lies in the upper half-z plane. From the bound (10) and the growth condition (5), we infer that

$$
\begin{equation*}
\int_{\Gamma} \frac{\ln (1+|f(t)|)}{\left(1+|t|^{3 / 2}\right)}|d t|<\infty \tag{11}
\end{equation*}
$$

This gives an upper bound on the form factor along $\Gamma$. The desired lower bound now follows as a consequence of the theorems in Boas ${ }^{3}$ applied to the function $C(z)=f\left[(z-i \alpha)^{2}\right]$.

It would be of interest to extend this result to other amplitudes.

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# MEASUREMENT OF PHOTON TIME-OF-ARRIVAL DISTRIBUTION IN PARTIALLY COHERENT LIGHT* 

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We have measured the distribution in time of arrival of photons in a highly monochromatic and spatially coherent light beam from a low-pressure $\mathrm{Hg}^{198}$ discharge lamp. This distribution could provide fairly direct information about the spectral profile of the light source and the shape of the resultant photon wave packets. By using a time-to-height converter and pulse-height analyzer instead of the usual coincidence circuit, ${ }^{1,2}$ all arrival times in the range 0 to 30 nsec were recorded during the same run. This completely eliminated datanormalization and lamp-stabilization problems. Since the resolution time of the photomultipliers used was about four times the coherence time of the light, the shape of the measured time distribution reflects the tube resolution but the height or number of excess counts provides a measure of the width of the mercuryline spectral profile. The time distribution agrees satisfactorily with a curve calculated from the Fourier transform of the spectral distribution as measured with a long Fabry-

Perot interferometer.
Light from a standard $\mathrm{Hg}^{198}$ discharge tube excited by about 10 W of $3-\mathrm{kHz}$ rf power was made spatially coherent by a $0.14-\mathrm{mm}$ aperture near the lamp and a $2.4-\mathrm{mm}$ aperture 1.4 m from the first (Fig. 1). An interference filter between the two apertures selected the blue $4358-\AA$ mercury line. The light leaving the second aperture was divided by a $45^{\circ}$ beamsplitting mirror and fell on the photocathodes of two RCA 7746 photomultiplier tubes. The photocathodes were each 4.0 in . away from the beam splitter. Pulses from the photomultipliers were sent to discriminators which standardized any input pulse over 100 mV into an output pulse 300 mV high and 30 nsec long. The discriminator output pulses were put through coincidence circuits acting as gates, the gating signals coming from a second set of discriminators and coincidence circuit that insured that the two photomuliplier output pulses were greater than 125 mV in height and within 30 nsec of each other. The gating arrangement


FIG. 1. The optical and electronic experiment arrangement. The two coincidence circuits immediately preceding the time-to-height converter were used as gates to reduce time slewing in the $100-\mathrm{mV}$ discriminators.
insured that the pulses used were well above threshold for the timing discriminators, improving their timing resolution, and that pulse pairs greater than 30 nsec apart were rejected early in the logic system. The output of the two coincidence circuits used as gates, with one output delayed by 15 nsec , were sent to a time-to-height converter whose output consisted of pulses whose height was proportional to the time overlap of the two input pulses. These pulses were sorted according to height and stored in the memory of a pulse-height analyzer. All of the fast-time resolution circuits were standard commercial modules. ${ }^{3}$

In an idealized experiment in which the beam falling on the photomultipliers was completely spatially coherent and completely polarized, and the photomultipliers had infinitely short time resolution and no noise, the expected number of counts recorded as a function of the difference in time of arrival of two photons at the two photocathodes would be ${ }^{4}$

$$
R(\Delta t)=R_{0}\left(1+|\gamma(\Delta t)|^{2}\right)
$$

where $R_{0}$ is the product of the number of single photons per second counted by each of the two photomultipliers and $\gamma(\Delta t)$ is the Fourier transform of the spectral intensity profile of
the light source; that is,

$$
\gamma(\Delta t)=\int g(\omega) e^{i \omega \Delta t} d \omega
$$

where $g(\omega)$ is the normalized intensity distribution of the light source. For the type of timeanalysis system used in this experiment, $R(\Delta t)$ should be multiplied by the probability that no count occurs in the interval 0 to $\Delta t$ before one occurs at $\Delta t$, but since the mean time between counts at each photomultiplier was $10^{5} \mathrm{nsec}$, this probability was essentially unity for times from 0 to 30 nsec .

For a Doppler-broadened mercury line,

$$
g(\omega)=\frac{1}{(2 \pi)^{1 / 2}{ }_{\omega}} \exp \left[\frac{-\left(\omega-\omega_{0}\right)^{2}}{2 \sigma_{\omega}^{2}}\right],
$$

a Gaussian function of $\Delta \omega=\omega-\omega_{0}$, with standard deviation $\sigma_{\omega}$. Measurement of the intensity distribution of the $4358-\AA$ line with a FabryPerot interferometer gave a full width at halfmaximum of $0.008 \AA$ or a $\sigma_{\omega}$ of $3.4 \times 10^{9} \mathrm{rad} /$ sec. A Gaussian $g(\omega)$ leads to a Gaussian $|\gamma(\Delta t)|$ or

$$
\gamma(\Delta t)=\exp \left(-\Delta t^{2} / 2 \sigma_{t}^{2}\right) \exp \left(i \omega_{0} \Delta t\right)
$$

where $\sigma_{t}=1 / \sigma_{\omega}$. Finally, if we let $\sigma_{T}=\sigma_{t} / \sqrt{2}$,

$$
|\gamma(\Delta t)|^{2}=\exp \left(-\Delta t^{2} / 2 \sigma_{T}{ }^{2}\right),
$$

and the expression for the counting rate versus time delay becomes

$$
R(\Delta t)=R_{0}\left[1+\exp \left(-\Delta t^{2} / 2 \sigma_{T}{ }^{2}\right)\right],
$$

leading to the familiar curve of constant background with a Gaussian peak near $\Delta t=0$.
In a real experiment, this curve of $R(\Delta t)$ is modified in two ways. The finite resolution time of the photomultipliers must be folded into the Gaussian peak and the effects which tend to decorrelate the photon counts must be taken account of. Previous experiments have been able to take accurately into account the latter but not the former effect. ${ }^{1,2,5}$
In the present experiment, the photomultiplier resolution time was measured by allowing simultaneous single photons to strike the two photocathodes. The simultaneous photons were produced by $2-\mathrm{MeV}$ electrons from a $\mathrm{Sr}^{90}-$ $\mathrm{Y}^{90}$ source travelling through a half inch of Lucite and producing about 50 photons of Čerenkov radiation. The small solid angle and efficiency for detection of these photons at the two photocathodes insure that the coincidences
counted are indeed single-photon events. The measured time distribution for these simultaneous photons is shown in Fig. 2(c). The full width at half maximum of 2.3 nsec is due mostly to transit time variations in the first few dynodes of the photomultipliers. This photomultiplier response function for simultaneous events can be written as

$$
P(\Delta t)=\exp \left(-\Delta t^{2} / 2 \sigma_{p}^{2}\right)
$$

where $\sigma_{p}=0.91 \mathrm{nsec}$. With the photomultiplier resolution function folded in, the expected counting rate becomes

$$
R(\Delta t)=R_{0}\left[1+\frac{\sigma}{\sigma_{p^{\prime}}} \exp \left(\frac{-\Delta t^{2}}{2 \sigma_{p^{\prime}}{ }^{2}}\right)\right]
$$

where $\sigma_{p^{\prime}}=\left(\sigma_{T}{ }^{2}+\sigma_{p}{ }^{2}\right)^{1 / 2}=0.95 \mathrm{nsec}$. This keeps the total number of counts under the peak constant, since the peak is widened by $\sigma_{p^{\prime}} / \sigma_{T}$ and lowered by $\sigma_{T} / \sigma_{p^{\prime}}$. In the present case, $\sigma_{T} /$ $\sigma_{p^{\prime}}=0.22$.
When decorrelation effects are taken into account, $R(\Delta t)$ becomes

$$
R(\Delta t)=R_{0}\left[1+D \frac{\sigma_{T}}{\sigma_{p^{\prime}}} \exp \left(\frac{-\Delta t^{2}}{2 \sigma_{p^{\prime}}{ }^{2}}\right)\right]
$$

$D$, the decorrelation coefficient, is contributed to by the lack of polarization of the beam ( 0.50 ), the integral of the spatial coherence of the beam ( 0.67 ), the signal-to-noise ratios of the photomultipliers ( 0.74 ), and the polarization selection of the $45^{\circ}$ beam splitter ( 0.96 ). These effects combine to give a value of $D=0.24$. Allowing for photomultiplier resolution and decorrelation effects, the expected counting rate is

$$
R(\Delta t)=R_{0}\left[1+0.053 \exp \left(-\Delta t^{2} / 2 \sigma_{p^{\prime}}{ }^{2}\right)\right]
$$

The data are presented in Fig. 2(a) together with a plot of the expected $R(\Delta t)$. The data and expected curves agree satisfactorily, giving good evidence for the bunching of photons in a beam of Gaussian light. The shape of the curve is governed mostly by the photomultiplier response function and its height is directly proportional to $\sigma_{T}$ or the coherence time of the light. It is also possible to identify $\sigma_{t}$ with the width of the Gaussian wave packets of the mercury-light photons.

For comparison, Fig. 2(b) shows the results of runs taken with an incandescent light source,


FIG. 2. (a) The distribution of time of arrival of photons in a spatially coherent beam of light from a lowpressure $\mathrm{Hg}^{198}$ lamp. The solid line is a curve calculated from the spectral distribution of the light and the photomultiplier response function. The error bar shown is statistical and applies to all points. One channel corresponds to 0.25 nsec . (b) The distribution of time of arrival of photons in a beam of light from an incandescent lamp with all conditions otherwise the same as in (a). (c) The response (calibration) of the detection system to simultaneous single photons from a Čerenkov source. Again, one channel corresponds to 0.25 nsec . The full width of the curve at half-maximum is 2.3 nsec .
for which the coherence time is so short that the peak height is reduced until $R(\Delta t)$ is indistinguishable from a straight line.

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iment and Lansing Research Corporation for the loan of a Fabry-Perot interferometer.

[^1][^2]
# CLASSIFICATION OF THE $Y_{1}{ }^{*}(1765)$ AND $Y_{0}{ }^{*}$ (1520) RESONANCES <br> IN THE 1134 REPRESENTATION OF SU(6) * 

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The $Y_{1}{ }^{*}(1765) \frac{5}{2}-$ resonance exhibits ratios of meson-baryon ( $\underline{8} \times \underline{8}$ ) decay widths to meson-baryon resonance ( $\underline{8} \times \underline{10}$ ) decay widths such as to indicate that it can be classified in the 1134 product representation of $\mathrm{SU}(6)$. A comparison of the decay modes of the $Y_{0}{ }^{*}(1520)$ with those of the $Y_{1}{ }^{*}(1765)$ shows that it, too, may be classified in the 1134 .

The two-particle decay widths of both baryon and meson resonances may be used to determine (a) in which $\operatorname{SU}(3)$ multiplet they occur, and (b) whether they may be accommodated in some bigger multiplet of a higher symmetry scheme. In the present work, we correlate the various decays of the $Y_{1}{ }^{*}(1765) J^{P}=\frac{5_{2}}{}{ }^{-}$ and $Y_{0}{ }^{*}(1520) J P=\frac{3}{2}-$ resonances on the basis of relativistic $\operatorname{SU}(6)$ and show that their decays are compatible with an assignment to the 1134 representation of $\operatorname{SU}(6)$.
The various $Y_{1}{ }^{*}(1765)$ relative decay widths, listed in Table I, have been extracted from the recent work of Uhlig et al. ${ }^{1}$ A total decay width of 105 MeV is used. The decay widths of the $Y_{0}{ }^{*}(1520)$, also listed in Table I, have been taken from the compilation of Rosenfeld et al. ${ }^{2}$ The $Y_{1}{ }^{*}$ decays via two modes of particular interest to us, namely, meson-baryon $(\underline{8} \times \underline{8})$ and meson-baryon resonance $(\underline{8} \times 10)$. The baryon resonances are those of the $\overline{J P}=\frac{3}{2}+$
decuplet. A comparison of decay widths with the predictions of $\mathrm{SU}(3)$ within one mode of decay determines the $\operatorname{SU}(3)$ nature of the decaying state. The role of a higher symmetry scheme is to relate the two modes. As mentioned in Ref. 1 and as may be seen from Table I, the meson-baryon decay modes are consistent with an assignment of the $Y_{1}{ }^{*}(1765)$ to an $\operatorname{SU}(3)$ octet, rather than to a 27,10 , or $10^{*}$. In our classification attempt we ignore the absence of the other members of the $\frac{5}{2}^{-}$octet, being content to try to pin down the assignment of the state at hand. This may be a fruitful philosophy for treating members of the as yet incomplete $\frac{3}{2}^{-}$and $\frac{5}{2}^{+}$multiplets. In trying to embed our octet state in a representation of a higher symmetry scheme, we choose as the underlying theoretical basis the relativistic $\operatorname{SU}(3)$ theory ${ }^{3}$ in which the particles at rest are grouped according to nonchiral $\mathrm{U}(6) \otimes \mathrm{U}(6)^{4,5}$ and identified in $\operatorname{SU}(6) S \cdot{ }^{6}$ Their decays are cal-


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