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PROPAGATION OF ZERO SOUND IN LIQUID He^3 AT LOW TEMPERATURES*

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We have observed the propagation of sound in liquid He^3 at 0.32 atm and at frequencies of 15.4 and 45.5 MHz down to a temperature T^* of 2 mdeg on the magnetic temperature scale valid for powdered cerium magnesium nitrate (CMN) in the form of a right circular cylinder with diameter equal to height. As the temperature rises the sound attenuation increases, reaches a maximum, and then decreases. At low temperatures the attenuation is proportional to T^{*2} and is independent of frequency. At high temperatures the attenuation is proportional to ω^2/T^{*2}, where ω is the angular frequency. The sound propagation velocity is relatively temperature independent at both high and low temperatures but near the attenuation maximum the velocity changes.

In 1957 Landau[1] predicted that at sufficiently low temperatures a new type of sound, which he called zero sound, could be propagated in liquid He^3. Based on Landau's idea, a more detailed theory of the velocity and attenuation of sound in both the hydrodynamic (first sound) and zero-sound regions was worked out by Khalatnikov and Abrikosov.[2] At temperatures sufficiently high that quantum effects are unimportant, it is predicted that the attenuation of zero sound be proportional to T^2 and independent of frequency. In the first-sound region it is predicted that the attenuation is proportional to ω^2/T^2, corresponding to classical viscous attenuation with viscosity proportional to T^{-2}. Both of these temperature and frequency dependences are observed in the present experiments. In what follows we shall show that there is quantitative agreement with theory on velocity changes and semiquantitative agreement on attenuation, so that the present experiments clearly demonstrate the phenomenon of the propagation of zero sound.

The first experiments to support strongly the existence of zero sound in He^3 are those of Keen, Matthews, and Wilks,[3] who measured the effective acoustic impedance of liquid He^3 and found a transition from a high-temperature to a low-temperature regime at about 0.092°K for a frequency of 1000 MHz. It was not possible for them to measure the propagation characteristics of the sound nor, as it will be shown here, to deduce correctly by means of theory the velocity change between first and zero sound. However, on the basis of the present experiments there is no doubt that the phenomenon observed by them was indeed the transition from first to zero sound.

Only a few brief remarks on the present experiments can be made here. Most details are thoroughly discussed elsewhere.[4] The cell used for the measurements is illustrated schematically in Fig. 1. The CMN acts both as refrigerant and as thermometer. The sound propagation in the He^3 occurs in a small gap between a fused quartz ultrasonic delay line and a receiving crystal of X-cut quartz of fundamental frequency 15 MHz. The delay line serves to allow the transient in the receiving crystal, caused by electrostatic pickup from the initial pulse at the transmitting crystal, to damp sufficiently before the sound pulse reaches the gap. The gap length used for the experiments reported here was 1.38×10^{-2} cm and was provided by three small quartz flats. The princi-

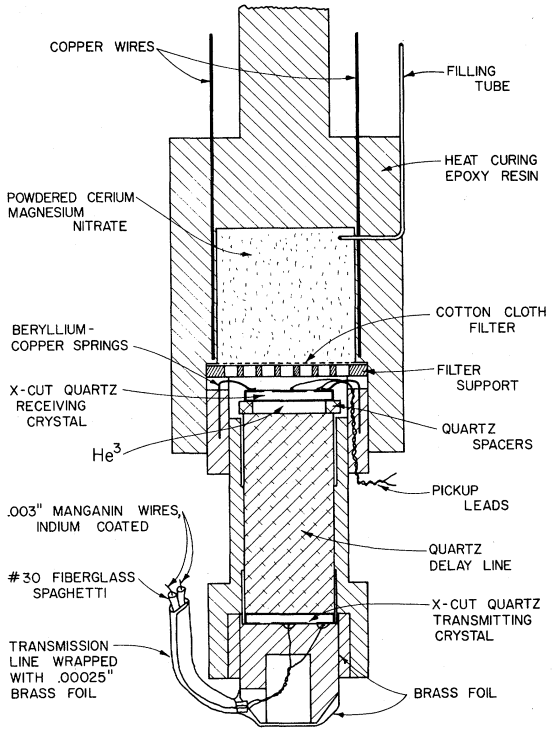

FIG. 1. Schematic drawing of sound cell. Electrical isolation of the receiving from the transmitting circuit was accomplished by wrapping both transmitting and receiving leads with one layer of 0.000 64-cm brass foil, surrounding the bottom of the transmitting crystal with brass foil, and having the top half of the delay line, which was gold plated, electrically grounded to the shields on both the transmitting and receiving transmission lines, which were grounded together. Neither the electrical shield for the receiving circuit nor the gold plating with connecting wire on the quartz delay line is shown in the drawing.

pal experimental difficulty was electrostatic shielding of the receiving from the transmitting circuits, consistent with a very small residual heat leak to the sound cell.

The CMN thermometer used here is of the type which measured the self-diffusion coefficient in He^3 [4,5] to be proportional to T^{*-2} to below 3 mdeg K. Cryogenic conditions were extremely good. Approximately two hours were required to warm from 2 to 3 mdeg K. There were no limitations on the number of data points which could be obtained.

Measurements were made by triggering simultaneously an ultrasonic pulser and a sweep-delay circuit in an oscilloscope. A delayed, expanded sweep, showing the first few cycles of the received pulse at times shorter than those in which multiple reflections in the gap might cause difficulties, was photographed. Attenuation measurements were made by selecting definite parts of the pulse and measuring the amplitude of the oscillations. A calibrating pulse was frequently used to compensate for changes in the over-all gain of the electronic apparatus. Velocity measurements were made by measuring the relative phase between the received sound signal and a "clock" signal, the latter obtained from an apparatus at room temperature similar sonically to the low-temperature apparatus except that there was no gap. Absolute velocities were calculated from the relative phase measurements using the sound velocity measurements of Abel, Anderson, and Wheatley[6] and using the measured thickness of the quartz spacer. Frequencies were determined by measuring the period of the received oscillations as displayed on the oscilloscope. In order to obtain an attenuation coefficient it was necessary to know the zero-attenuation limit for the received signal. This was obtained by plotting the logarithm of the gain-corrected signal height either against T^{*2} at low T or against T^{*-2} at high T and extrapolating to zero.

The results of the measurements are shown on Fig. 2 where both amplitude-attenuation coefficient and velocity are displayed. With the present gap and signal available it was not possible at 45.5 MHz to measure values of α greater than 200 cm^{-1}, which accounts for the absence of high attenuation-coefficient data for this frequency. Attenuation coefficients below about 10 cm^{-1} are particularly subject to error and hence should not be weighted heavily. The straight lines on the attenuation plot are

$$\alpha_0 = A_0 T^{*2}, \tag{1}$$

and

$$\alpha_1 = A_1 \omega^2 T^{*-2}, \tag{2}$$

where $A_0 = 1.57 \times 10^6$ (1/cm $K^{\circ 2}$) and $A_1 = 2.68 \times 10^{-18}$ (sec^2 $K^{\circ 2}$/cm). Values of A_0 and A_1 obtained in different runs are given in Table I. The value of A_0 for run 1 is subject to greater error than the others due to calibrational difficulties.

The velocity measurements on Fig. 2 correspond to an average first-sound velocity c_1 of 187.9 m/sec (by definition from Ref. 6) and an average zero-sound velocity c_0 of 194.4 m/sec for a pressure of 0.32 atm. The measured value of $(c_0 - c_1)/c_1$ is then 0.035 ± 0.003.

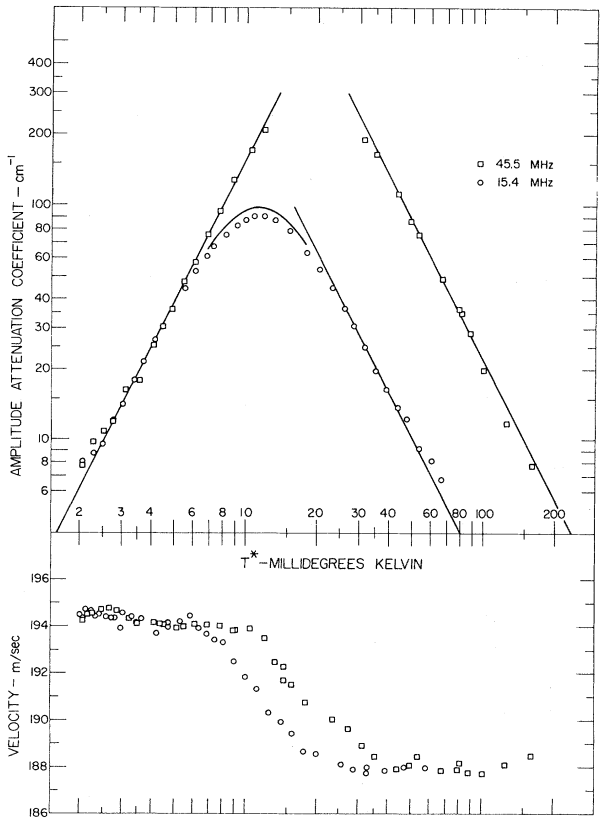

FIG. 2. Amplitude attenuation coefficient and sound propagation velocity as a function of magnetic temperature in pure liquid He^3 at 0.32 atm and for frequencies of 15.4 and 45.5 MHz. Each point shown on the graph is the average of several raw data points. The straight line drawn through the low-temperature attenuation data represents Eq. (1), while the straight lines drawn through the high-temperature data represent Eq. (2), with $\omega/2\pi$ equal to 15.4 and 45.5 MHz. With the present gap it was not possible to measure the 45.5-MHz attenuation coefficient above $\alpha = 200$ cm^{-1}. The smooth curve just above the attenuation data for 15.5 MHz is a plot of Eq. (6) with $\omega/2\pi = 15.4$ MHz and α_0 and α_1 given by Eqs. (1) and (2).

Also important is the temperature T_{max}^* at which one finds maximum attenuation or maximum rate of change with temperature of the velocity. Under the simplest assumptions this temperature corresponds to $\omega\tau = 1$, where τ is the relaxation time for velocity, which is assumed to have a T^{-2} temperature dependence. However, more generally one would expect $\omega^{1/2}/T_{max}^*$ to be a constant at the maximum. Table II shows the results of an analysis of this point, both for the present experiments and for those of Ref. 3. Inspection of the table

Table I. Values of the coefficients A_0 and A_1 of Eqs. (1) and (2) for several runs.

Run	$\omega/2\pi$ (MHz)	A_0 [10^6 (K° ^{2} cm)$^{-1}$]	A_1 [10^{-18} sec^2 K° ^{2}/cm]
1	15.4	1.44	2.74
2	15.4	1.57	2.65
3	45.5	1.58	2.66
4	15.4	1.62	2.65

shows that, in the case of attenuation measurements, in which T_{max}^* is well determined, the frequency dependence is quantitatively verified. It is difficult to estimate T_{max}^* from the velocity measurements, so the scatter is greater. However, it seems clear from these data that the transition observed in Ref. 3 is indeed to be attributed to a transition from first to zero sound.

Our results can also be compared with the theory of Khalatnikov and Abrikosov[2] in which the velocity of zero sound is found to be given by the implicit equation

$$w(s_0) = \frac{1+\frac{1}{3}F_1}{F_0(1+\frac{1}{3}F_1)+s_0^2F_1}, \tag{3}$$

where $w(s_0) = \{(s_0/2)\ln[(s_0+1)/(s_0-1)]\}-1$, $s_0 = c_0/v_F$, and v_F is the Fermi velocity. Recently determined[7] values of the Fermi-liquid parameters are $F_0 = 10.77$, $F_1 = 6.25$, and $v_F = 53.8$ m/sec for a pressure of 0.28 atm (close to the present one). At this pressure $c_1 = 187.2$ m/sec. Solving Eq. (3) for s_0 one finds $s_0 = 3.597$ [and $w(s_0) = 0.027\,033$]. This leads to $[(c_0-c_1)/c_1]_{0.28\ atm} = 0.034$, in remarkable agreement with the measured value at 0.32 atm of 0.035 ± 0.003. Using a theory for energy transfer from a solid to liquid He^3, Keen, Matthews, and Wilks[3] found $(c_0-c_1)/c_1 = 0.10 \pm 0.03$, a larger effect than observed here. Assuming the essential correctness both of the experiments of Ref. 3 and of the present ones, the discrepancy in the value of $(c_0-c_1)/c_1$ must be attributed to an inadequacy of the theory explaining energy transfer into the liquid.

In regard to attenuation, at high temperatures one expects[2]

$$\alpha_1 = \frac{2}{15}\frac{v_F^2}{c_1^3}(1+F_1/3)\omega^2\tau_1. \tag{4}$$

For the values given above for the parameters in Eq. (4), one finds $\tau_1 T^2 = 1.5\times10^{-12}$ sec K°^{2}.

In the zero-sound region one expects[2]

$$\alpha_0=\frac{1}{c_0\tau_0}\left\{1-\left[\frac{1+\frac{1}{3}F_1}{s_0}[1+w(s_0)]-s_0w(s_0)(F_1-3)\right]\left[\frac{1+\frac{1}{3}F_1}{w(s_0)}\left(\frac{s_0}{s_0^2-1}-\frac{1+w(s_0)}{s_0}\right)-2F_1s_0w(s_0)\right]^{-1}\right\}. \quad (5)$$

From the values given above for the parameters in Eq (5), one finds $\tau_0T^2=1.1\times10^{-12}$ sec $K^{\circ 2}$, somewhat less than the value of τ_1T^2. It is possible that τ need not be the same for both first and zero sound.

It should also be possible to relate the attenuation and velocity measurements by a Kramers-Kronig relation, according to the work of Ginzburg.[8] If the high- and low-frequency attenuation coefficients are given by Eqs. (1) and (2), then one can estimate the absorption coefficient α at any frequency by

$$1/\alpha=1/\alpha_0+1/\alpha_1. \quad (6)$$

Using this assumption one finds from Eq. (14) of Ref. 8 that

$$\frac{c_0-c_1}{c_1}=\frac{c_0}{w}(\alpha_0\alpha_1)^{1/2}. \quad (7)$$

Evaluation of the data using Eq. (7) leads to $(c_0-c_1)/c_1=0.040$, about 14% higher than the mean experimental value. The attenuation according to Eq. (6) is shown on Fig. 2, indicating that at 15.4 MHz Eq. (6) overestimates the attenuation by about 10% at the peak. Since the oversimplified attenuation coefficient given by Eq. (6) gives too large an attenuation near the maximum, one would expect Eq. (7) to overestimate the fractional velocity change.

According to our assumptions the measurement of α_1 is equivalent to a measurement of viscosity, since one expects

$$\alpha_1=\tfrac{2}{3}(\omega^2/\rho c_1^3)\eta, \quad (8)$$

where ρ is the mass density and η is the coefficient of viscosity. The present experiments give $\eta T^2=2.3$ μP $K^{\circ 2}$. Using, in principle, the same experimental method, we[6] earlier found $\eta T^2=2.8$ μP $K^{\circ 2}$, about 20% higher. As may be seen from Table I, random error in the present experiments is small compared to 20%. Other quantities which enter in determining the absolute value of η are the frequency and the path length in the helium. The former is known to about 1% while the latter was determined independently by measuring the phase shifts in received sound on varying the pressure and by using values of c_1 versus pressure as determined in Ref. 6. The path length agreed within 2% with that obtained by mechanical measurement. It is possible that a 20% difference in "observed" ηT^2 could be caused by two effects. The first is that in Ref. 6 the scatter in the data was large, certainly of order 10% or more. Secondly, ηT^2 was determined by drawing a T^{-2} line through the low-T data, though the data did not determine this temperature dependence. Any line drawn in this way will overestimate ηT^2. It is interesting that the value of η determined here is in excellent agreement with a calculation[7] of this quantity using Fermi-liquid parameters. A more serious discrepancy on the value of η is with that determined by another method by Betts, Keen, and Wilks,[9] who find $\eta T^2=(3.8\pm0.6)$ μP $K^{\circ 2}$. However, there is a substantial scatter in their data, which go to 0.04°K, and it is possible that ηT^2 is again artificially high as a result of forcing a T^{-2} fit at too high a temperature.

One prediction of Landau[1] with respect to zero sound could not be tested in this experiment. In the present temperature range the attenuation coefficient is proportional to T^2. However, at lower temperatures quantum ef-

Table II. Test of the assumption that at the temperature T_{max}^* of maximum attenuation or of maximum rate of change of velocity the quantity $\omega^{1/2}/T_{max}^*$ is a constant.

	Absorption		Velocity		
$\omega/2\pi$ (MHz)	T_{max}^* (°K)	$(\omega/2\pi)^{1/2}/T_{max}^*$ [10^5 $(\sec^{1/2} K^{\circ})^{-1}$]	T_{max}^* (°K)	$(\omega/2\pi)^{1/2}/T_{max}^*$ [10^5 $(\sec^{1/2} K^{\circ})^{-1}$]	Experimenter
1000			0.092	3.4	a
45.5	0.0193	3.5	0.018	3.7	Present work
15.4	0.0113	3.5	0.011	3.6	Present work

[a]Ref. 3.

fects are important and the attenuation is predicted to be modified, eventually at very low T varying as ω^2 and independent of T. The temperature region at which the change occurs is given by $T=\hbar\omega/2\pi k$, which is approximately 0.34 mdeg K for $\omega/2\pi = 45$ MHz.

Finally, it would be very interesting to observe zero sound in a weakly interacting Fermi liquid where distortion of the Fermi surface could be relatively large compared with pure He^3, where the effective interactions are strong and distortions of the Fermi surface are small. At first sight the dilute solutions of He^3 in He^4 would be ideal for this purpose, second sound in the solution being analogous to first sound in the He^3. However, for zero sound to propagate [see Eq. (3)], one must have approximately $F_0+F_1>0$. Calculation of F_0+F_1 for a 5.0% solution of He^3 in He^4 using the potential suggested by Bardeen, Baym, and Pines[10] gives $F_0+F_1=-0.25$. Hence it appears unlikely that zero sound can be propagated in the weakly interacting Fermi gas in a dilute solution.

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LINEAR INSTABILITY THEORY OF LASER PROPAGATION IN FLUIDS*

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The phenomenon of anomalous Stokes gain,[1] observed when a laser beam travels through a Raman cell longer than a certain critical length, has recently been the subject of intensive discussions.[2-4] In the present note, it is proposed that this anomalous gain is due to rapidly growing spatial and temporal instabilities which arise when intense laser beams interact with the medium. By incorporating the light-medium coupling terms in the linearized hydrodynamical equations of momentum and energy, we will show that for a fluid such instabilities are indeed predicted by the solutions of the appropriate dispersion relation. The process considered here is essentially that of stimulated Brillouin scattering, but with the reverse effect of the sound wave on the laser wave—by the varying induced dipole moment density—taken into account.

The light wave is coupled to the fluid—assumed to be optically transparent, nonmagnetic, and electrically nonconducting—by the mechanisms of electrostriction, thermal energy deposition, and the high-frequency Kerr effect.[5] For a linearly polarized light wave E, the electrostrictive force density $\vec{f}_{es}$ acting on a volume element of uniform composition with dielectric constant ϵ and density ρ is given by

$$\vec{f}_{es}=\frac{\rho}{8\pi}\nabla\left\{E^2\left(\frac{\partial\epsilon}{\partial\rho}\right)_T\right\}-\frac{E^2}{8\pi}\left(\frac{\partial\epsilon}{\partial T}\right)_\rho\nabla T, \qquad (1)$$

where ∇T is the gradient of the temperature. The modified Navier-Stokes equation for a volume element of velocity $\vec{v}$ becomes

$$\rho\frac{d\vec{v}}{dt}=-\nabla p+\vec{f}_{visc}+\vec{f}_{es}. \qquad (2)$$

Here, p is the pressure without the field, and the viscous force density is $\vec{f}_{visc}=(\eta'+\eta)\nabla(\nabla\cdot\vec{v})$