SPACE-CHARGE-LIMITED CURRENTS IN NONMETALLIC SOLIDS

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The theory for space-charge-limited currents[1-5] in insulators and semiconductors can be extended rigorously to embrace the essentially time-dependent cases and to include charge-carrier diffusion.[6] Dynamical and diffusion effects are expected to be of practical importance in certain operating regimes for some thin nonmetallic crystal elements,[7] and extraordinary transient voltage-current relations are predicted for them by the theory outlines here.

With trapping negligible, the one-dimensional single-carrier current flow in an ideal nonmetallic solid is governed by the conduction-continuity and Poisson equations[1]

$$\frac{\partial n}{\partial t}=\frac{\partial}{\partial x}\left(\mu n\frac{\partial \varphi}{\partial x}+D\frac{\partial n}{\partial x}\right), \tag{1}$$

$$\partial^2\varphi/\partial x^2=-(q/\epsilon)n, \tag{2}$$

for the potential field $\varphi=\varphi(x,t)$ and carrier concentration $n=n(x,t)$ of particles with constant drift mobility μ, effective charge q, and diffusivity D in a medium of permittivity ϵ; by definition, μ and q have the same sign (minus for electrons, positive for holes) and the Einstein relation takes the form $D=\mu kT/q$. Combining Eqs. (1) and (2) and integrating twice with respect to x, one obtains the inhomogeneous nonlinear equation

$$\frac{\partial\varphi}{\partial t}-\tfrac{1}{2}\mu\left(\frac{\partial\varphi}{\partial x}\right)^2-D\frac{\partial^2\varphi}{\partial x^2}=-\epsilon^{-1}xJ+(\text{trivial gauge function of } t \text{ alone}), \tag{3}$$

where $J=J(t)$ is the total Maxwell current (drift plus diffusion plus displacement) per unit area. Equation (3), an inhomogeneous Burgers equation[8] for $\partial\varphi/\partial x$, can be integrated exactly[9]; it is satisfied by

$$\varphi=-\epsilon^{-1}x\int_0^t J(t')dt'+\frac{2D}{\mu}\ln\psi \tag{4}$$

if ψ (>0) satisfies the homogeneous linear equation

$$\frac{\partial\psi}{\partial t}+\frac{\mu}{\epsilon}\int_0^t J(t')dt'\frac{\partial\psi}{\partial x}-D\frac{\partial^2\psi}{\partial x^2}=0. \tag{5}$$

Hence, in the case of an unbounded x domain, (4) satisfies Eq. (3) with

$$\psi=\tfrac{1}{2}(\pi Dt)^{-1/2}\int_{-\infty}^{\infty}\exp\{-(4Dt)^{-1}[\xi-x+(\mu/\epsilon)\int_0^t(t-t')\times J(t')dt']^2+(\mu/2D)\varphi_0(\xi)\}\,d\xi, \tag{6}$$

in which $\varphi_0(x)\equiv\varphi(x,0)$ is the prescribed initial potential field.[10]

Of greater practical interest is a finite x domain $0\leq x\leq L$, appropriate to a thin insulating crystal of length L. With an Ohmic injecting contact at $x=0$, the boundary condition

$$\left(\frac{\partial\varphi}{\partial x}\right)_{x=0}=0 \text{ implying}$$

$$\left(\frac{\psi^{-1}\partial\psi}{\partial x}\right)_{x=0}=\frac{\mu}{2\epsilon D}\int_0^t J(t')dt' \tag{7}$$

is a satisfactory approximation,[11] compatible with a prescribed potential difference

$$V=V(t)\equiv\varphi(L,t)-\varphi(0,t) \tag{8}$$

with $\mu V>0$ for $t>0$ that is established by an external voltage source. Subject to a field-free initial condition $\varphi(x,0)=0$ [requiring $V(0)=0$], the space-charge-limited solution[12] to Eq. (5) with (7) and (8) can be obtained by application of well-known linear methods, although the exact solution is not expressible in closed form. However, an approximate version of the solution to Eq. (5) with (7) and (8), valid with good accuracy provided that the applied voltage (8) does not change too rapidly after an initial transient rise, i.e., $|\dot V|\ll|\mu|V^2/L^2$ for $t\gtrsim L^2/\mu V$, is given by

$$\begin{aligned}\psi&=\exp\{(\mu/2\epsilon D)x\int_0^t J(t')dt'+(2\mu J/\epsilon)^{1/2}x^{3/2}/3D\} \quad \text{for } 0\leq x\leq\lambda,\\ &=\exp\{(\mu/2\epsilon D)x\int_0^t J(t')dt'+\mu Vx/2DL\} \quad \text{for } \lambda\leq x\leq L,\end{aligned} \tag{9}$$

with

$$J=q\mu\epsilon V^2/8L^2\lambda \tag{10}$$

and

$$\lambda\equiv\min\{(\mu/\epsilon)\int_0^t(t-t')J(t')dt'+2(Dt)^{1/2},\,L\}. \tag{11}$$

A transient voltage-current relation follows from (10) and (11), the current density determined implicity by the applied voltage; by introducing some obvious approximations, the explicit dependence is found to take the approximate form

$$J \cong q\mu\epsilon V^2/16L^2(Dt)^{1/2} \text{ for } 0 \leq t \lesssim t_1,$$
$$\cong q\epsilon V/8Lt \text{ for } t_1 \lesssim t \lesssim t_2,$$
$$\cong q\mu\epsilon V^2/8L^3 \text{ for } t_2 \lesssim t, \tag{12}$$

where the characteristic times are

$$t_1 \equiv 4DL^2/\mu^2V^2 \cong 4(2.5 \text{ cm}^2/\text{sec})(10^{-2} \text{ cm})^2 \times (10^2 \text{ cm}^2/\text{V sec})^{-2}(10 \text{ V})^{-2} = 10^{-9} \text{ sec},$$
$$t_2 \equiv L^2/\mu V \cong (10^{-2} \text{ cm})^2/(10^2 \text{ cm}^2/\text{V sec})(10 \text{ V}) = 10^{-7} \text{ sec}$$

for typical physical magnitudes. Hence, in the case of an applied voltage that rapidly attains a finite constant value in a time of the order t_1, the associated current density increases in magnitude from $J(0) = 0$[13] to a maximum value about equal to $J(t_1) \cong q\epsilon\mu^2V^3/32DL^3$ ($\cong \pm 1$ A/cm^2, for typical physical magnitudes) and then decreases in magnitude asymptotically to the steady or quasisteady value[1] $J(t_2) \cong q\mu\epsilon V^2/8L^3$ ($\cong \pm 10^{-2}$ A/cm^2, again for typical physical magnitudes). The very large diffusion-dominated transient current density, two orders of magnitude greater than the steady space-charge-limited value for J, should be observable with fast electronic circuitry and near-perfect insulating crystals substantially free of large-cross-section traps.

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[1]W. Shockley and R. C. Prim, Phys. Rev. 90, 753 (1953).

[2]R. W. Smith and A. Rose, Phys. Rev. 97, 1531 (1953).

[3]S. M. Skinner, J. Appl. Phys. 26, 498, 509 (1955).

[4]M. A. Lampert and A. Rose, Phys. Rev. 121, 26 (1961).

[5]A. Many and G. Rakavy, Phys. Rev. 126, 1980 (1962).

[6]Whereas charge-carrier diffusion plays a relatively passive role (subordinate to charge-carrier drift) for the steady currents,[1,4] it is necessary to include diffusion in a rigorous dynamical theory for space-charge-limited currents. The $D = 0$ idealization[5] is a singular limiting case for the dynamical theory, Eq. (3) being parabolic with $t = \text{const}$ characteristics for positive (physical) D but hyperbolic with charge-carrier flow-line characteristics for the academic $D = 0$. Moreover, subject to an Ohmic injecting-contact boundary condition (7), the diffusion governs the flow of charge carriers for small x with the drift current vanishing at the contact. Thus, the usefulness of a $D = 0$ theory is quite limited, the actual smallness of physical D notwithstanding.

[7]M. A. Lampert, private communication.

[8]J. M. Burgers, Proc. Acad. Sci. Amsterdam 53, 247 (1950).

[9]J. D. Cole, Quart. Appl. Math. 9, 225 (1951).

[10]Ordinarily, the integral (6) can be evaluated with the method of steepest descents, which gives the more explicit but approximate solution

$$\varphi = \varphi_0(\lambda) - \tfrac{1}{2}[\varphi_0'(\lambda)]^2\mu t - (D/\mu)\ln[1 - \mu t\varphi_0''(\lambda)] - \epsilon^{-1}x\int_0^t J(t')dt',$$

where $\lambda = \lambda(x,t)$ is a real root of the equation

$$x - \lambda + \varphi_0'(\lambda)\mu t - (\mu/\epsilon)\int_0^t (t-t')J(t')dt' = 0.$$

[11]M. A. Lampert and A. Rose, Phys. Rev. 113, 1236 (1959).

[12]The macroscopic theory also admits the "condenser" solution to Eq. (5) with (7) and (8), approximately of the form

$$\psi = \tfrac{1}{2}[1 + \exp\{(\mu/\epsilon D)x\int_0^t J(t')dt'\}]$$

with

$$J = (1 - e^{-\mu V/D})^{1/2}(\epsilon/L)\dot{V}$$

for

$$|\dot{V}| \ll |\mu|V^2/L^2.$$

In the case of an ineffective Ohmic contact that fails to provide a free supply of charge carriers for the conduction band of the insulating crystal, the latter solution supersedes (9).

[13]For the $D = 0$ idealization, (10) and (11) produce $J(0) = 3\epsilon V(0)/2L$, a condenser-type[12] voltage-current relation. Naturally, the $D = 0$ theory[5] cannot predict the large diffusion-dominated transient current density.