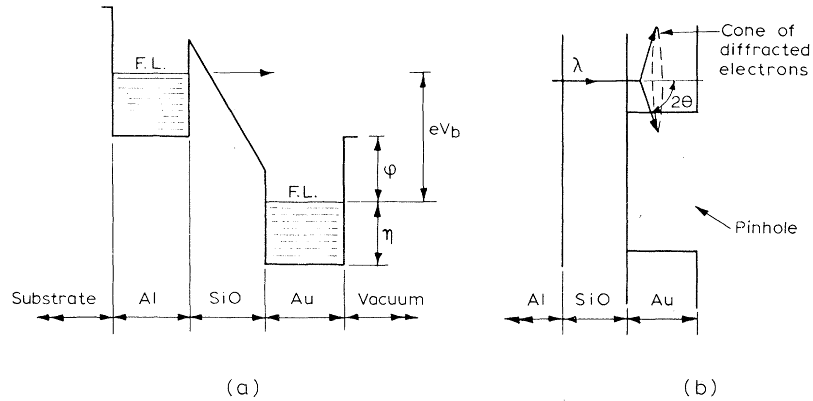

FIG. 2. Illustrations of (a) energy diagram of the cathode showing the electron transport, and (b) physical diagram of part of cathode, in the vicinity of a pinhole with the cone of diffraction of the electrons superimposed.

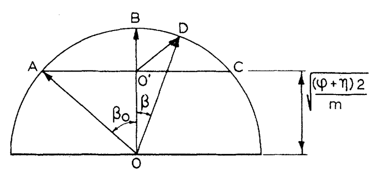

FIG. 3. Velocity diagram of $v_{\parallel}$. Vectors from O to semicircle represent all possible velocity vectors of electrons arriving at pinhole edge. Vectors drawn from O' to the arc ABC represent all the possible velocity vectors normal to the pinhole edge outside the metal.

the image at the screen and not the shape.

It will be apparent that the arc ABC reflects the image of the diffracted electrons at the phosphor screen.[4] The radius of arc, r, is given by

$$r = |v_{\parallel}|t = \frac{24.54s}{dV_a^{1/2}}\left[1 - \frac{36}{d^2(V_b+\eta)}\right]^{1/2} \text{cm}, \qquad (1)$$

where s = cathode-screen separation in cm, d is the lattice spacing in angstroms, and V_a is the acceleration potential. Thus according to (1), increasing V_b or decreasing V_a causes the radii of the rings to increase, as observed, while the experimentally determined radii are in reasonable agreement with the theoretical values calculated using (1).

The encouragement and support given by Dr. P. White is gratefully acknowledged.

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[1]J. G. Simmons and R. R. Verderber, to be published.

[2]All energies are measured in volts.

[3]F. Seitz, Modern Theory of Solids (McGraw-Hill Book Company, Inc., New York, 1940), p. 146.

[4]If the electron had passed through the upper surfaces the images would have been complete circles. This process is inhibited because $v_{\perp} < [2(\varphi+\eta)/m]^{1/2}$, and also the thickness of the Au film is much greater than the attenuation length of electrons in Au.

[5]Work function of Au is 4.82 eV: Handbook of Chemistry and Physics, edited by C. D. Hodgman (Chemical Rubber Company, Cleveland, Ohio, 1952), 37th ed., p. 2344.

FLUCTUATIONS IN A PLASMA CONFINED BY A TOROIDAL OCTUPOLE MAGNETIC FIELD*

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The stability of a collisionless plasma confined by a toroidal octupole magnetic field was studied to check the validity of the $\oint dl/B$ stability theory. Stability analysis using a fluid model predicts that the plasma will be stable with respect to interchange perturbations when $\oint dl/B$ decreases from the center of the confinement region.[1] The central confinement region of the Wisconsin toroidal octupole[2,3] satisfies the $\oint dl/B$ condition for stability. However, outside a flux surface ψ_c near the outer conducting vacuum wall, the $\oint dl/B$ principle predicts instability (Fig. 1).

A hydrogen plasma injected into the confin-

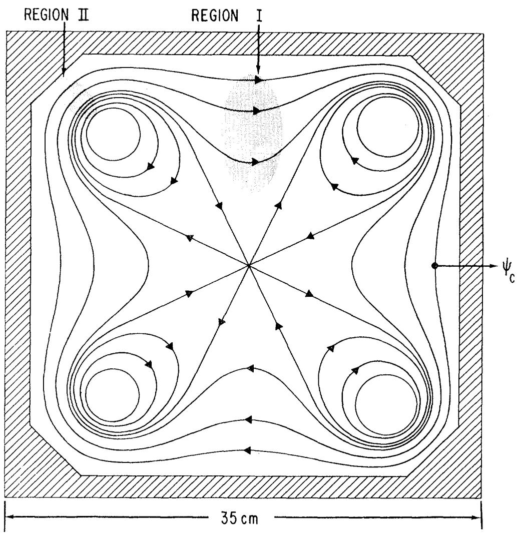

FIG. 1. View of the minor cross section of the toroidal octupole magnetic field pattern. $\oint dl/B$ is a minimum on the flux surface labeled ψ_C.

ing region had a peak density of $(1\text{-}2)\times10^9$ cm^{-3}, $T_e \simeq 10$ eV, and $T_i \simeq 50$ eV. Under these conditions, the plasma was collisionless during the 10^{-3} sec that the plasma was being lost on the hoop supports and there was a minimum of 6.6 ion gyroradii across the stable confinement region. Since the plasma was collisionless and $\beta = 2\mu_0 nkT/B^2 \ll 1$, the plasma velocity across the magnetic field was given by $\vec{V} = \vec{E}\times\vec{B}/B^2$. During the confinement time, the electric field fluctuations were very small in the $\oint dl/B$ stable region but became very large in the $\oint dl/B$ unstable region.[4] In the unstable region, the floating potential fluctuations were 1-2 V ($T_e \simeq 10$ eV), the characteristic frequencies were near 100 kHz ($eB/M_i \simeq 1.5$ MHz), the perpendicular wavelength was about 3 cm, and the parallel wavelength was essentially infinite. The phase velocity of the fluctuations perpendicular to the field in a reference frame where $E_0 = 0$ was in the same direction as the ion gradient-B drift in the region of unfavorable curvature behind the current-carrying hoop (region II). These observations identify the instability in the $\oint dl/B$ unstable region as the interchange or flute instability.

We now consider the stability of the plasma in the region of unfavorable curvature (region II) taking into account the finite ion gyroradius and finite density but neglecting connection along a line of force to the neighboring region of favorable curvature (region I). The results of Mikhailovski[5] predict that for the parameters of this experiment, the plasma in the region of bad curvature (region II) would be unstable to interchange perturbations if there were no $\oint dl/B$ stabilization. Detailed measurements were made in the region of locally unfavorable curvature outside the hoops (region II). If the $\oint dl/B$ principle were not valid, then a flute instability could grow in this region in about 0.2 μsec. However, the plasma was again observed to be stable in the region predicted by the $\oint dl/B$ principle.

Measurements of ion saturation currents were similar to the electric field measurements, a smooth decay in the predicted stable region and 20-50% fluctuations in the predicted unstable region. The density profile in this unstable region is shown in Fig. 2. Any density gradient in the unstable region was quickly removed by the flute instability. However, the plasma in this region was not immediately lost due to the presence of the conducting walls. As the flutes approach the conducting wall by an $\vec{E}\times\vec{B}$ drift, they must stop since the tangential component of $\vec{E}$ must be zero at the conducting wall. However, ions within an ion gyroradius from the wall were absorbed reducing the density there. This situation was also observed by Ioffe[6] in a mirror experiment which was $\int dl/B$ unstable everywhere in the confinement region. However, the plasma in the $\oint dl/B$ stable region of the toroidal octupole can be stably confined as evidenced by the steep density profile inside ψ_C.

Measurements of density fluctuations, δE, and the density gradient ∇n enabled us to cal-

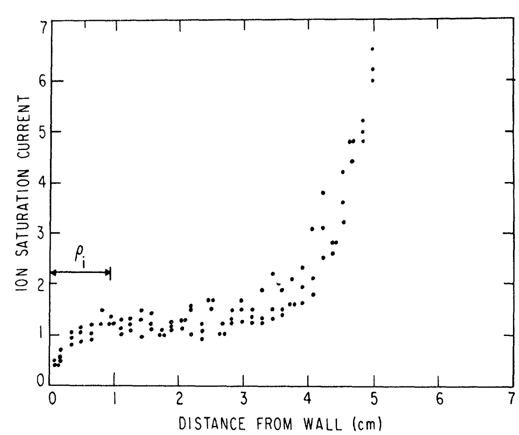

FIG. 2. Ion saturation current (proportional to plasma density) in the $\oint dl/B$ unstable region near the conducting wall.

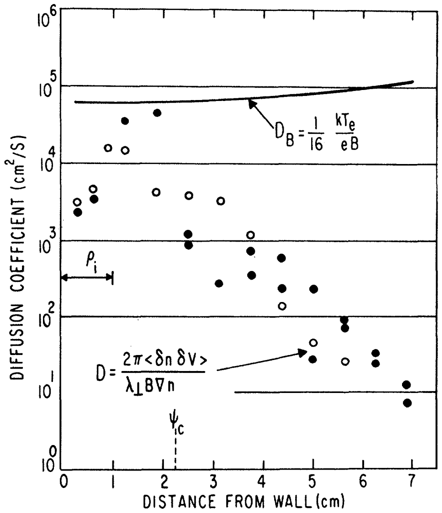

FIG. 3. Calculated diffusion coefficient in the $\oint dl/B$ unstable region near the conducting wall.

culate an effective turbulent diffusion coefficient D. The time-average flux of plasma to the wall caused by the fluctuations[7] is given by

$$\langle nv\rangle = \langle \delta n \delta E\rangle/B = 2\pi\langle \delta n \delta V\rangle/\lambda_\perp B = -D\nabla n,$$

where δV is the floating potential fluctuation and $\lambda_\perp$ is the wavelength of the fluctuation perpendicular to the magnetic field. The effective diffusion coefficient as a function of distance from the wall is shown in Fig. 3; here we have assumed that δn and δE were exactly in phase thereby giving a maximum diffusion coefficient. In the $\oint dl/B$ unstable region where the fractional change in $\oint dl/B$ is +5%, the turbulent diffusion coefficient was the same order of magnitude as the Bohm diffusion coefficient. However, in the stable confinement region where the fractional change in $\oint dl/B$ is −10%, the observed diffusion coefficient was four orders of magnitude below that given by the Bohm formula. It should be noted that the confinement of a similar plasma ($n \simeq 5\times10^9$ cm^{-3}, $T_e \simeq 10$ eV) in the Model-C Stellarator[8] was limited by a diffusion coefficient that was approximately the Bohm value, and potential fluctuations the order of kT_e/e were present throughout the plasma.

The confinement of plasma in the toroidal octupole appeared to be limited by thermal flow of particles into the supports for the current-carrying hoops, since the lifetime of ions of a given energy was inversely proportional to their velocity over the entire ion-energy spectrum.[3] The lifetime of the ions averaged over their spectrum was approximately 600 μsec. The calculated mean lifetime assuming isotropic bombardment of the supports and the high-permeability tube for the ion-energy analyzer was 1000 μsec.

If it is assumed that the Bohm diffusion coefficient $D = kT_e/16eB$ is a local quantity, then the lifetime of the plasma in the nonuniform magnetic field of the toroidal octupole can be calculated. Since the diffusion of plasma across surfaces of magnetic flux ψ is slow compared to the thermal speed of particles along the field lines, the density will be a function only of ψ. We now arrive at a diffusion equation given by

$$\frac{\partial n}{\partial t} = +\frac{\pi^2}{4}\frac{(kT_e/e)}{(dV/d\psi)}\oint R^2 dl\,\frac{\partial^2 n}{\partial \psi^2},$$

where $dV/d\psi$ is the volume of a unit flux tube and R is the distance from the major axis of the toroid to the element dl on a field line. The lifetime for the fundamental mode $n = n_0 \times\cos(\pi\psi/2\psi_c)$ is 160 μsec for the parameters of this experiment. The observed density profile $n \simeq n_0\exp(-3\psi/\psi_c)$ was peaked very sharply near the center confinement region again suggesting loss due to supports rather than Bohm diffusion.

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