INSTABILITY OF HELIUM II IN POTENTIAL FLOW BETWEEN ROTATING CYLINDERS*

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The attenuation of second sound has been used to locate hydrodynamic flow transitions in steady-state potential flow of helium II between concentric rotating cylinders. Absence of vorticity is observed until the first transition.

Potential flow between concentric rotating cylinders occurs when $\operatorname{curl}\vec{v}=0$, where the primary flow velocity $\vec{v}$ coincides with the velocities $R_1\Omega_1$ and $R_2\Omega_2$ at the inside and outside cylinders. Defining a Reynolds number at the inside cylinder by $N_1=\nu^{-1}R_1^2\Omega_1$, where ν is the kinematic viscosity, the condition for potential flow is that $N_1=N_2$, where N_2 is the Reynolds number for the outside cylinder. Secondary flow, due to centrifugal forces, occurs when the Taylor boundary is crossed (Fig. 1). The stability of the superfluid and normal-fluid components of helium II between rotating cylinders has been treated theoretically by Chandrasekhar and Donnelly.[1] They assumed the two fluids were coupled by a mutual friction which was proportional to $\operatorname{curl}\vec{v}_S$, where $\vec{v}_S$ is the superfluid velocity. When the superfluid is in primary potential flow, then $\operatorname{curl}\vec{v}_S=0$, and the two fluids are uncoupled. Lord Rayleigh[2] showed that for an (uncoupled) inviscid fluid, the Taylor boundary between primary and secondary flow coincides with potential flow.

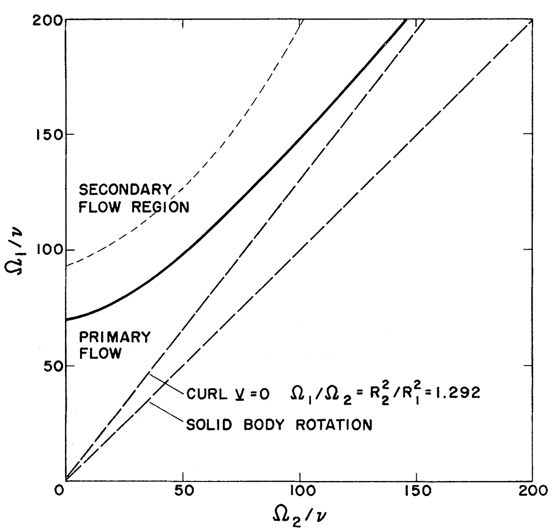

FIG. 1. Stability diagram taken from G. I. Taylor, Phil. Trans. Roy. Soc. London A223, 289 (1923). Ω_1 and Ω_2 are the angular velocities of the inside and outside cylinders; the value for R_2^2/R_1^2 is for the cylinders used by Taylor. The heavy line is the Taylor boundary. The dashed line in the secondary flow region has been added, and represents the second boundary, at which circumferential waves are generated on the toroids comprising the Taylor cells.

We have studied experimentally potential flow of helium II between concentric rotating cylinders, using the annular space as a radial-mode second-sound resonant cavity. We measured the reduction of Q of the resonant cavity due to mutual friction between the normal fluid and vortex lines (when present) in the superfluid. The absence of vortex lines in primary potential flow makes second sound a sensitive detector of the onset of secondary flow, and also makes it possible to measure the average of $\operatorname{curl}\vec{v}_S$ in secondary flow.

The annular resonant cavity, between vertical anodized aluminum cylinders, was 3 in. long, and had i.d. equal to 2.143 in., and o.d. equal to 2.624 in. R_2^2/R_1^2 was equal to 1.500, and potential flow was achieved by a gear train which turned the inside cylinder $\frac{3}{2}$ as fast as the outside cylinder (in the same direction). Two different configurations at the ends of the cylinders were used: First, the outside cylinder was provided with flat parallel ends extending over and under the inside cylinder (with 0.035-in. clearance) to $\frac{5}{8}$-in. bearings at the top and bottom. For the second configuration, the inside cylinder was provided with flat rings at the top and bottom, which closed the ends of the cavity, except for a 0.015-in. gap between the rings and the outside cylinder. The second-sound heater and detector were 1-in. wide Aquadag bands, centered lengthwise on each cylinder. The second-sound Q of the cavity[3] varied with temperature and alignment from 1000 to 3200.

The attenuation of second sound by superfluid vorticity is given by Hall and Vinen[4]:

$$\frac{4\pi f}{Q_0}\left[\frac{A_0}{A(\Omega_1)}-1\right]=B\,|\operatorname{curl}\vec{v}_S\,|. \tag{1}$$

A_0 and $A(\Omega_1)$ are the amplitude of the second-sound signal when the cavity is at rest and when

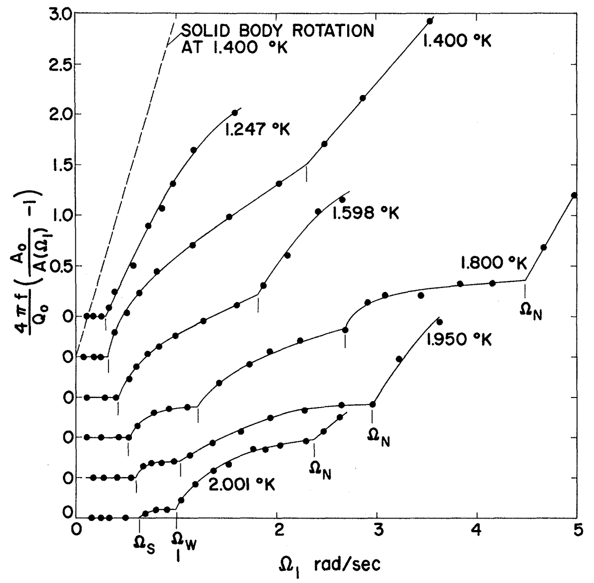

FIG. 2. The attenuation of second sound in steady-state potential flow of helium II between rotating cylinders. $\Omega_1/\Omega_2 = R_2^2/R_1^2 = 1.500$. The dashed line shows the attenuation due to solid-body rotation, for comparison.

rotating; f is the second-sound frequency; Q_0 is the Q of the cavity at rest; and B is the Hall and Vinen perpendicular mutual friction coefficient. Our steady-state results are shown in Fig. 2, where the left-hand side of Eq. (1) is plotted against Ω_1, for measurements at six temperatures. The measurements show that helium II can be rotated in potential flow without any attenuation of second sound when $\Omega_1 < \Omega_S$. For comparison, the attenuation observed with solid-body rotation at 1.400°K is shown as a dashed line. Results similar to Fig. 2 have been obtained by Donnelly[5] with a rotating cylinder viscometer. Donnelly's measurements were made while rotating only the inside cylinder, which is a different form of Couette flow. Donnelly observed two breaks, which he associated with the Taylor boundaries for the superfluid and for the normal fluid.

Since the superfluid in potential flow is in neutral stability, the question arises as to why we observe reproducible breaks in the curves in Fig. 2. The first two breaks at 1.800 and 1.950°K were observed with both configurations of the cylinder ends, and occurred at the same angular velocities, showing that the breaks are not strongly modified by end effects. However, the breaks are dependent on the past history of the liquid: If the helium has been stirred since it was cooled below the lambda transition, the attenuation curve is smoothed out and the breaks are barely discernible.

The curves in Fig. 2 are understood better if account is taken of transient phenomena associated with each point. To advance to the next point, the inside cylinder was accelerated for 1 to 3 sec at the rate of 0.1 rad/sec^2 (the outside cylinder accelerated $\frac{2}{3}$ as fast). Since the acceleration front in the liquid advances slowly, it seems likely that acceleration of the cylinders was followed by a period during which liquid close to the inside cylinder was in unstable flow (above the Taylor boundary in Fig. 1). After acceleration of the cylinders, the second-sound amplitude decreased for 20 to 30 sec. At maximum attenuation, the transient value of $B|\mathrm{curl}\,\vec{v}_S|$ was 1 to 3 sec^{-1}. As potential flow was re-established at the new angular velocity, the transient vorticity died out, over a time interval of 6 to 8 min. When $\Omega_1 < \Omega_S$, the second-sound amplitude recovered to A_0. We suppose that when $\Omega_1 > \Omega_S$, vortex lines generated during the transient period were retained in the superfluid steady-state flow. This would change the hydrodynamics in two ways: First, the superfluid would then be coupled to the normal fluid, and second, the superfluid would have eddy viscosity.

Our experiment does not tell us whether the steady-state vortex lines are parallel to the $\mathrm{curl}\,\vec{v}_S$ which exists during acceleration (parallel to the axis of rotation), or whether they form rings around the inside cylinder, as would be generated in secondary flow (Taylor cells). The succession of breaks on each curve (four at 1.800°K) presumably is associated with different modes of secondary flow, some of which involve the normal fluid.

Hydrodynamic similitude in Couette flow is expressed by use of Reynolds numbers. For the normal-fluid Reynolds numbers N_n, we use the velocity and radius of the inside cylinder, and the normal-fluid viscosity and density. For a superfluid Reynolds number N_S, we have assigned the superfluid an eddy viscosity of 30 μP, a value mentioned by Hall in his study on angular acceleration of liquid helium II.[6] Table I shows the correlation we have been able to achieve, as the superfluid and normal-fluid densities change with temperature. The average variation in N_S (at Ω_S) from the average value is about 8%, and the average variation in N_n (at Ω_n) from the average value is

Table I. Temperature dependence of various quantities, obtained from Fig. 2. The superfluid Reynolds number N_s is calculated assuming an eddy viscosity of 30 μP. Curl $\vec{v}_s$ is evaluated at $\Omega_1 = 1$ rad/sec.

Temperature (°K)	Ω_s (rad/sec)	N_s at Ω_s	Ω_n (rad/sec)	N_n at Ω_n	Curl $\vec{v}_s$ (sec^{-1})
1.247	0.30	10 500			0.81
1.400	0.32	10 800			0.65
1.598	0.42	12 800			0.52
1.800	0.52	13 100	4.48	104 100	0.33
1.950	0.60	11 400	2.95	100 000	0.18
2.001	0.63	10 400	2.38	87 200	0.086
Average values		11 500		97 100	

about 7%.[7] From values of B which we have determined by solid-body rotation,[8] we have calculated the steady-state average values of $|\mathrm{curl}\,\vec{v}_s|$ at $\Omega_1 = 1$ rad/sec. The values given in Table I may be compared with 2 sec^{-1}, the value for solid-body rotation at 1 rad/sec.

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[1]S. Chandrasekhar and R. J. Donnelly, Proc. Roy. Soc. (London) A241, 9 (1957).

[2]Lord Rayleigh, Proc. Roy. Soc. (London) A93, 148 (1917).

[3]We define Q as $f/\Delta f$, where f is the second-sound frequency, and Δf is the full width of the resonance at $\sqrt{2}/2$ of the maximum amplitude.

[4]H. E. Hall and W. F. Vinen, Proc. Roy. Soc. (London) A238, 204 (1956).

[5]R. J. Donnelly, Phys. Rev. Letters 3, 507 (1959).

[6]H. E. Hall, Phil. Trans. Roy. Soc. London A250, 359 (1957). The eddy viscosity in turbulent superfluid is much larger than 30 μP; see P. P. Craig, in Proceedings of the Eighth International Conference on Low Temperature Physics, Washington, D. C., edited by R. O. Davies (Butterworths Scientific Publications, Ltd., London, 1963), p. 102.

[7]The break at Ω_w has not been correlated by a Reynolds number, but at the highest four temperatures in Fig. 1, the centrifugal force on unit volume of normal fluid at the inside cylinder, $\rho_n r_1 \Omega_1^2$, is reasonably constant if $\Omega_1 = \Omega_w$. The agreement may be fortuitous.

[8]P. J. Bendt, Phys. Rev., to be published.

MEASUREMENT OF A BARRIER FOR THE EXTRACTION OF EXCESS ELECTRONS FROM LIQUID HELIUM*

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In this Letter we describe an experiment which has demonstrated the existence of a small potential barrier to the extraction of negative ions (excess electrons) from liquid helium into the vapor. We envision this work as a sequel to the results of Sommer[1] and of Woolf and Rayfield[2] which have shown that there exists a barrier to the penetration of electrons into the liquid. The present results indicate a profound difference in the behavior of positive and negative ions crossing the liquid surface, as first noticed by Careri, Fasoli, and Gaeta,[3] and are consistent with the model of the excess electron as a localized state in liquid helium which has been studied by Jortner et al.[4] and by Hiroike et al.[5]

The experiment is performed by condensing liquid into one of the cells shown in Fig. 1 to a height h above the grid. The cell is then sealed and the current extracted from the liquid is measured as a function of temperature, keeping constant the extracting and collecting fields E_x and E_c. The characteristic so measured is shown in Fig. 2, where temperature-independent (TI) and temperature-dependent (TD) regions are observed.