STRETCH SCHEME, A MICROSCOPIC DESCRIPTION OF ROTATIONS IN NUCLEI

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A good angular-momentum wave function containing the maximum possible intrinsic angular momenta leads to rotational spectra. The rotational excitation energies arise from the residual two-body force.

It is well known that strong deformations occur only in nuclei which are sufficiently far away from shell closure for both protons and neutrons. An understanding of the nuclear deformations thus must contain prominently, in addition to correlations between like particles, also correlations between protons and neutrons. Two identical nucleons in the same shell have the maximum binding energy in a state coupled to $J=0$, i.e., if they are paired off to $J=0$, as a consequence of the maximum overlap of their wave functions in the angular coordinates. On the other hand, a proton and a neutron have a large binding energy (large overlap) both when they are paired off to $J=0$ and when they are aligned to the maximum possible angular momentum $J_{max}=2j$, which here is allowed by the Pauli principle. The pairing correlations are dominant in nuclei with a single closed shell and lead to a pairing wave function which gives an approximate description of the ground state, and whose elementary excitations, the quasiparticle states, give a first approximation to the low-lying excited states. Likewise, the aligning correlations between protons and neutrons lead directly to a wave function, the "stretch" wave function, which is appropriate to an approximate description of the ground state of deformed nuclei and whose elementary excitations are associated with both the "rotations" and the low-lying "intrinsic" states. In this picture, both of these excitations are directly related to the two-body force, and the "rotational" energy here appears purely as a potential energy. The stretch scheme is related to the "aligned coupling scheme" of Bohr and Mottelson.[1] However, there are characteristic differences between the two. Since the deformed stretch wave function is rotationally invariant, no projection techniques have to be employed, and the physical description of the nuclear rotations in terms of individual nucleon motion is transparent throughout.

We consider here the simplest case of $2N$ protons in the shell j and $2N$ neutrons in the shell k. The stretch wave function is then constructed in the following way. First, N protons and N neutrons are coupled to the maximum possible angular momentum, say C, allowed by the Pauli principle. This chain wave function is unique, and it contains the maximum possible number of aligned proton-neutron pairs (see Fig. 1). Next, the remaining N protons and N neutrons are coupled to a similar chain. Finally, these two chains are coupled to a state with a total angular momentum zero. This way the wave function contains correlations between all particles, and the overlap between the wave

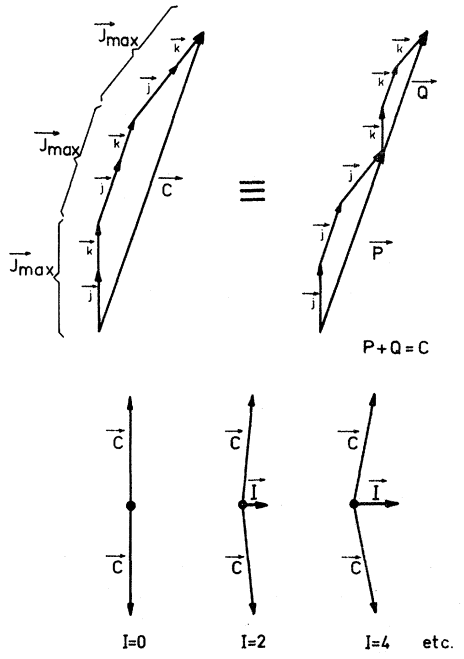

FIG. 1. Top part: the structure of a chain. The coupling of aligned pairs to maximum angular momentum C is unique; both coupling schemes shown are identical. Bottom part: the stretch state and its rotational excitations. In the rotational states with $I\neq 0$, the effective overlap between the wave functions of the two chains decreases with increasing I in that more and more across-the-chains pairs are broken.

functions of the two chains is maximized.

The stretch wave function has the following properties: (i) The stretch state is nondegenerate. It has a large binding energy and it is stable against distortions since any other 0^+ state requires the breaking of at least two pairs. (ii) By construction, the state is spherically symmetric, e.g., $\langle 0|Q_0|0\rangle = 0$, where Q_0 is the quadrupole operator. Still it has an intrinsic quadrupole moment, viz. $\langle 0|Q_0^2|0\rangle \neq 0$, which is, on the contrary, large. (iii) The stretch wave function exhibits two kinds of elementary excitations. The ones, the "rotational" excitations, consist in the coupling of the two chains C to angular momenta $I \neq 0$; the others, the "intrinsic" excitations, consist in the breaking of a chain leading to a chain angular momentum $C' < C$.

The excitation energy of the rotational states results from the loss of overlap between the wave functions of the two chains (see Fig. 1). An analogous loss of overlap accounts for the excitation energy of the "intrinsic" excitations. The kinetic energy of all the elementary excitations of the stretch state remains unchanged. Still, the "rotational" excitations deserve their name since in the states with $I \neq 0$ the angular momentum vectors $\vec{C}$ of the two chains precess about the direction of $\vec{I}$ with angular velocities which increase with increasing I.

Since the wave function must be symmetric under exchange of the two chains, only the even angular momenta, i.e., $I = 2, 4, 6, \cdots$, are allowed. The energies of the rotational states are of the almost obvious but not trivially derivable form

$$E_I = A + \sum_{\alpha\beta\gamma} \begin{Bmatrix} C & C & I \\ \alpha & \beta & \gamma \end{Bmatrix}^2 g(\alpha,\beta,\gamma). \qquad (1)$$

Here A is the contribution to the energy from nucleons in the same chain, while the sum arises from the interactions between nucleons of the two chains. In other words, $A/2$ is the self-energy of each chain, while the second term represents an effective chain-chain interaction energy. Asymptotically, for large C and small I, (1) becomes

$$E_I = E_0 + I(I+1)E_R. \qquad (2)$$

In this formula higher terms in $[I(I+1)/C(C+1)]$ have been dropped. The form (2) is a simple expression of the fact that the interaction energy between the chains must be of the form $\vec{C}_1 \cdot \vec{C}_2$. This is proportional to $\cos\alpha \approx 1 - \alpha^2/2$, where α is the angle between $\vec{C}_1$ and $\vec{C}_2$, and has the magnitude $\alpha \approx I/C$.

A complete model calculation has been carried through in which the energies were computed in the form (1). The force used was

$$V_{12} = \exp[-(r_{12}/\mu)^2] \sum_{S,T} V_{ST} P_{ST}, \qquad (3)$$

with $V_{10} = -40$ MeV, $V_{01} = -24$ MeV, $V_{00} = -24$ MeV, $V_{11} = 25$ MeV, which is the "COP" force of Gillet, Green, and Sanderson.[2] Harmonic-oscillator wave functions were used; the potential well parameter α, and the range parameter of the force μ, were chosen as $\mu\alpha = 1$. Typical results are presented in Table I and in Fig. 2. The quantity $E_R(I)$ in Table I has been introduced by using (2) as the defining equation. The deviations from a pure rotational character, i.e., $\rho = 1$ in Fig. 2, are very similar to those found in the experiments: The parameters $E_R(I)$ become more constant as the number of particles (holes in the nuclei of Fig. 2) increases. However, the absolute values of the inertial parameters (see Table I) cannot be expected to be given correctly by this simple model since only very few particles and only one single-particle shell has been considered in the calculations so far.

The relationship between the stretch wave function and the Bohr-Mottelson aligned scheme[1] is given by the identity

$$\Phi_{\text{aligned}}(p)\Phi_{\text{aligned}}(n) = \sum_I \langle CCC-C|I0\rangle \Psi_0[I],$$

where $\Psi_0[I]$ are the rotational wave functions with $M = 0$ and Φ_{aligned} are the Slater determinants for protons and neutrons, respectively, in which the m-states are filled beginning with the largest possible values, i.e., in the sequence $m = \pm j, \pm(j-1), \pm(j-2)$, etc. Thus the stretch rotational wave functions correspond to the

Table I. Results for $j = h_{11/2}$ and $k = h_{11/2}$ for different nucleon numbers. The ground-state binding energy E_0 is relative to the independent-particle shell-model energy. The rotational energy parameter $E_R(2)$ is computed from the state $I = 2$.

$4N$	E_0 (MeV)	$E_R(2)$ (keV)
4	−2.5	28.4
8	−5.1	10.6
12	−8.4	5.8

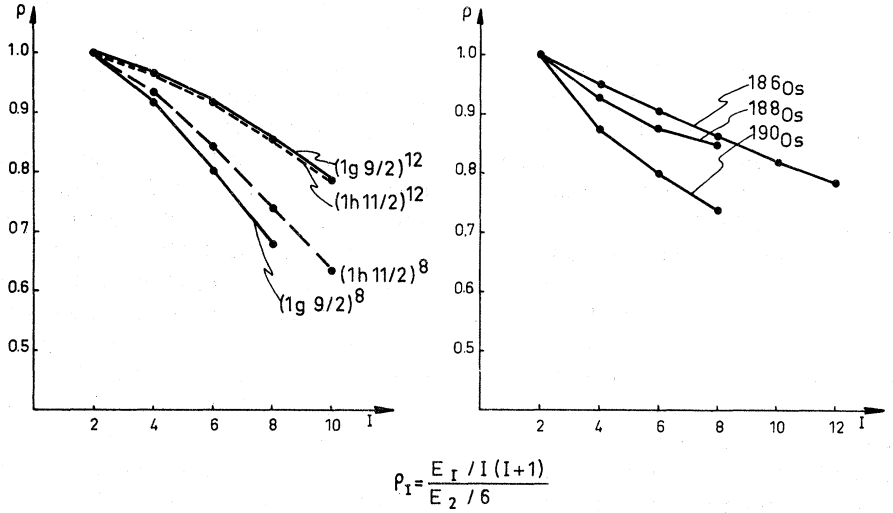

FIG. 2. Departures from a purely rotational spectrum. Left part, stretch; right part, experiments. The theoretical and experimental curves are qualitatively very similar, in spite of the restriction to a single configuration in the calculation.

projected aligned-scheme wave functions, and stretch wave functions built up with holes correspond to positive intrinsic deformations while those built up with particles correspond to negative intrinsic deformations. Therefore we have used the experimental data of hole nuclei in Fig. 2.

The intrinsic excitations, i.e., those states which result from breaking a single chain, are not unique for a given total angular momentum I. A description of the intrinsic excited states thus requires a treatment of the configuration mixing between these excitations. The calculations for this case have not yet been completed.

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[1]B. Mottelson, in Proceedings of the Enrico Fermi International School of Physics, Course XV, edited by G. Racah (Academic Press, Inc., New York, 1962), p. 45.

[2]V. Gillet, A. Green, and E. Sanderson, Saclay Report No. SPT/V.G. 27 bis, March, 1966 (to be published).

MASSES OF N^{12}, Al^{24}, P^{28}, Cl^{32}, AND Sc^{40}

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The masses of N^{12}, Al^{24}, P^{28}, Cl^{32}, and Sc^{40} have been determined by measuring the threshold energies for the reaction $C^{12}(p,n)N^{12}$, $Mg^{24}(p,n)Al^{24}$, $Si^{28}(p,n)P^{28}$, $S^{32}(p,n)Cl^{32}$, and $Ca^{40}(p,n)Sc^{40}$.

Protons from the Yale model MP tandem Van de Graaff accelerator were momemtum analyzed by a 90° double-focusing magnet with a nominal radius of curvature of 52 in. The beam trajectories in the magnet were limited by entrance and exit slits each set with a total separation of 0.030 in. Under these conditions the energy spread in the beam was $\leq 0.03\%$ [as measured at the $Li^7(p,n)Be^7$ threshold] and the energy stability over a two-hour interval was ≤ 200 eV (as measured at a proton energy of 6 MeV).

The analyzing magnet was calibrated[1,2] using various charge states of O^{16} in the reaction $D(O^{16},n)F^{17}$ and the well-known (p,n) thresholds for targets of Li^7, Cu^{65}, C^{13}, F^{19}, Al^{27}, Ni^{60}, Fe^{54}, and Ni^{58}. Neutrons above the Li^7, Cu^{65}, C^{13}, F^{19}, Al^{27}, and Ni^{60} (p,n) thresholds and the $D(O^{16},n)F^{17}$ thresholds were detected with