QUADRUPOLE MOMENT OF THE FIRST EXCITED STATE OF SPHERICAL NUCLEI*

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Two mechanisms are described which can account qualitatively for enhanced static quadrupole moments of the first excited (2^+) state of spherical nuclei. The associated calculations indicate the breakdown of the usual approximations based on pairing theory plus random-phase approximation.

There is mounting evidence that the conventional vibrational picture[1] of spherical nuclei is in need of revision. The most striking support for this assertion comes from recent measurements of the quadrupole moment of the first excited (2^+) state of Cd^{114} [2] and of other nuclei.[3] Qualitatively one may summarize the results of these experiments by recognizing that the quadrupole moment (diagonal element of the quadrupole operator) exhibits the same degree of enhancement compared to single-particle values as does the $E2$ transition amplitude (off-diagonal element of the same operator). This is somewhat reminiscent of the situation in the region of strongly deformed nuclei, though the magnitudes involved hardly indicate that one has reached anything like the strong-coupling limit characteristic of the latter.

We report here some results of two distinct calculations. Each of these attempts to correct in a different way what is perhaps the salient fault of the current microscopic theory, namely its assumption that the density distribution of the first 2^+ state is on the average the same as that of the spherical ground state. For this preliminary study, we have used the conventional model Hamiltonian of pairing plus quadrupole force.[4] We wish to emphasize that our aim here has not been to produce a finished theory. Rather it has been to illuminate as clearly as possible the inadequacy of current concepts and methods and to illustrate contributory mechanisms, not contained in the conventional approximations, which go in the required direction. We shall restrict ourselves largely to a description of these mechanisms.[5]

In the first calculation we assume as usual that in zero order both 0^+ and 2^+ are characterized by the density distribution of pairing theory.[6] The off-diagonal matrix element of the density operator coupling these two states is then also determined in the standard way by the random phase (RPA) or quasiboson approximation. The new observation is that this fluctuating density implies by means of a self-consistent perturbation theory that there are corrections to the excited-state density, second order in the RPA amplitudes, which necessarily describe a deformation.[7] Numerically, these corrections turn out to be quite large. We have found, for Cd^{114} for example, that reasonable parameters can be chosen such that we fit simultaneously the 2^+ energy $\omega_2 = 0.558$ MeV, the reduced transition probability $B(E2, 0^+ \rightarrow 2^+) = (0.51 \pm 0.02) \times 10^{-48} e^2\ \mathrm{cm}^4$, and the quadrupole moment,[2] $Q_{2+} = (-0.50 \pm 0.25) \times 10^{-24}\ \mathrm{cm}^2$. For reasons discussed below, however, this accord is not to be taken seriously, since the theory is at best incomplete.

For illustrative purposes, we exhibit in Table I the results obtained for a range of values

Table I. Representative quadrupole-force parameters (X_{np}, $X_p = X_n$) obtained by fitting first 2^+ energy, effective charge (e_{eff}) needed to describe $E2$ transition to the ground state, and quadrupole moments (Q_2) determined by RPA on the one hand and by a self-consistent calculation on the other. Additional parameters used in the calculation were proton single-particle energies $h(a)$: $h(f_{5/2}) = -0.10$ MeV, $h(p_{3/2}) = 0.60$, $h(p_{1/2}) = 1.6$, $h(g_{9/2}) = 2.5$; neutron single-particle energies: $h(d_{5/2}) = 0.00$, $h(g_{7/2}) = 1.70$, $h(s_{1/2}) = 1.30$, $d(\frac{3}{2}) = 2.80$, $h(11/2) = 2.50$; pairing force parameters: $G_p = (26/A)$ MeV, $G_n = (23/A)$ MeV.

X_{np} (MeV)	$X_p = X_n$ (MeV)	e_{eff}	$Q_{2^+}(Cd^{114})$ (10^{-24} cm^2) RPA	Present theory
0.50	1.21	0.77	−0.046	0.98
0.60	1.16	0.79	−0.066	−0.092
0.64	1.12	0.80	−0.72	−0.50
0.70	1.01	0.81	−0.082	−1.01

of the quadrupole-force parameters, defined as by Kisslinger and Sorensen,[4] namely

$$X_{ij}=(5/4\pi)[N_i+\tfrac{3}{2}][N_j+\tfrac{3}{2}]b^4\chi_{ij}, \tag{1}$$

where i,j refers to neutron (n) or proton (p) and $X_{nn}=X_n$, $X_{pp}=X_p$. Here N_i is the relevant harmonic-oscillator principal quantum number, b the fundamental length of the oscillator problem, and the χ_{ij} the force constants that occur in the quadrupole interaction. The single-particle energies and pairing-force parameters also coincide with those chosen by the same authors. Similar results were found utilizing the parameters of Tamura and Udagawa.[8] The calculations were done as follows: For a given choice of X_{np}, $X_n=X_p$ was chosen to yield the experimental excitation energy, ω_2, when calculated by means of the RPA. The same approximation yields a formula for $B(E2,0^+\rightarrow 2^+)$ which fixed e_{eff}. Finally Q_{2+} was determined according to the RPA on the one hand or according to our self-consistent formula on the other. The result is that with our method one sees rather sharp transitions completely absent from the RPA. We thus find that the static quadrupole moment, nominally of the second order of smallness compared to the $0^+\rightarrow 2^+$ quadrupole-transition amplitude, is, in fact, of the same order as the latter. It is, moreover, extremely sensitive to the parameters chosen. Among other consequences, we can no longer justify the linearization procedure which gives the RPA. Qualitatively similar results have been obtained for other nuclei.

We turn to the second calculation and consider one of the 2^+ states $|2q\rangle$. If we remove a particle labeled by the shell-model quantum numbers $\alpha=(nlj_am_\alpha)$ (destruction operator $\mathfrak{a}_\alpha$) we can characterize the possible results of such an action by the set of amplitudes

$$\psi_{JM\nu}(\alpha,2q)=\langle JM\nu|\mathfrak{a}_\alpha|2q\rangle, \tag{2}$$

where the $|JM\nu\rangle$ are states ("hole-core" states) in the neighboring odd nucleus. Several situations can now obtain. (Let us consider a set of single-particle subshells $j_a, j_b, j_c, \cdots$.) If indeed the amplitudes (2) for a given state $|JM\nu\rangle$ are appreciable for only one choice of j_a, i.e., if $|JM\nu\rangle\cong|JM(j_a2)\rangle$, then we are effectively in the weak-coupling limit and may therefore expect

$$\sum_{J=|j_a-2|}^{j_a+2}\langle 2q|\mathfrak{a}_\alpha^\dagger|JM(j_a2)\rangle\langle JM(j_a2)|\mathfrak{a}_\alpha|2q\rangle \cong\langle 2q|\mathfrak{a}_\alpha^\dagger\mathfrak{a}_\alpha|2q\rangle \cong\langle 0|\mathfrak{a}_\alpha^\dagger\mathfrak{a}_\alpha|0\rangle, \tag{3}$$

and this is indeed borne out by subsequent calculation. On the other hand there may be important mixing, so that a given state $|JM\nu\rangle$ can be reached easily by coupling various holes to the excited core. In this event the angular-momentum coupling of one particle is intimately related to the others present, and the coherent buildup of a quadrupole deformation may ensue. In the second calculation, we have obtained coupled equations of motion for amplitudes of type (2) in what may loosely be termed a Hartree-Bogolyubov calculation for the 2^+ state, allowing precisely for the angular-momentum coupling of the type described to arise self-consistently.[9]

In the present instance no states of the even nucleus other than the 2^+ intervene. There is then no real possibility of calculating either ω_2 or $B(E2,0^+\rightarrow 2^+)$. For fixed single-particle energies and pairing forces, we studied the value of Q_{2+} (which is essentially the self-consistent potential of the problem) as a function of a single quadrupole-force parameter. Results found for Cd^{114} are given in Table II,

Table II. Quadrupole moments calculated by means of a self-consistent theory of the 2^+ state. With the exception of line two (see below) the parameters χ_{ij} were all set equal in this calculation. The others parameters are the same as for Table I. The χ values for cases (a) and (b) were both fitted to the 2^+ energy in the RPA. Case (b) corresponds to line three of Table I. Case (c) was chosen to give a 2^+ energy 25% smaller than experiment. An effective charge of 0.8 was used in all cases.

	$\frac{5}{4\pi}b^4\chi$	$Q_{2^+}(Cd^{114})$ (10^{-24} cm^2)
(a)	0.034	−0.001 21
(b)	See caption	−0.001 26
(c)	0.035 23	−0.002 92
	0.037	−1.83
	0.038	−1.86
	0.042	−1.92
	0.051	−1.95

where a sharp transition is again in evidence. The first three lines correspond to essentially spherical solutions, the remaining entries to deformed solutions. It is seen that the transition occurs over a range of a few percent change in the value of the quadrupole coupling strength.[10] In this range the energy of the 2^+ state shows equally extreme sensitivity to the quadrupole coupling strength. In fact, the deformed solutions of Table II occur for values where the RPA no longer yields physically acceptable results ($\omega_2^2 < 0$ for $\chi > 0.036\,27$).[11] This is probably sufficient to rule out the present mechanism as the sole or dominant one necessary to produce the experimental results sought, though not as a contributory mechanism in a more complete calculation.

We thus wish to emphasize that neither of the calculations reported is to be taken as a literal attempt at a final theory. We do believe that the work indicates strongly that both phonon mixing (first calculation) and particle mixing (second calculation) provide mechanisms which can help account for the buildup of permanent quadrupole deformations in the low-lying excited states of so-called spherical nuclei. Encouraged by our findings, we have undertaken a serious program of calculation which incorporates the mechanisms discussed above as well as including the coupling to more highly excited states. In effect we are carrying out the application of the full generalized Hartree-Fock method to the intermediate-coupling situation.[12,13]

The details of the calculations reported in this Letter will be submitted for publication shortly.

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[2]J. de Boer, R. G. Stokstad, G. D. Symons, and A. Winther, Phys. Rev. Letters 14, 564 (1965); P. H. Stelson, W. T. Milner, J. L. C. Ford, Jr., F. K. McGowan, and R. L. Robinson, Bull. Am. Phys. Soc. 10, 427 (1965).

[3]Proceedings of the Conference on Coulomb Excitation, Copenhagen, Denmark, May, 1966 (unpublished). The authors are indebted to R. A. Sorensen for informing them of the results of this conference and to P. H. Stelson for a private communication of unpublished measurements.

[4]L. S. Kisslinger and R. A. Sorensen, Rev. Mod. Phys. 35, 853 (1963).

[5]The only other effort known to us to discuss the new situation is that of T. Tamura and T. Udagawa, Phys. Rev. Letters 15, 765 (1965). These authors propose that the quadrupole moment may be understood if we regard the 2^+ state as a mixture of one- and two-phonon excitations. Though there is small overlap between this idea and the mechanisms discussed by us, we do not doubt that the effect in question exists and must be taken into account in the more complete theory now under study. In the latter, a self-consistent intermediate-coupling theory, one is no longer able to distinguish cleanly the various mechanisms, and it remains to be seen to what extent one can salvage the concept of phonon.

[6]That such an assumption must be made to derive the random-phase approximation is not readily evident in many of the derivations extant. It is quite explicit in the derivation based on the generalized Hartree-Fock approximation given by A. K. Kerman and A. Klein, Phys. Rev. 132, 1326 (1963).

[7]The method of carrying out the self-consistent perturbation theory is not essentially different from that described by G. Do Dang and A. Klein, Phys. Rev. 147, 689 (1966), for taking into account the blocking effect in excited states of the pairing theory. Here the "blocking" occurs with respect to both particle number and angular momentum. Because we deal with fundamentally nonlinear equations, the perturbations, rather than being determined explicitly in each order, are found to satisfy linear equations. This is a further manifestation of the self-consistency aspect of the problem. We ultimately find explicit equations for the reduced quadrupole matrix elements from which the convergence properties can be studied numerically. The enhancement we find arises from the self-consistency.

[8]T. Tamura and T. Udagawa, Nucl. Phys. 53, 33 (1964).

[9]By contrast we recall that in the first calculation it was triggered by the amplitude associated with the $B(E2, 0^+ \rightarrow 2^+)$.

[10]We found that any attempt to follow the transition curve in detail required excessive computing time, since convergence proved exceedingly slow in this region.

[11]The reader should bear in mind that in contrast to the conventional Hartree-Fock or Hartree-Bogolyubov method, our calculation conserves angular momentum throughout.

[12]A somewhat simplified version of the general method has been studied by A. I. Sherwood and A. Goswami (to be published). Their approach corresponds essentially to what would naturally be the first iteration of a sequence designed to achieve full self-consistency. Whereas in the full calculation one obtains simultaneously both even- and odd-state energies, the former

from self-consistency requirements, the latter as eigenvalues, in the simplified version even-state energies are taken from experiment or from RPA calculations. In this simplified theory there is no intimation of large diagonal matrix elements.

[13]Of course the method of intermediate coupling as a way of studying the properties of odd nuclei—assuming that one knows the properties of the neighboring even nucleus—has been widely used in several versions since the work of D. C. Choudhury, Kgl. Danske Videnskab Selskab, Mat.-fys. Medd. 28, No. 4 (1954). For a recent application see D. C. Choudhury and E. Kujawski, Phys. Rev. 144, 1013 (1966). A version of the phenomenological theory, more sophisticated in principle, in which allowance is made for a deformation of the 2^+ state as well as for its transition amplitude to the ground state, has been developed by V. K. Thankappan and W. W. True, Phys. Rev. 137, B793 (1965). It is interesting (and was of some inspiration to the present authors) that in a study of the energy levels of Cu^{63}, the best fit was obtained for diagonal and off-diagonal quadrupole matrix elements of Ni^{62} of comparable size. It is our opinion, however, that for a truly accurate description of the one-phonon state and of its parent or daughter odd-nucleus states, one will have to study transitions to states symmetrically disposed with respect to the states of interest and not just transitions to the ground states.

SYMMETRY BETWEEN PARTICLE AND ANTIPARTICLE POPULATIONS IN THE UNIVERSE

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In cosmological theories questions have often been raised as to whether the symmetry property between particles and antiparticles is reflected in the over-all particle and antiparticle populations of the universe.[1] In this Letter we shall show that, according to currently accepted world models and the observed microscopic properties of matter, had the total number of particles and antiparticles been equal at the time when the universe was still in a highly condensed state, all but a small fraction of particle pairs would have annihilated into photons or π mesons as the universe expanded, and the radiation energy density of the present universe would have been much greater than what is observed.[2] A detailed report on the early evolution of the universe will be published elsewhere.

The following conclusion is based on the assumption that sometime in the past the temperature and density of the universe were so high that an equilibrium between particle-pair creation and radiation can be assured. In the case of nucleon pairs this requires a temperature of the order of 10^{12} °K.

In the early epoch of an expanding world model, the radiation energy dominates. For the case of no interaction between radiation and particles, Alpher, Follin, and Herman[3] give the following expressions for the Hubble constant H_0, temperature T, and the particle number density n as functions of the time after creation t:

$$H_0(t) = 1/2t, \tag{1}$$

$$T(t) = T' t^{-1/2}, \tag{2}$$

and, if the total number of particles is conserved,

$$n(t) = n' t^{-3/2}. \tag{3}$$

Different cosmological theories will give somewhat different values of T' and n'. Alpher et al. give $T' = 1.5\times10^{10}$ °K. The evaluation of n' is different from the evaluation of T', because the present universe is a matter universe for which

$$T(t) \propto t^{-2/3}, \tag{4}$$

$$n(t) \propto T^3 \propto t^{-2}. \tag{5}$$

Hence n' must be evaluated from the equation

$$n(t) = n'(T/T')^3. \tag{6}$$

Using the present value of $n = 10^{-7}$ and the constant T', one finds that $n' = 10^{22}$, if the age of the universe is taken to be $\sim10^{10}$ years. The proper inclusion of interactions between particles and antiparticles will alter only slightly the numerical character of our result.[4]

Using Eqs. (1) and (2), the Boltzmann transport equation describing the nonequilibrium processes in an expanding universe is solved for the case in which (a) the initial particle-