[5]C. G. Wohl, F. T. Solmitz, and M. L. Stevenson, Phys. Rev. Letters 17, 107 (1966).

[6]W. M. Smart, A. Kernan, G. E. Kalmus, and R. P. Ely, Jr., Lawrence Radiation Laboratory Report No. UCRL-16961 (unpublished).

[7]R. D. Tripp, in Proceedings of the International School Physics "Enrico Fermi," Course XXXIII (to be published); CERN Report No. CERN 65-7, 1965, revised (unpublished). We use the same notation and sign conventions as this reference.

[8]M. Taher-Zadeh, D. J. Prowse, P. E. Schlein, W. E. Slater, D. H. Stork, and H. K. Ticho, Phys. Rev. Letters 11, 470 (1963).

[9]S. L. Glashow and A. H. Rosenfeld, Phys. Rev. Letters 10, 192 (1963).

[10]The program VARMIT was written at Lawrence Radiation Laboratory by E. R. Beals. The input to the program is the χ^2 function and the analytic partial derivations of χ^2 with respect to all the variables. The minimum is found by the variable metric method of W. C. Davidson, Argonne National Laboratory Report No. ANL 5990, 1959; revised (unpublished). Approximately 2 min of time is required on the CDC 6600 computer to find the minimum of a function of 15 variables.

[11]An analysis of about 20 events in the reaction $\pi^+ + p \rightarrow \Sigma^+ + \pi^\mp + \pi^+ + K^+$ at 3.23 BeV/c favors $J^P = \frac{3}{2}^-$ for $Y_1^*(1660)$; Y. Y. Lee, D. D. Reeder, and R. W. Hartung, Phys. Rev. Letters 17, 45 (1966).

CHARGE INDEPENDENCE IN HIGH-ENERGY SCATTERING FROM DEUTERONS*

Colin Wilkin
Brookhaven National Laboratory, Upton, New York

(Received 20 July 1966)

It is shown that the usual shadow correction for high-energy scattering of particles by deuterons violates charge independence in the case of pions. Alternative formulas are derived which are applicable to this case.

Many years ago, Glauber, using an eikonal method, showed that the total cross section for scattering off deuterons could be approximated as the sum of the cross sections off protons and neutrons minus a so-called shadow term. In the original derivation[1] of the shadow term, as well as in subsequent refinements,[2,3] it is implicitly assumed that the incident particle has no internal degrees of freedom; in particular, no isospin. For pion-deuteron scattering this is not true and so, in this case, Glauber's arguments have to be generalized to include the effects of charge exchange scattering. Such a modification is clearly necessary because the use of the Glauber formula for pions would violate charge independence.

The consequences of charge independence are most easily seen if we represent the shadow term diagramatically[4,5] as in Fig. 1. There is an additional term with the protons and neutrons interchanged. The sum of these two diagrams may be evaluated[5] by taking only contributions from intermediate π^-, p, and n on their mass shells. In this way we obtain[6] an amplitude

$$F(\vec{q}) = \frac{i}{2\pi k}\int S(\vec{q}\,')f_{\pi^- p}^{\text{el}}(\tfrac{1}{2}\vec{q}+\vec{q}\,') \times f_{\pi^- n}^{\text{el}}(\tfrac{1}{2}\vec{q}-\vec{q}\,')d^2q', \quad (1)$$

where q is the three-momentum transfer to the deuteron, $S(q)$ the deuteron form factor, and $f_{\pi^- p}^{\text{el}}(q)$ is the elastic $\pi^- p$ amplitude for momentum transfer $t = -q^2$. By use of the optical theorem, this can be converted into an equation for a total cross section $\delta\sigma$

$$\delta\sigma = -\frac{2}{k^2}\text{Re}\int S(\vec{q}\,')f_{\pi^- p}^{\text{el}}(\vec{q}\,')f_{\pi^- n}^{\text{el}}(-\vec{q}\,')d^2q'. \quad (2)$$

If it is further assumed that the amplitudes are purely imaginary and also that they are not rapidly varying functions of q near the forward direction, one can obtain a simplified formula for the $\pi^- d$ total cross section.

$$\sigma_{\pi^- d} = \sigma_{\pi^- p} + \sigma_{\pi^- n} - (1/4\pi)\sigma_{\pi^- p}\sigma_{\pi^- n}\langle r^{-2}\rangle. \quad (3)$$

This equation, which is the one most commonly used in the analysis of experimental data, contains a parameter $\langle r^{-2}\rangle$ representing the

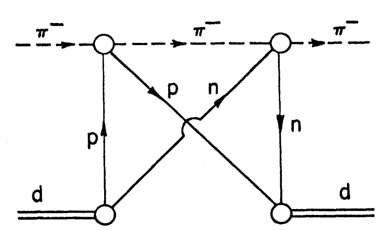

FIG. 1. Contribution to the Glauber shadow term.

average inverse-square separation of the particles in the deuteron. One can calculate in a completely analogous manner the total cross section of π^0's in deuterium

$$\sigma_{\pi^0 d}=\sigma_{\pi^0 p}+\sigma_{\pi^0 n}-(1/4\pi)\sigma_{\pi^0 p}\sigma_{\pi^0 n}\langle r^{-2}\rangle. \quad (4)$$

Now since the deuteron is in a state of $I=0$, isospin invariance requires that

$$\sigma_{\pi^- d}=\sigma_{\pi^0 d}. \quad (5)$$

It is easily seen that the shadow term in general violates condition (5) and hence charge independence.

The reason for the discrepancy is that certain diagrams, such as that of Fig. 2, have been ignored. For the π^- case there is only one diagram of this type. Its contribution is, in analogy with Eq. (1),

$$F'(q)=-\frac{i}{4\pi k}\int S(\vec{q}\,')F_{\pi^- p\to\pi^0 n}(\tfrac{1}{2}\vec{q}+\vec{q}\,')\times F_{\pi^0 n\to\pi^- p}(\tfrac{1}{2}\vec{q}-\vec{q}\,')d^2q'. \quad (6)$$

As compared with Eq. (1), a minus sign arises because a neutron-proton pair are interchanged at a deuteron vertex, where the wave function is antisymmetric. The charge-exchange amplitude occurring in Eq. (6) is expressible in terms of the elastic amplitudes.

$$f_{\pi^- p\to\pi^0 n}=(2)^{-1/2}(f_{\pi^- n}^{\text{el}}-f_{\pi^- p}^{\text{el}}). \quad (7)$$

The shadow term (1) should therefore be modified to read

$$F(\vec{q})=\frac{i}{2\pi k}\int S(\vec{q}\,')\{f_{\pi^- p}^{\text{el}}(\tfrac{1}{2}\vec{q}+\vec{q}\,')f_{\pi^- n}^{\text{el}}(\tfrac{1}{2}\vec{q}-\vec{q}\,')-\tfrac{1}{4}[f_{\pi^- n}^{\text{el}}(\tfrac{1}{2}q+q')-f_{\pi^- p}^{\text{el}}(\tfrac{1}{2}q+q')]\times[f_{\pi^- n}^{\text{el}}(\tfrac{1}{2}q-q')-f_{\pi^- p}^{\text{el}}(\tfrac{1}{2}q-q')]\}d^2q', \quad (8)$$

and the simplified formula (3) becomes

$$\sigma_{\pi^- d}=\sigma_{\pi^- p}+\sigma_{\pi^- n}-(1/4\pi)\times[\sigma_{\pi^- p}\sigma_{\pi^- n}-\tfrac{1}{4}(\sigma_{\pi^- p}-\sigma_{\pi^- n})^2]\langle r^{-2}\rangle. \quad (9)$$

For π^0's there are two diagrams of type (2):

$$\sigma_{\pi^0 d}=\sigma_{\pi^0 p}+\sigma_{\pi^0 n}-(1/4\pi)\times[\sigma_{\pi^0 p}\sigma_{\pi^0 n}-\tfrac{1}{2}(\sigma_{\pi^- p}-\sigma_{\pi^- n})^2]\langle r^{-2}\rangle. \quad (10)$$

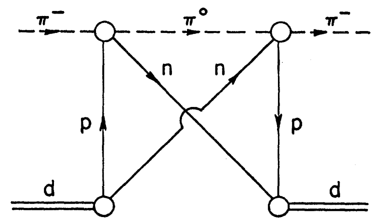

FIG. 2. Contribution to the modified shadow term.

By inspection it is seen that (9) and (10) are equal so that charge independence is satisfied.

Similar formulas will, of course, also apply for kaons and other $I=\frac{1}{2}$ particles. In this case

$$f_{K^- p\to\bar{K}^0 n}=[f_{K^- n}^{\text{el}}-f_{K^- p}^{\text{el}}], \quad (11)$$

from which the shadow term is

$$F(q)=\frac{i}{2\pi k}\int S(q')\{f_{K^- p}^{\text{el}}(\tfrac{1}{2}q+q')f_{K^- n}^{\text{el}}(\tfrac{1}{2}q-q')-\tfrac{1}{2}[f_{K^- n}^{\text{el}}(\tfrac{1}{2}q+q')-f_{K^- p}^{\text{el}}(\tfrac{1}{2}q+q')]\times[f_{K^- n}^{\text{el}}(\tfrac{1}{2}q-q')-f_{K^- p}^{\text{el}}(\tfrac{1}{2}q-q')]\}d^2q'. \quad (12)$$

The "experimentalist's" form is

$$\sigma_{K^- d}=\sigma_{K^- p}+\sigma_{K^- n}-(1/4\pi)\times[\sigma_{K^- p}\sigma_{K^- n}-\tfrac{1}{2}(\sigma_{K^- p}-\sigma_{K^- n})^2]\langle r^{-2}\rangle. \quad (13)$$

Appropriate modifications have to be made to many other formulas in Refs. 1-3, e.g., those concerning deuteron break-up reactions. The effects are particulary large in such charge exchange reactions as $\pi^+ + d\to\pi^0+p+p$. The modifications are, however, straightforward and will not be discussed further here.

The correction term that we have derived here is proportional to a charge-exchange amplitude squared. Therefore, by Pomeranchuk's theorem, it must disappear in the high energy limit. In practice, it vanishes quite rapidly for elastic scattering or total cross sections, being negligible above about 2 BeV/c, depending on the particular reaction involved. At the other extreme, in the vicinity of the 3-3 pion nucleon resonance, the large shadowing that the Glauber formula predicts is reduced by a factor $\frac{1}{3}$ by the new correction. This will be easy to check when there are good measurements of the pion-deuteron total cross sections in this region.

It may be noticed from Eq. (9) that if there were a pure $I=\frac{1}{2}$ resonance with no background, then in its neighborhood the Glauber term would be zero. On the other hand, the new correction would not be zero and would actually correspond to antishadowing.

In principle, one should also consider diagrams such as that of Fig. 1, but with the intermediate pion replaced by a ρ meson.[5] Below a few BeV their contribution is strongly suppressed because the minimum momentum transfer q allowed is

$$q \approx (\mu_\rho^2 - \mu_\pi^2)/2k. \tag{14}$$

According to the eikonal philosophy, for not-too-high energies, this momentum transfer makes the ρ contribution incoherent with the pion over the deuteron radius and, therefore, unimportant. In any case, their inclusion is extremely difficult because it would require knowledge of the phase of the amplitude $A_{\pi N \to \rho N}$. One would then have to resort to such unreliable models as those of Regge or SU(6). It is amusing to note that if one describes the ρ-production amplitude by a one-pion exchange, then the isospin factors are such as to cancel the ρ-contribution completely.

In summary, we have derived a modified Glauber formula which is charge independent, but is as easy to apply as the original. The fact that this modification was not proposed much earlier shows that in such problems a diagrammatic approach may have something to add to an eikonal one.

The author wishes to thank Dr. A. Mueller and Dr. R. F. Peierls for useful discussions and much patience. He is especially indebted to Professor L. Wolfenstein for clarifying many of the ideas. An interesting conversation with Professor R. J. Glauber is also appreciated.

---

*Work performed under the auspices of the U. S. Atomic Energy Commission.

[1]R. J. Glauber, Phys. Rev. 100, 242 (1955).

[2]V. Franco and R. J. Glauber, Phys. Rev. 142, 1195 (1966). In this reference some consideration has been given to the case where the incident particle has spin.

[3]D. R. Harrington, Phys. Rev. 135, B358 (1964).

[4]H. N. Pendleton, Phys. Rev. 131, 1833 (1963).

[5]E. S. Abers, H. Burkhardt, V. L. Teplitz, and C. Wilkin, Nuovo Cimento 42A, 365 (1966).

[6]We shall, throughout, keep to the notation of Ref. 2. Moreover, our Eqs. (1), (2), and (3) are equivalent to Eqs. (3.8), (3.12), and (4.31), respectively, in Ref. 2.

"REGENERATION" EFFECTS IN ω-φ PRODUCTION*

Marc Ross†
Massachusetts Institute of Technology, Cambridge, Massachusetts

and

Leo Stodolsky
Brookhaven National Laboratory, Upton, New York
(Received 14 July 1966)

According to current ideas,[1] the ω and φ mesons are a coherent mixture of a pure SU(3) octet state φ_0 and a pure SU(3) singlet ω_0. The eigenstates for the freely propagating ω and φ turn out to be approximately[2] $\varphi = (\sqrt{2}\varphi_0 - \omega_0)/\sqrt{3}$ and $\rho = (\varphi_0 + \sqrt{2}\omega_0)/\sqrt{3}$. Now there is no reason to expect these particular linear combinations to be respected in general; for example, in high-energy scattering we might suppose that unitary spin exchange, like isospin exchange, is small, in which case φ_0 and ω_0 are the eigenstates of the scattering. Under these circumstances, the "elastic" scattering amplitudes for φ_0 and ω_0, (if they are not accidentally degenerate), produce "inelastic" reactions of the type $\varphi \to \omega$ by "diffraction dissociation"[3] or, in the terminology of the K^0 mixing theory, by "regeneration" of an ω by an incoming φ.

We would like to point out that effects closely analogous to the K^0 mixing phenomena should, in general, exist for high-energy ω-φ production and to examine cases in which they might be observed.[4] Even though the masses of φ and ω differ by a full 240 MeV, we are justified in using the particle-mixture analogy, since the question of coherence is relative to the phase