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ALGEBRA OF CURRENTS AND THE K_1^0-K_2^0 MASS DIFFERENCE*

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The K_1^0-K_2^0 mass difference is calculated exactly from some recently suggested nonleptonic decay models by use of the techniques of the algebra of currents. We find $M(K_2^0) - M(K_1^0) = 0.49/\tau(K_1^0)$, which agrees in both sign and magnitude with the recent experiments.

Recently several authors have proposed[1-6] effective Hamiltonians for the nonleptonic hadron decays, which are quite different in structure from the usual current-current model. Through the use of the techniques of the algebra of currents, these models have led to a good description of the nonleptonic decays.[1,2,6] In the present note we shall show, by using the same techniques, that these models also lead to an exact evaluation of the K_1^0-K_2^0 mass difference. Both the sign as well as the magnitude of the mass difference are found to be in good agreement with the experiments. It may be emphasized here that an unambiguous evaluation of this mass difference has not so far been possible with the current-current model.

The second-order self-energy of K_1^0 or K_2^0 due to the weak interaction is given by[7]

$$\Delta E = \mathrm{Re}\frac{(2\pi)^3}{2i}\int d^4x[\langle K_j^0 | T(H_w(x)H_w(0)) | K_j^0\rangle - \langle 0 | T(H_w(x)H_w(0)) | 0\rangle], \quad (1)$$

so that the K_2^0-K_1^0 mass difference is given by

$$\Delta m = \Delta E(K_2^0) - \Delta E(K_1^0) = -\mathrm{Re}(2\pi)^3 i\int d^4x \langle K^0 | T(H_w(x)H_w(0)) | \bar{K}^0\rangle. \quad (2)$$

Since the parity-conserving and the parity-nonconserving parts of H_w do not interfere in Eq. (2), we may consider their contributions separately.

Now, to get at Δm by the techniques of the algebra of currents, we shall start by defining the following amplitude:

$$M_{\mu\nu} \equiv i\int d^4x\, e^{-iqx} \langle K^0 | T(J_\mu(x) J_\nu(0)) | \bar{K}^0 \rangle, \tag{3}$$

where $J_\mu(x)$ stands for the seventh component of the vector or the axial-vector current densities [$\mathfrak{F}_{\mu,7}$ or $\mathfrak{F}_{\mu,7}{}^5$ in Gell-Mann's notation and $(1/2i)(V_{\mu 2}{}^3 - V_{\mu 3}{}^2)$ and $(1/2i)(A_{\mu 2}{}^3 - A_{\mu 3}{}^2)$ in the standard SU(3) tensor notation]. Then integrating by parts we have

$$q_\mu q_\nu M_{\mu\nu} = i\int d^4x\, e^{-iqx} \langle K^0 | T([\partial_\mu J_\mu(x)][\partial_\nu J_\nu(0)]) | \bar{K}_0 \rangle + i\int d^4x\, e^{iqx} \langle K^0 | \delta(x_0)[J_0(x), \partial_\mu J_\mu(0)] | \bar{K}^0 \rangle$$
$$+ q_\nu \int d^4x\, e^{-iqx} \langle K^0 | \delta(x_0)[J_0(x), J_\nu(0)] | \bar{K}^0 \rangle. \tag{4}$$

The first term on the right-hand side of Eq. (4) in the limit $q \to 0$ will be shown to be directly related to Δm. Before this is actually done, we shall discuss each of the terms in Eq. (4) for the cases $J_\mu = V_\mu$ and $J_\mu = A_\mu$ separately. It is simple to see that the second and the third terms on the right vanish in the limit $q \to 0$. The third term involves an equal-time commutator between a "charge" and a current density which is calculable from the quark model and cannot make the $\Delta S = 2$ transition. In any case this term also vanishes due to the factor q_ν. For the second term we make the plausible assumption that $\partial_\mu J_\mu$ transforms as a component of an octet; so that the equal-time commutator again cannot lead to the $\Delta S = 2$ transition between K^0 and $\bar{K}^0$. Hence, these two terms will be dropped from further discussion.

The left-hand side of Eq. (4) vanishes for the case $J_\mu = A_\mu$ in the limit $q \to 0$, since no single-particle (scalar) state degenerate with the K^0 is available. Hence the limit of the first term on the right of Eq. (4) for $J_\mu = A_\mu$ vanishes:

$$\lim_{q \to 0} R_A(q) \equiv i\int d^4x \langle K^0 | T([\partial_\mu A_\mu(x)][\partial_\nu A_\nu(0)]) | \bar{K}^0 \rangle = 0. \tag{5}$$

For the case where J_μ stands for the vector current density V_μ, since $\partial_\mu V_\mu$ is nonvanishing in the first order of the SU(3)-symmetry breaking, the first term on the right,

$$R_V(q) \equiv i\int d^4x\, e^{-iqx} \times \langle K^0 | T([\partial_\mu V_\mu(x)][\partial_\nu V_\nu(0)]) | \bar{K}^0 \rangle, \tag{6}$$

is at least of the second order in the symmetry breaking. Hence, the term $q_\mu q_\nu M_{\mu\nu}$ must be evaluated to the same order. Since we eventually have to go to the limit $q \to 0$, we shall not discuss the multiparticle contributions. The single-particle-state contributions due to the π^0 and the η^0 must thus be calculated to the second order, and only at the end shall we go to the limit[9] $q \to 0$. We then have

$$q_\mu q_\nu M_{\mu\nu} = \frac{1}{(2\pi)^3} \frac{1}{M_K} [F_+(\pi^0 \to K^0) F_+(\bar{K}^0 \to \pi^0) \{(M_K{}^2 - M_\pi{}^2) + O((M_K{}^2 - M_\pi{}^2)^2) f_\pi(q)\}$$
$$+ F_+(\eta^0 \to K^0) F_+(\bar{K}^0 \to \eta^0) \{(M_K{}^2 - M_\eta{}^2) + O((M_K{}^2 - M_\eta{}^2)^2) f_\eta(q)\}]. \tag{7}$$

In Eq. (7), the F_+ stands for the usual form factors in the matrix elements of the vector current between the states indicated. Also $f_\pi(q)$ and $f_\eta(q)$ are some q-dependent functions, whose explicit forms are not relevant since they occur in terms of higher order in the SU(3)-symmetry breaking, and so must be taken as zero for consistency. Making use of the SU(3) values of the F_+, we obtain

$$\lim_{q \to 0} q_\mu q_\nu M_{\mu\nu} = -\frac{1}{(2\pi)^3} \frac{1}{8M_K} (4M_K{}^2 - 3M_\eta{}^2 - M_\pi{}^2). \tag{8}$$

We thus obtain $R_V(0)$ in terms of deviations from Gell-Mann–Okubo mass formula

$$\lim_{q \to 0} R_V(q) = -\frac{1}{(2\pi)^3} \frac{1}{8M_K} (4M_K{}^2 - 3M_\eta{}^2 - M_\pi{}^2). \tag{9}$$

In the model[5,6] where the interaction Hamiltonian density for nonleptonic decays is taken[10] to be

$$H_w(x) = i\lambda[\partial_\mu(V_{\mu 3}{}^2 - V_{\mu 2}{}^3) + \partial_\mu(A_{\mu 3}{}^2 - A_{\mu 2}{}^3)],$$
$$= 2\lambda\partial_\mu(\mathfrak{F}_{\mu,7} + \mathfrak{F}_{\mu,7}{}^5), \quad (10)$$

we find directly, comparing Eqs. (2), (6), and (9), that

$$\Delta m = (\lambda^2/2M_K)(4M_K{}^2 - 3M_\eta{}^2 - M_\pi{}^2). \quad (11)$$

It should be noted that, as seen from Eq. (5), only the parity-conserving part of H_w gives a nonvanishing contribution to Δm. The parameter λ may be fixed from the $K_1{}^0 \to 2\pi$ decay rate,[6] so that we have

$$(\Delta m)\tau(K_1{}^0) = \frac{16\pi}{3} \frac{f_\pi{}^2 M_K(4M_K{}^2 - 3M_\eta{}^2 - M_\pi{}^2)}{(M_K{}^2 - M_\pi{}^2)^2 (M_K{}^2 - 4M_\pi{}^2)^{1/2}}, \quad (12)$$

where f_π stands for the decay constant of the $\pi_{\mu 2}$ decay and has the value $f_\pi \simeq M_\pi$.

In the alternative model[1-4] of nonleptonic decays,

$$H_w(x) = g_S \mathcal{S}_7{}^5 + g_P \mathcal{S}_6, \quad (13)$$

where $\mathcal{S}_7{}^5$ is the seventh component of an octet of pseudoscalar densities transforming like $i\bar{q}\lambda_i\gamma_5 q$, and $\mathcal{S}_6$ is the sixth component of a similar octet of scalar densities. Using partially conserved axial-vector current in the form

$$\partial_\mu \mathfrak{F}_{\mu,7}{}^5 = -(f_\pi/\sqrt{2})M_K{}^2\varphi_7, \quad (14)$$

where

$$(M_\pi{}^2 - \Box)\varphi_7 = b\mathcal{S}_7{}^5 \quad (15)$$

and b is a scale factor, we see from Eq. (2) that the contribution to Δm from the parity-nonconserving part again vanishes. For the parity-conserving part, assuming that the SU(3) symmetry-breaking interaction transforms as $\mathcal{S}_8$ (belonging to the same octet as $\mathcal{S}_6$), we may write

$$\partial_\mu \mathfrak{F}_{\mu,7}(x) = i\delta m[\mathcal{S}_8(x), F_7(x_0)] = \delta m \frac{\sqrt{3}}{2}\mathcal{S}_6(x), \quad (16)$$

where δm is the strength parameter for the mass-splitting interaction. Then we have

$$R_V(q) = i(\delta m)^2 \tfrac{3}{4}\int d^4x\, e^{-iqx}$$
$$\times \langle K^0 | T(\mathcal{S}_6(x)\mathcal{S}_6(0)) | \bar{K}^0\rangle. \quad (17)$$

Thus we have from Eq. (2)

$$\Delta m = -\frac{4}{3}\frac{g_P{}^2}{(\delta m)^2}(2\pi)^3 R_V(0). \quad (18)$$

In this model the $K_1{}^0$ decay rate is given by[2]

$$\Gamma(K_1{}^0) = \frac{1}{32\pi}\frac{g_S{}^2}{(\delta m)^2}\frac{1}{f_\pi{}^2 M_K{}^2}(M_K{}^2 - M_\pi{}^2)^2$$
$$\times (M_K{}^2 - 4M_\pi{}^2)^{1/2}. \quad (19)$$

From Eqs. (9), (18), and (19) we obtain again the result (12), assuming $g_S = g_P$, as suggested[11] by the calculations on the ratio of $(K \to 3\pi)/(K \to 2\pi)$.

We thus find that both the models lead to the identical expression[12] for $(\Delta m)\tau(K_1{}^0)$ as given in Eq. (12). Finally, we obtain the numerical result

$$\Delta m \tau(K_1{}^0) = 0.49. \quad (20)$$

We find that $K_2{}^0$ is heavier than $K_1{}^0$ in agreement with recent experimental measurements.[13] The magnitude of the mass difference also is in good agreement with experiments.[14]

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[10]We have chosen here equal strengths for the parity-conserving and parity-nonconserving parts of the Hamiltonian, required by the calculations on the ratio $(K \to 3\pi)/(K \to 2\pi)$. For a detailed discussion on this point see G. S. Guralnik, V. S. Mathur, and L. K. Pandit, to be published.

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ERRATUM

EQUIVALENT REPRESENTATIONS IN SYMMETRIZED TENSORS. Donald R. Tompkins [Phys. Rev. Letters 16, 1058 (1966); 17, 622(E) (1966)].

In order to be general, Eq. (2) [hence, Eq. (5) also] must either contain modified idempotents $(PQ')_i^{\mu} \equiv (PQ)_i^{\mu} + $(additional terms) or else be replaced by

$$e = \sum_{i,\,\mu} (N^{\mu}/G)^2 (PQ)_i^{\,\mu},$$

where the sum is over all tableaux of all patterns of $\mathcal{S}_r$. The above equation displays a resolution into two-sided ideals rather than a Peirce resolution. Either alternative only adds terms to the special resolution given in the Letter; so the arguments and results are not changed.

In Eqs. (6) replace

$$B_n^{\,\mu} \equiv (N^{\mu}/G)(PQ)_n^{\,\mu} S_{n2} T_{i_1 \cdots i_r}.$$

Nth basis $(n = N^{\mu})$:

by

$$B_n^{\,\mu} \equiv (N^{\mu}/G)(PQ)_n^{\,\mu} S_{n2} T_{i_1 \cdots i_r}.$$

$\vdots$

nth basis $(n = N^{\mu})$:

and in Ref. 5 replace $(a, b, c) = (1, \cdots, n)$ by $(a, b, c) = (1, \cdots, m)$.