Table I. Values of the Grüneisen constant, γ, as obtained from our Eq. (2).

	γ(calculated)	γ(thermal)	γ(compressibility data)
Li	0.60[a]	1.17[d]	1.53,[g] 0.63[h]
Na	1.15[a]	1.25[d]	1.18[g]
K	1.14[a]	1.34[d]	1.29[g]
Rb	1.14[a]	1.48[d]	0.90[g]
Sn	1.81[b]	2.14,[d] 2.03[e]	2.01,[i] 1.85[j]
In	2.61	2.21[e]	2.24[j]
Se	2.26	2.63[f]	
Al	2.83	2.17[d]	1.96[g]
Tl	2.56	2.73,[d] 2.96[e]	2.13[j]
Ni	2.63[c]	1.88,[d] 1.91[e]	2.18,[i] 1.81[j]

[a]From the values of constant C_1 in Eq. (1) as given by Kraut and Kennedy.[2]

[b]Below polymorphic transition.

[c]Melting-point data from H. M. Strong and F. P. Bundy, Phys. Rev. 115, 278 (1959). Compression data from Ref. 7. Plot lies above the range of Fig. 1.

[d]From thermal measurements (Ref. 8).

[e]From thermal measurements (Ref. 10).

[f]Calculated from available thermal data.

[g]From pressure dependence of compressibility data (Ref. 9).

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[j]From shock-wave experiments (Ref. 10).

ical test of our Eq. (2) will become possible.

It should be emphasized that the melting equation in the present form does not involve any adjustable parameter. Equation (2) can be employed with reasonable confidence, using the values of γ, which can be easily obtained from thermal and other measurements. This point is of importance in geophysical and other applications.

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Note added in proof.–We have been informed that related work has been published by J. Gilvarry, Phys. Rev. Letters 16, 1089 (1966); this work is unfortunately not yet available to us.

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ULTRASONIC EVIDENCE FOR STRONG COUPLING OF ELECTRONS TO TRANSVERSE PHONONS IN SINGLE-CRYSTAL INDIUM*

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When the phonon angular frequency ω and wave number q satisfy the conditions $\omega > 10^{10}$ sec^{-1} and $ql \gg 1$, where l is the electron mean free path, then the "electromagnetic" and collision-drag interactions may be neglected, and free-electron theory predicts that transverse phonons and the conduction electrons in a metal are not coupled.[1] Electron-phonon coupling associated with shear deformation of the Fermi surface can become effective only for arbitrary Fermi surfaces.[2] Since the latter mechanism is ordinarily assumed to be very small relative to electromagnetic interaction, the free-electron prediction of essential decoupling of transverse phonons from conduction electrons in the simpler metals has been regarded generally as a very reasonable one in the regime of total breakdown of electromagnetic screening. We should like to report experimental evidence for strong "residual" coupling of low-temperature transverse phonons to the conduction electrons in indium.[3] To our knowledge,

these data,[4] together with the results corresponding to the longitudinal modes reported earlier,[5] represent the first such description of Fermi surface "dynamical properties" determined experimentally for a metal. The data describe the set of deformation integrals over the zones of electron-phonon interaction for essentially the complete array of transverse, as well as longitudinal, pure propagation modes in single-crystal indium.

It has been suggested by Chambers[6] and Pippard[2,7] that ultrasonic attenuation measurements could produce useful information on the coupling parameter describing Fermi surface deformation due to interaction with phonons. It was noted,[2,7] however, that while the longitudinal-wave electronic attenuation, α_L, is for the case $ql > 1$ describable in terms of a deformation parameter K_{ij}, through the relation

$$\alpha_L = \frac{\hbar q}{4\pi^2 \rho v_L} \oint RK_{ij}^2 d\psi, \qquad (1)$$

the measured transverse-wave absorption coefficient α_t is—even when $ql \gg 1$—only very indirectly relatable to corresponding deformation parameters, through an integral taken over the entire Fermi surface. In Eq. (1) the integral is taken about the effective zones of the Fermi surface; ρ is the density of the metal, v_L is the longitudinal-phonon velocity for the particular mode indicated by propagation and polarization indices i and j, respectively, and R is the product of the principal radii of curvature. It was shown[8] by one of the present authors (JRL), however, that in the absence of electromagnetic interaction and for $ql \gg 1$ (a) the transverse-wave electronic attenuation reduces to a purely shear-deformation contribution, α_D, of the same form as Eq. (1), i.e.,

$$\alpha_D = \frac{\hbar q}{4\pi^2 \rho v_t} \oint RK_{ij}^2 d\psi, \qquad (2)$$

where v_t is the transverse wave velocity, and (b) α_D can readily be determined experimentally in a type-I superconductor satisfying $ql \gg 1$, since breakdown of screening is accomplished at ordinary ultrasonic frequencies "immediately" below T_c as a consequence of the Meissner effect. [The remaining electronic attenuation is given by Eq. (2).] Consequently it becomes possible to measure the "deformation coupling" associated with transverse as well as longitudinal phonons.

On this model the shear-deformation interaction, and its variation over the effective zones of the indium Fermi surface, has been studied experimentally by means of pulse-ultrasonic techniques. Due to the anomalously high electronic attenuation, arising in part from the very low shear-wave velocities[9] in indium, total electronic attenuation values ranged from 50 to 172 dB/cm for the various shear-wave modes, at frequencies ranging from 30 to 150 Mc/sec. The strain-amplitude effect in indium was corrected by extrapolating to zero the attenuation values measured as a function of amplitude at low amplitudes.[4,5] The single-crystal samples were made approximately 1 mm in thickness in order to permit accurate measurements at the aforementioned attenuation levels and to minimize strain-amplitude effects, which at the lowest peak voltage applied (~0.5 V) did not exceed roughly 10%. The value of ql exceeded 100 at a frequency of 90 Mc/sec.

In Fig. 1 are shown representative shear-wave data (here uncorrected for strain-amplitude effect) on electronic attenuation versus temperature. The electronic attenuation is separable into regions α_D and α_t^E (cf. Ref. 8), where α_t^E is the so-called "rapid fall" contribution attributable to electromagnetic interaction. Since the attenuation α_t^E "turns off," not immediately at T_c but within a very narrow temperature range (dependent on frequency), there is from $t \equiv T/T_c = 1$ to $t \sim 0.99$, roughly, a region of overlap between the electromagnetic and deformation contributions. The deformation contribution at $t = 1$ is determined by extrapolating the BCS function for the temperature dependence of the ultrasonic attenuation from just below the break point up to $t = 1$. Techniques for expanding the temperature scale near T_c and continuously recording the $\alpha(t)$ trace in the overlap region will be discussed elsewhere.

In Table I is summarized the magnitude of the observed shear-deformation absorption, α_D, compared with that for the total transverse phonon-electron interaction, α_t. The ratio α_D/α_t is frequency independent.[8] It is immediately apparent that deformation is, surprisingly, a very significant part of the total electronic interaction with low-temperature transverse phonons in indium; coupling to the electron system is strong in indium even in the absence of electromagnetic interaction. It may be noted incidentally from Table I that there is no sig-

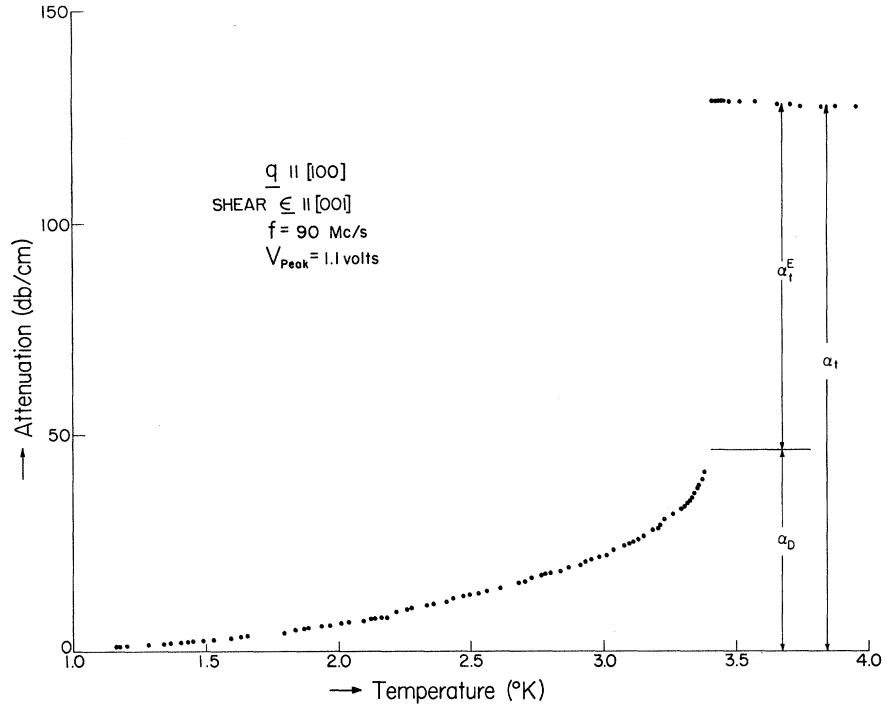

FIG. 1. Temperature dependence of shear-wave ultrasonic attenuation by electrons in indium.

nificant anisotropy observed in α_D/α_t for the indicated phonon orientations. In Table II are listed experimentally determined deformation integrals corresponding to pure longitudinal and transverse phonons in single-crystal indium. It is seen that the deformation integrals are considerably smaller for the transverse phonon-electron interaction and, as we have noted elsewhere,[4,5] significant anisotropy is present only among the longitudinal cases, viz., for $q \parallel [110]$. (The longitudinal-wave anisotropy has been observed also by Bliss and Rayne.[10])

On comparison with the predictions of the free-electron model, which neglects shear-deformation interaction, the data summarized in Tables I and II are seen to be reasonable quantitatively. It has been shown[8] that, for

Table I. Observed shear-deformation contribution compared with total transverse phonon-electron interaction.

Orientation	α_D/α_t
$\vec{q} \parallel [110]$, $\vec{\epsilon} \parallel [1\bar{1}0]$	0.35 ± 0.04
$\vec{q} \parallel [110]$, $\vec{\epsilon} \parallel [001]$	0.32 ± 0.04
$\vec{q} \parallel [001]$, $\vec{\epsilon} \perp [001]$	0.34 ± 0.04
$\vec{q} \parallel [100]$, $\vec{\epsilon} \parallel [001]$	0.33 ± 0.04

$ql \gg 1$,

$$\alpha_t - \alpha_D = \alpha_t^E, \tag{3}$$

where α_t^E represents that part of the total electronic attenuation which is associated with electromagnetic interaction and α_D is given by Eq. (2). The experimentally observed α_D value represents the correction term which must be applied to the observed attenuation coefficient α_t in order to extract the absorption coefficient α_t^E; and since, as has been noted, the shear-deformation contribution is neglected in the free-electron model, it is α_t^E which must be compared with the result of the free-electron calculation,[11] which we shall call α_t^f. Although the experimentally derived values for α_t^E cannot be expected, in principle, to be precisely the same as the free-electron predictions, it is not unreasonable to expect them to be in fair agreement. The electromagnetic interaction, even for $ql \gg 1$, is not restricted to the effective zones but involves a complex function of the deformation parameter averaged over the whole Fermi surface.[7] The assumption is seen in Table III to be a good one. It is worth noting that in the absence of the α_D correction, the observed values for α_t differ from the free-electron prediction α_t^f by $\approx 30\%$.

Table II. Deformation integrals for longitudinal and transverse phonons in single-crystal indium.

Orientation	Frequency (Mc/sec)	α_D (dB/cm)	α_L [a] (dB/cm)	$\oint RK_{ij}^2 d\psi$ ($\times 10^{30}$ cm^{-4})
$\vec{q} \parallel [110]$, $\vec{\epsilon} \parallel [1\bar{1}0]$ transverse	90	60.0		68.3
$\vec{q} \parallel [110]$, $\vec{\epsilon} \parallel [001]$ transverse	90	46.6		56.4[b]
$\vec{q} \parallel [001]$, $\vec{\epsilon} \perp [001]$ transverse	90	54.2		65.8
$\vec{q} \parallel [100]$, $\vec{\epsilon} \parallel [001]$ transverse	90	52.5		63.8
$\vec{q} \parallel [110]$ longitudinal	150		53.4	306
$\vec{q} \parallel [001]$ longitudinal	270		62.5	164
$\vec{q} \parallel [100]$ longitudinal	270		59.0	162

[a]See Ref. 5.
[b]Calculated from lower frequency data.

Table III. Comparison of attenuation due to electromagnetic interaction with free-electron prediction.

Orientation	Frequency (Mc/sec)	α_t (dB/cm)	α_D (dB/cm)	α_t^E (dB/cm)	α_t^f (dB/cm)
$\vec{q} \parallel [001]$, $\vec{\epsilon} \perp [001]$	90	159	54.2	104.8	111
$\vec{q} \parallel [100]$, $\vec{\epsilon} \parallel [001]$	90	158	52.5	105.5	111
$\vec{q} \parallel [110]$, $\vec{\epsilon} \parallel [1\bar{1}0]$	90	172	60.0	112	118
$\vec{q} \parallel [110]$, $\vec{\epsilon} \parallel [001]$	90	145[a]	46.6[a]	98.4[a]	111

[a]Calculated from lower-frequency data.

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