very reasonable definition of a direct reaction can be made by taking only that part of

Ψ in which the target nucleus is in its ground state, $\chi_0(\xi)$. In the sudden approximation[1,2] the neutron and proton scatter independently in their respective optical wells at energies given by the center-of-mass motion of the deuteron and their relative kinetic energy in the deuteron. Thus the approximate form for Ψ is

$$\langle \chi_0 \Psi \rangle = \int d^3k_p' f(\vec{k}_d, \vec{k}_p') \Psi_p^{(+)}(k_p', r_p) \Psi_n^{(+)}(Q', r_n), \quad (4)$$

where $\vec{k}_d$ is the deuteron momentum, $\vec{Q}' = \vec{k}_d - \vec{k}_p'$, and f specifies the momentum distribution in the deuteron. The wave functions are defined by optical-model potentials.

When the operation of $(E_p - T_p - V_p)$ is commuted with the integration over k_p', the matrix element M_2 becomes

$$M_2 = C \int d^3k_p' f(\vec{k}_d, \vec{k}_p') \langle\langle \varphi_f \chi_0 \rangle, \Psi_n^{(+)}(Q', r_n) \rangle \times \langle \Psi_p^{(-)}(k_p, r_p), [E_p - E_p'] \Psi_p^{(+)}(\vec{k}_p', r_p) \rangle, \quad (5)$$

where the k_p' integration must be carried out before the r_p integration. The result should still be nonzero, but if the r_p integration is carried out first the proton matrix element contains a factor $\delta(E_p - E_p')$ so that the result is zero. It is therefore inadmissable to expand Ψ in eigenstates for the proton and interchange orders of integration. The situation is analogous to that in potential scattering.[6]

We now show directly that matrix element (5) has a zero "direct-reaction" component by examining the approximations leading to representation (4). The matrix element M_2, with Ψ projected onto the target ground state, may be written as

$$M_2 = C \langle \Psi_p^{(-)} \varphi_f, (E_p - T_p - V_p) \Psi \rangle - C \sum_{\lambda \neq 0} \langle \Psi_p^{(-)}(k_p, r_p) \langle \varphi_f \chi_\lambda \rangle, [E_p - T_p - V_p] \langle \chi_\lambda \Psi \rangle \rangle, \quad (6)$$

where an intermediate sum over a complete set of target states $|\chi_\lambda\rangle$ has been inserted. The second set of terms depends on the probability that the final nuclear states does not contain the target nuclear state and thus we neglect them for the direct-reaction matrix element.

The exact form for Ψ is

$$\Psi = [1 + (E - T_n - T_p - V_n - V_p - V_{np} - H_0 + i\epsilon)^{-1}(V_n + V_p)] \Psi_0, \quad (7)$$

where Ψ_0 is the initial plane-wave deuteron state. The first approximation made in Refs. 1 and 2 is to neglect V_{np} in intermediate states, i.e., in the denominator in Eq. (7). We may trivially add $E_f - T_n - V_n - H_0$ to the operator in matrix element (6) to show that it then reduces to

$$M = C \langle \Psi_p^{(-)} \varphi_f, V_{np} \Psi_0 \rangle. \quad (8)$$

This is the Born-approximation form with final distortion only. More generally, Tobocman[7] has shown that, if we had projected the deuteron ground state out of Ψ before expanding in powers of V_{np}, the first term would be the full distorted-wave Born-approximation expression.

The further "sudden" approximation made in Refs. 1 and 2 is to expand Ψ_0 in plane waves for the neutron and proton and then, for each individual term, to replace E in the energy denominator in Eq. (7) by $E' = E_0 + E_p' + E_Q'$. Here, E_0 is the energy corresponding to χ_0 and E_p', E_Q' are specified by the kinetic energies of the plane waves. A wave function similar to that given by Eq. (4) is obtained except that it is now no longer projected on χ_0. The components not projected on χ_0 have been neglected in Eq. (6).

The easiest way to show that this approximation gives a zero matrix element is to show that the correction term to it is just the Born-approximation form (8). The correction term contains the operator

$$[E - T_n - T_p - V_n - V_p - H_0][(E - T_n - T_p - V_n - V_p - H_0)^{-1} - (E' - T_n - T_p - V_n - V_p - H_0)^{-1}](V_n + V_p),$$

which reduces to

$$(E' - E)(E' - T_n - T_p - V_n - V_p - H_0)^{-1}(V_n + V_p).$$

We may now act with the inverse operator on the final state so that the factor $(E' - E)$ can-

cels and, after integrating over k_p', we are left with

$$M = C\langle \Psi_p^{(-)} \varphi_f, (V_n + V_p)\Psi_0 \rangle.$$

This is the prior form of the Born approximation and is equal to the post form (8) by Hermiticity. The operator T_p is Hermitian between the final-state wave function and Ψ_0 (in contrast to Ψ), since their overlap is bounded at infinity in the variable r_p. Thus operation of $H-E$ on the two wave functions gives equal prior and post forms of the Born approximation with distorted waves in the final state.

In the above we have avoided integrating over r_p before integrating over k_p'. Thus the "sudden" approximation leads to zero for the direct-reaction amplitude even when M_2 is evaluated correctly.

After making the two approximations, the neglected terms in Eq. (6) are given by Eq. (5). Another way to show that these do not correspond to a direct reaction is to examine the neutron matrix element in Eq. (5). We may regard $\Psi_n^{(+)}(Q', r_n)$ as the projection on χ_0 of the complete wave function $\Psi_i(\xi, r_n)$ for a neutron incident on the target nucleus with wave function χ_0. Both φ_f and Ψ_i then represent eigenstates of $H_0 + T_n + V_n$ with different energies and are thus orthogonal. Introducing the complete set of states χ_λ, we have

$$\langle\langle \varphi_f, \chi_0\rangle, \langle \chi_0, \Psi_i\rangle\rangle = -\sum_{\lambda \neq 0} \langle\langle \varphi_f, \chi_\lambda\rangle, \langle \chi_\lambda, \Psi_i\rangle\rangle.$$

Thus the comments after Eq. (6) apply and M_2, given by Eq. (5), does not represent a direct reaction.

In conclusion we see that the "sudden" approximation for Ψ is invalid for rearrangement collisions when used in matrix elements such as given by Eq. (3). It could be used for Ψ in matrix element (2), but then the transition to Eq. (3) is no longer possible and further evaluation is difficult. The use of the sudden approximation in elastic scattering[8] is not invalidated by the above arguments.

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[6]In potential theory the scattering amplitude is proportional to $\langle [T \exp(-ik\cdot r)]\Psi\rangle - \langle \exp(-ik\cdot r)[T\Psi]\rangle$ as may easily be verified directly. It is then obviously invalid to expand Ψ in the second matrix element in plane waves and reverse orders of integration.

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THRESHOLD PHOTONEUTRON CROSS SECTIONS FOR IRON AND BISMUTH*

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Photoneutron cross sections were measured for iron and bismuth for photon energies up to 15 keV above threshold by the neutron time-of-flight technique. A number of nuclear levels were observed. Nuclear properties which can be derived from the data include ground-state gamma-ray transition widths, gamma-ray strength functions, and average level spacings.

This Letter presents the results of a first attempt to make use of the time-of-flight technique to measure photoneutron cross sections near threshold with good resolution. Such measurements allow one to explore in detail the properties of many individual nuclear levels just above the neutron separation energy in all stable nuclei, and thus to investigate several classes of systematics of the excited nucleus. Also, the multipolarity and strength of the electromagnetic transitions which excite the nuclear states can be determined when the technique is applied properly. Such information is difficult to obtain in the 6- to 10-MeV energy range.

Previous studies of neutron-capture gamma-ray spectra,[1] scattering of nearly monoenergetic photons,[2] and resonance fluorescence with a continuous bremsstrahlung source[3] all suffer from inadequate gamma-ray resolution;