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REMEASUREMENT OF THE LAMB SHIFT IN H, $n = 2$*

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In a previous paper,[1] we reported a value of the Lamb shift $\mathcal{S}$ in the first excited state of atomic hydrogen. The result, based upon a level-crossing measurement, was $\mathcal{S} = 1058.07 \pm 0.10$ MHz. This value is of interest because (1) it lies 0.30 ± 0.14 MHz above the original result of Lamb and co-workers,[2] (2) it lies 0.43 ± 0.23 MHz above the most recent theoretical value.[3] The former discrepancy raises questions above a possible increase in the value of the fine structure constant α,[4] which is related in turn to the as yet unresolved problem of proton-structure corrections to the ground-state hyperfine structure (hfs) of atomic hydrogen.[5] The latter discrepancy may pose some fundamental questions in quantum electrodynamics.[6] In the present Letter, we report the completion of an independent experiment confirming our previous measurement of $\mathcal{S}$. We thereby establish a consistent systematic difference between our results and those of the Lamb experiments.

The level-crossing technique is described in Ref. 1. We review it briefly. Figure 1 shows the Zeeman effect on the $2\,^2S_{1/2}$ and $2\,^2P_{1/2}$ levels of atomic hydrogen, including hfs. The $2S$ levels are metastable. Using this fact, Lamb and co-workers measured $\mathcal{S}$ by inducing rf electric field transitions between the $2S$ state α and the short-lived $2P$ states e and f (Ref. 2). In our experiment, we measure $\mathcal{S}$ by locating crossing points between the $2S$ level β and $2P$ level e. Of the four hfs crossings which occur near 574 G, those two (marked A and B in Fig. 1) which obey $\Delta m_I = 0$ are observable[7] by coupling levels β and e via a static electric field. Our measurement of the $\beta \to e$ transition resonance

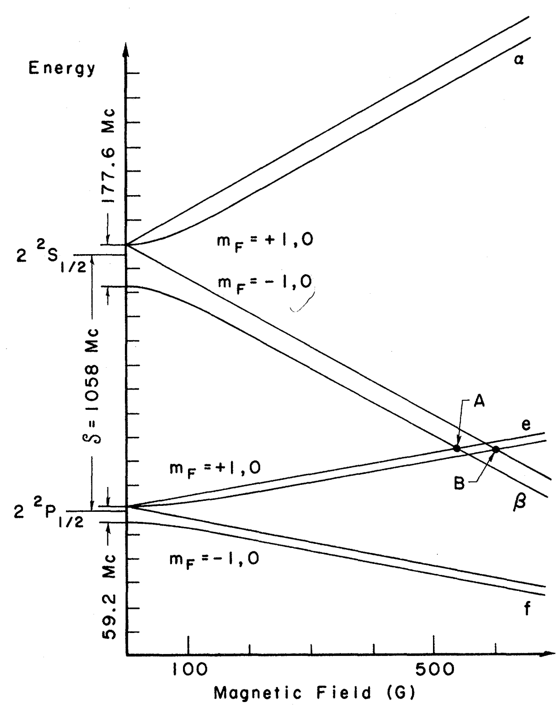

FIG. 1. Zeeman effect on the $J = \frac{1}{2}$ levels in H, $n = 2$. The levels $\alpha, \cdots, f$ are Lamb's designation. The present experiment is concerned with the measurement of the crossing point A, between levels $\beta(m_F = 0)$ and $e(m_F = +1)$.

corresponding to crossings A and B is essentially a Lamb-Retherford experiment at zero frequency.

Crossings A and B in atomic hydrogen occur near 538 and 605 G. We denote the crossings by H(538) and H(605), respectively, and the levels involved by (A) $\beta_A = \beta(m_F = 0) \rightarrow e_A = e(m_F = +1)$; ($B$) $\beta_B = \beta(m_F = -1) \rightarrow e_B = e(m_F = 0)$. Our previous work (Ref. 1) determined $\mathcal{S}$ by measurement of the H(605) resonance. The present work measures $\mathcal{S}$ from the H(538) resonance.

In contrast to the Lamb experiments, where the resonances were barely resolved superpositions of hfs transitions, our experiment resolves the hfs and allows measurement of the H(538) resonance alone. To do this, we produce a beam containing β states of the β_A variety only. After initially quenching all β's, we regenerate the β_A state by inducing rf magnetic dipole transitions $\alpha(m_F = 0) \rightarrow \beta_A$ at a low magnetic field (≈3 G). This beam has a β_A-to-β_B "purity" ratio of greater than 1500 to 1. Except for the new rf transition region to produce β_A's, our apparatus is identical to that described in Ref. 1.

Figure 2 shows a typical H(538) quenching resonance studied in the present work. We compare experimental points with a line shape derived from the Bethe-Lamb theory of the quenching of the 2S state.[8] The only parameters used in the fit are the percentage of quenching and the magnetic field corresponding to peak quenching. In the H(605) work, we did not average the line shape over the beam velocity distribution. In the present work on H(538), we gain a substantial improvement in line-shape fit by averaging over a v^2 velocity distribution. For example, our observed half-widths now agree with theory to $(-0.3 \pm 0.8)\%$.[9] In general, the observed $\beta_A \rightarrow e_A$ quenching panoram-

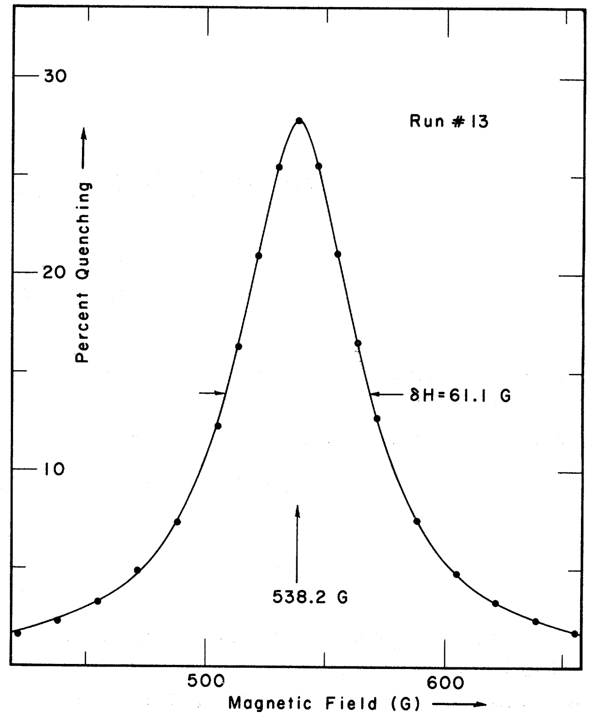

FIG. 2. Typical H(538) crossing-point quenching-resonance panoramic, at applied electrostatic field $E_Q \approx 0.6$ V/cm. The experimental points are compared with a velocity-averaged line shape derived from the Bethe-Lamb theory of the lifetime of the 2S state in external fields.

Table I. Line-center data for H(538) resonance. H_C' is the observed center measured by proton nmr frequency (in water sample). $\mathcal{S}$ is the calculated Lamb shift, after correction of H_C' for line-center shift. All errors quoted are one standard deviation.

Run No.	No. of center measurements	H_C' (kHz)	Fractional weight	Line-center shift (kHz)	Calculated Lamb shift $\mathcal{S}$ (MHz)
1a	7	2292.273 ±0.308	0.0380	0.382	1058.158 ±0.142
1b	8	2291.876 ±0.298	0.0384	0.372	1057.994 ±0.138
2	23	2291.903 ±0.177	0.2245	0.319	1058.028 ±0.081
3	12	2292.238 ±0.141	0.3462	0.346	1058.159 ±0.066
4	24	2291.470 ±0.205	0.1643	0.341	1057.834 ±0.095
5	24	2291.917 ±0.192	0.1886	0.333	1058.027 ±0.089

ics confirm the expectation of a highly symmetric resonance line shape. Line-asymmetry corrections amount to only about 150 ppm of the observed line centers, and are mainly due to variation of the $2S-2P$ Stark matrix element with magnetic field.

We accumulate line-center data by measuring fractional quenching at the upper and lower $\frac{3}{4}$ points of the resonance line. Measured line slopes are used to calculate equalized quenching values. Table I shows data for five independent runs. Subtracting various line-center shifts, we quote a weighted average corrected line center for the H(538) resonance:

$$H_C = 2291.627 \pm 0.229 \text{ kHz}, \tag{1}$$

in units of proton nmr frequency in a water sample. From H_C we calculate a value of $\mathcal{S}$, as

$$\mathcal{S} = 1058.04 \pm 0.10 \text{ MHz}, \tag{2}$$

with precision about twice the standard deviation of the mean of five independent runs.

This result is an improvement over the previous H(605) result because (1) it is based on five rather than two high-precision runs; (2) line-shape agreement with theory is substantially improved; (3) we have corrected various minor errors,[10] and have conducted a comprehensive review of possible line-asymmetry effects. In particular, we believe the calculated line-center shifts in Table I are good to ±20% (30 ppm in H_C) as an upper limit of error. Finally, following a suggestion by Lamb, we have investigated a previously unnoticed effect which can shift the line center. If the beam is "tilted" with respect to the axis of the Helmholtz coil, then it experiences a motional electric field which can asymmetrize the quenching,[11] thereby causing a line-center shift. The observed center H_C' shifts up or down depending on whether the beam-tilt field adds to or subtracts from the applied quenching field. We can measure this shift by comparing H_C' values for positive and negative polarity of the applied quenching field. Runs 1a and 1b were done in this way, and exhibit the beam-tilt shift in an extreme case. In all other runs, this shift was averaged to zero by reversing the applied quenching field polarity at 25 cps. We are not certain of the size of this shift in the H(605) work, although indirect evidence indicates it was not larger than ±0.05 MHz in $\mathcal{S}$.

Combining the H(605) and H(538) work, we quote a value for the Lamb shift $\mathcal{S}$ in H, $n = 2$, as

$$\mathcal{S} = 1058.05 \pm 0.10 \text{ MHz}. \tag{3}$$

The stated precision is more than twice the standard deviation of the mean. We consider it to be an upper limit of error. The agreement between the H(605) and H(538) results to well within this error establishes a consistent systematic difference of $+0.28 \pm 0.14$ MHz in $\mathcal{S}$ between the present experiment and the Lamb experiments.[12]

Relevant details, especially concerning possible line-center shifts, will appear in a forthcoming publication.

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[2]S. Triebwasser, E. S. Dayhoff, and W. E. Lamb, Jr., Phys. Rev. 89, 98 (1953). Their value was $\mathcal{S} = 1057.77 \pm 0.10$ MHz.

[3]G. W. Erickson and D. R. Yennie, Ann. Phys. (N.Y.) 35, 271 (1965). The theoretical value is $\mathcal{S} = 1057.64 \pm 0.21$ MHz. This will be reduced, however, due to a recent calculation by M. Soto (unpublished). Soto quotes $\mathcal{S} = 1057.50 \pm 0.11$ MHz for hydrogen (private communication from M. Soto). The discrepancy between experiment and theory is thereby increased to 0.57 ± 0.14 MHz.

[4]E. R. Cohen and J. W. M. DuMond, Rev. Mod. Phys. 37, 537 (1965).

[5]V. W. Hughes, in Nucleon Structure, edited by R. Hofstadter and L. Schiff (Stanford University Press, Stanford, California, 1964), p. 235. Recent papers are C. K. Iddings, Phys. Rev. 138, B446 (1965); S. Fenster et al., Phys. Letters 19, 513 (1965); S. D. Drell and J. D. Sullivan, Phys. Letters 19, 516 (1965); S. Fenster and Y. Nambu, Progr. Theoret. Phys. (Kyoto), Suppl. Extra No., 250 (1965).

[6]Recent review articles on quantum electrodynamics include those of F. M. Pipkin, in Proceedings of the Oxford International Conference on Elementary Particles, Oxford, England, 1965 (Rutherford High Energy Laboratory, Chilton, Berkshire, England, 1966); R. Gatto, in High Energy Physics, edited by E. S. Burhop (Academic Press, Inc., New York, 1966), Chap. 15.

[7]Because of state mixing by off-diagonal elements of the hfs interaction, the $\beta(m_F = 0)-e(m_F = 0)$ crossing is not strictly forbidden by $\Delta m_I = 0$. The shift in either crossing point A or B is small in our experiment, and

may be treated as a perturbation. On the other hand, such "forbidden" crossings may provide information on the $n=2$ fine structure; see M. Leventhal, Bull. Am. Phys. Soc. 11, 327 (1966).

[8]W. E. Lamb, Jr., and R. C. Retherford, Phys. Rev. 79, 549 (1950).

[9]This confirms the assumed value of the $2P$ lifetime, $\tau = 1.595\times10^{-9}$ sec, to better than 1%.

[10]For example, we previously used an incorrect value of the free-electron–free-proton g-value ratio; see footnote 55 in Ref. 1. Correcting this, and incorporating the latest atomic constants, we find that $\mathcal{S}$ from H(605) should be lowered by 0.01 MHz.

[11]The "beam-tilt" effect is mainly responsible for the "beam notch" discussed in Sec. IIIA of Ref. 1. Considering this effect, we can now account for better than 99% of the β beam at the crossing point. This places an upper limit on stray-field quenching; we calculate that stray fields cannot be larger than $|E_S| = 0.1$ V/cm, as compared with the applied quenching field $|E_Q| = 0.6$ V/cm.

[12]We have also measured such a difference in deuterium. See R. T. Robiscoe and B. L. Cosens, Bull. Am. Phys. Soc. 11, 62 (1966).

SYMMETRY OF PARAELECTRIC DEFECTS IN ALKALI HALIDES*

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For an interpretation of many measurements on paraelectric impurities, such as OH^-, CN^-, NO_2^-, and Li^+ in alkali halides,[1] it is important to know the symmetry of the lattice imperfection created by the impurity. One of the most direct methods of determining the symmetry is by sound velocity and absorption measurements. The results of such studies on Li^+ in KCl are reported here.

For the inelastic properties, an "elastic dipole" can be defined (in analogy to the electric dipole for the dielectric properties) which is a measure of the elastic distortion of the lattice produced by the lattice imperfection and which is a second-rank tensor and can be represented by an ellipsoid (instead of a vector as for the electric moment). For symmetry reasons, the dipoles of the Li^+ (which forms an electric dipole because it is displaced from the center of the vacancy left by the replaced K^+ ion) are ellipsoids of revolution with the dipolar axis as axis of rotation. The stresses resulting from an acoustic wave in a cubic crystal can be represented by "symmetric components" which reflect the symmetry of the crystal.[2] One of these components is, e.g., the hydrostatic pressure $\frac{1}{3}(\sigma_1+\sigma_2+\sigma_3)$ which is totally symmetric (A_{1g} representation). Each of these stress components produces a strain component of the same symmetry, related by the corresponding elastic modulus (again written in symmetric form). In the cubic crystal there are 3 such moduli: $C_{11}+2C_{12}$, $C_{11}-C_{12}$, and C_{44}. The velocity of a wave of given directions of propagation and polarization can then be expressed in terms of these constants; they are given in Table I, column 3. Column 4 indicates the irreducible representations involved. The different symmetric stress components interact with a defect depending on its symmetry.[2] For instance the A_{1g} component (hydrostatic component) does not induce transitions

Table I. Interaction (indicated by "x") of various ultrasonic stresses with defects of different symmetries in a cubic host lattice. The velocity is expressed in terms of the appropriate moduli C_{ij} and the density ρ. "Stress symmetry" indicates the representations of the cubic group according to which the components of the stress tensor for a given acoustic mode transform.

Propagation direction	Polarization direction	Velocity	Stress symmetry	Equilibrium orientation of the defect ⟨100⟩	⟨111⟩	⟨110⟩
[100]	[100]	$\{[(C_{11}+2C_{12}) + 2(C_{11}-C_{12})]/3\rho\}^{1/2}$	E_g, A_{1g}	x		x
[100]	[010]	$(C_{44}/\rho)^{1/2}$	T_{2g}		x	x
[110]	[001]	$(C_{44}/\rho)^{1/2}$	T_{2g}		x	x
[110]	[1$\bar{1}$0]	$[(C_{11}-C_{12})/2\rho]^{1/2}$	E_g	x		x