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STUDY OF Y_1^* RESONANT AMPLITUDES BETWEEN 1660 AND 1900 MeV*

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A partial-wave analysis of the reaction $K^- + n \to \Lambda + \pi^-$ has confirmed the spin-parity assignments for $Y_1^*(1765)$ and $Y_1^*(2030)$ and measured the mass, width, and $\Lambda\pi$ branching ratio of $Y_1^*(1765)$ as 1776 ± 6 MeV, 129 ± 16 MeV and 0.14 ± 0.02, respectively. A tentative spin-parity assignment for $Y_1^*(1660)$ and $Y_1^*(1915)$ is also made. The resonant amplitudes $Y_1^*(1765)$ and $Y_1^*(1915)$ are in phase at the resonant energy and are 180° out of phase with $Y_1^*(1660)$ and $Y_1^*(2030)$.

The cross section for the pure isospin $I=1$ channel $K^- + p \to \Lambda + \pi^0$ in the c.m. energy interval 1660 to 1900 MeV shows a broad rise centered around 1780 MeV.[1] We have analyzed the angular distributions and polarizations in the reaction $K^- + n \to \Lambda + \pi^-$ in this energy interval in order to study Y_1^* resonant amplitudes in the $\Lambda\pi$ channel.

The known $I=1$ resonances between 1660 and 1900 MeV are $Y_1^*(1660)$ and $Y_1^*(1765)$. In addition, amplitudes due to $Y_1^*(1915)$ and $Y_1^*(2030)$ may be present in the energy interval under study.

$\underline{Y_1^*(1660)}$.—This resonance has $J=\frac{3}{2}$, $x_{\overline{K}N} \cong 0.15$ and $x_{\Lambda\pi} \cong 0.05$, where x_R is the branching ratio in the channel R.[2] The parity is uncertain.

$\underline{Y_1^*(1765)}$.—The assignment I, $J^P = 1, \frac{5}{2}^-$ has been deduced from a study of the reaction $K^- + \text{nucleon} \to Y_0^*(1520) + \pi^3$; $x_{\overline{K}N} = 0.5$, and $x_{\Lambda\pi}$ is not known.

$\underline{Y_1^*(1915)}$.—This resonance was recently discovered as a bump in the K^-n total cross section; $(J+\frac{1}{2})x_{\overline{K}N} = 0.31$, but J, P, and $x_{\Lambda\pi}$ are unknown.[4]

$\underline{Y_1^*(2030)}$.—A study of the reactions $K^- + p \to \Lambda + \pi^0$ and $K^- + p \to \overline{K}^0 + n$ in the K^- momentum interval 1220 to 1700 MeV/c has given I, $J^P = 1, \frac{7}{2}^+$, with $x_{\overline{K}N} = 0.25$ and $x_{\Lambda\pi} = 0.16$.[5]

The analysis described below leads to the following results: (i) The bump at 1780 MeV in the cross section for $K^- + \text{nucleon} \to \Lambda + \pi$ is due to a Y_1^* resonance of mass 1776 ± 6 MeV, width 129 ± 16 MeV, $J^P = \frac{5}{2}^-$, and $x_{\Lambda\pi} = 0.14 \pm 0.02$. We identify this resonance with $Y_1^*(1765)$ and confirm the previous $I\,J^P$ assignment.[3] (ii) We verify that the parity of $Y_1^*(2030)$ is positive. (iii) The parity of $Y_1^*(1660)$ is probably negative; a conclusive parity determination is not possible because the $Y_1^*(1660)$ amplitude is relatively weak in the $\Lambda\pi$ channel and there is insufficient data around 1660 MeV in this experiment. (iv) There are some indications that $J^P = \frac{5}{2}^+$ and $x_{\Lambda\pi} = 0.12 \pm 0.08$ for $Y_1^*(1915)$. (v) We observe that the relative phase φ of $Y_1^*(1765)$ and $Y_1^*(2030)$, each at the resonant energy, is 162 ± 9 deg; this phase difference is always 0 deg in the elastic channel. It also seems probable that $Y_1^*(1765)$ is in phase with $Y_1^*(1915)$ at the resonant energy, but 180 deg out of phase with $Y_1^*(1660)$. These observations can be related to the relative signs of the coupling constants $g_{\bar{K}NY^*}$ and $g_{\Lambda\pi Y^*}$, as discussed below.

Experimental Details. – A total of 22 000, 75 000, 63 000, and 91 000 pictures of K^--deuterium interactions at 815, 915, 1015, and 1110 MeV/c, respectively, were taken in the Lawrence Radiation Laboratory (LRL) 25-inch bubble chamber. The average beam intensity was $9K^-$ per picture.

We measured 23 580 events having the topology of the interaction sequence

$$K^- + d \to \Lambda + \pi_1^- + p_1, \quad \Lambda \to \pi^- + p, \tag{1}$$

with momentum of the proton p_1 between 0 and 230 MeV/c.

A total of 4117 events fitted the Reaction (1) hypothesis and lay within the assigned fiducial region. The momentum distribution of the protons (p_1) agrees with the Hulthen form of the deuteron wave function and, therefore, we assume that the observed interaction is $K^- + n \to \Lambda + \pi^-$, with the proton in the role of spectator.

The polarization of the Λ was calculated from the observed asymmetry of the protron from the Λ decay relative to the production normal $\hat{n} = \hat{K} \times \hat{\pi}_1 / |\hat{K} \times \hat{\pi}_1|$, with $\alpha_\Lambda = 0.66$. Experimental details and the angular and polarization distributions are given.[6]

Analysis. – The angular and polarization distributions may be expressed in the form

$$\frac{d\sigma}{d\Omega} = \lambda\!\!\!^{-2} \sum_m A_m P_m(\hat{K}\cdot\hat{\pi}), \tag{2}$$

$$\left(\frac{d\sigma}{d\Omega}\right)\vec{\mathrm{P}} = \hat{n}\cdot\lambda\!\!\!^{-2} \sum_m B_m P_m{}^1(\hat{K}\cdot\hat{\pi}), \tag{3}$$

where $P_m(\hat{K}\cdot\hat{\pi})$ is the Legendre polynomial of order m, $P_m{}^1(\hat{K}\cdot\hat{\pi})$ is the first associated Legendre polynomial, and $\lambda\!\!\!^-$ is the incident c.m. wavelength divided by 2π. The quantities A_m and B_m are functions of the complex transition amplitudes $T_l{}^{\pm}$ for states with $J = l \pm \frac{1}{2}$, and $\sigma = 4\pi\lambda\!\!\!^{-2} A_0$.[7]

The experimental data on the pion angular distribution and the Λ polarization were divided into ten intervals in c.m. energy. Coefficients A_m and B_m were determined by fitting the experimental distributions in each energy interval to Eqs. (2) and (3). Figure 1 shows A_m/A_0 and B_m/A_0 plotted against c.m. energy; the coefficients are divided by A_0 so that the figure shows only the information learned in this experiment. All the data can be fitted by an expansion to $m \leq 6$, indicating that amplitudes with $J > \frac{7}{2}$ are not required in this energy region. The absence of an A_7 coefficient shows that only one amplitude with $J = \frac{7}{2}$ is required to fit the data. The rapid energy variation of the A coefficients suggests that at least one resonant amplitude is strongly present. As already noted, the total cross section for $K^- + \text{nucleon} \to \Lambda + \pi$ shows a pronounced bump at 1780 MeV. This is most likely due to $Y_1^*(1765)$ with $J^P = \frac{5}{2}^-$. In support of this hypothesis we note that A_2 and A_4 are large and positive in the energy region where the total cross section peaks, whereas A_6 is insignificant. This observation suggests a $J = \frac{5}{2}$ amplitude, since the square of an amplitude with spin J makes a positive contribution to A_m for all even m with $m < 2J$.

To obtain more quantitative information on the amplitudes present in the $\Lambda\pi$ channel, we made a computer search for the set of partial-wave amplitudes which best fitted the polarization and differential cross sections. The measured angular distributions were converted to differential cross sections by using the published cross sections for the reaction $K^- + p \to \Lambda + \pi^0$.[1]

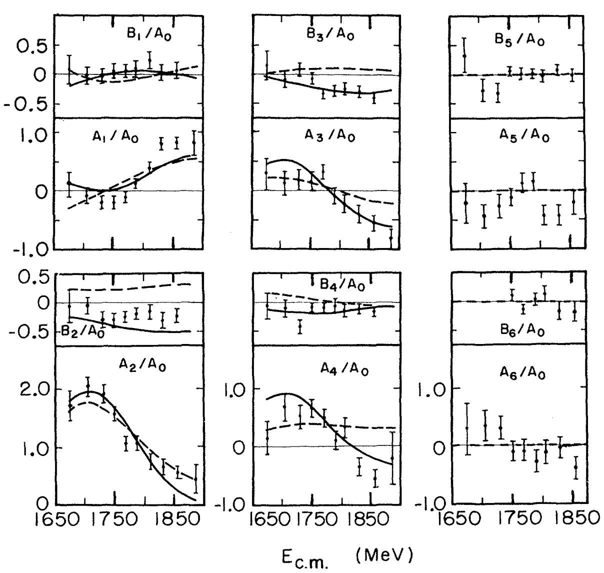

FIG. 1. Coefficients A_i/A_0 and B_i/A_0 obtained by fitting the angular and polarization distributions with the expansion $d\sigma/d\Omega = \lambda\!\!\!^{-2}\sum A_m P_m(\hat{K}\cdot\hat{\pi})$ and $(d\sigma/d\Omega)\vec{\mathrm{P}} = \hat{n}\lambda\!\!\!^{-2}\sum_m B_m P_m{}^1(\hat{K}\cdot\hat{\pi})$. The lower protion of each figure shows A_i/A_0, and the upper portion, B_i/A_0, plotted against c.m. energy. The continuous curves are calculated from solution 1a, with resonant D_5 amplitude; the dashed curves correspond to solution 1b, with resonant F_5 amplitude.

Table I lists the sets of amplitudes assumed in different fits. In fits 1, 2, and 3 we assume four nonresonant amplitudes S_1, P_1, P_3, and D_3, since analyses of the $\Lambda\pi$ channel in the c.m. interval 1600 to 1700 MeV have established the presence of S_1, P_1, and at least one $J=\frac{3}{2}$ background amplitudes.[8,1] The energy dependence of these amplitudes is not known, and we hypothesize that they are constant over the energy region. The magnitudes and phases of these amplitudes were allowed to vary in all the fits.

In order to test for the presence of $Y_1^*(1765)$ with $J=\frac{5}{2}$ in the $\Lambda\pi$ channel, the four nonresonant amplitudes were combined with a single $\frac{5}{2}^-$ (D-wave) resonant amplitude in fit 1a, and with a single $\frac{5}{2}^+$ (F-wave) resonant amplitude in fit 1b.

The resonant amplitudes had the Breit-Wigner form $T=\frac{1}{2}(\Gamma_{\bar{K}N}\Gamma_{\Lambda\pi})^{1/2}/(E_R-E-\frac{1}{2}i\Gamma)$ with $\Gamma_i \propto [q_i^2/(q_i^2+X^2)]^{l_i}(q_i/E)$ and $\Gamma=\sum_i\Gamma_i$. The summation is over all decay channels of the resonance; X is fixed at 350 MeV, and q_i and l_i are the momentum and orbital angular momentum of the decay products of the resonance of energy E in channel i.[9] In fit 1 and succeeding fits, the mass E_R, width Γ, and the magnitude $x_{\bar{K}N}x_{\Lambda\pi}$ $(=\Gamma_{\bar{K}N}\Gamma_{\Lambda\pi}/\Gamma^2)$ of the $J=\frac{5}{2}$ resonant amplitude at the resonant energy were allowed to vary.

The differential cross sections and polarizations predicted by each set of amplitudes 1a and 1b were compared with the experimental data and the χ^2 function computed. The χ^2 was a function of 11 variables—the magnitudes and phases of the four nonresonant amplitudes, and the mass, width, and magnitude of the resonant amplitude. One phase is arbitary, and this was fixed by making the $J=\frac{5}{2}$ resonant amplitude purely imaginary at $E=E_R$ (this convention was used in all fits). The program VARMIT[10] was used to search through the hyperspace of 11 variables for the minimum in χ^2.

The solutions that minimize χ^2 for the 1a and 1b hypotheses are shown in Fig. 2(a) and 2(b), and the final χ^2 is listed in Table I. The resonant D_5 amplitude is clearly favored over the resonant F_5 amplitude, but both solutions are highly improbable.

In fit 2 we added to the amplitudes in fit 1 the $J=\frac{7}{2}$ resonant amplitude due to $Y_1^*(2030)$. According to Refs. 4 and 5, we fixed the mass and width at 2035 and 160 MeV, respectively; our data are insensitive to these parameters since the resonant energy is far removed from the energy region under study. The data are sensitive, however, to the parity of $Y_1^*(2030)$, and this was checked by trying both the $J^P=\frac{7}{2}^+$ (F-wave) and $\frac{7}{2}^-$ (G-wave) hypothesis. The magnitude and phase of $Y^*(2030)$ were allowed to vary, thus increasing the number of variables from 11 to 13. The only acceptable solution is 2a, which requires negative parity for the $J=\frac{5}{2}$ resonant amplitude and positive parity for the $J=\frac{7}{2}$ resonant amplitude. Solution 2a gives 1777 ± 6 and 135 ± 16 MeV, respectively, for the mass and width of the $\frac{5}{2}^-$ resonance. Therefore, we identify this resonance with $Y_1^*(1765)$ and confirm the previous determination of I, $J^P=1$, $\frac{5}{2}^-$. The measured mass

Table I. Partial-wave amplitudes used for a least-squares fit to the experimental distributions in Fig. 1. The χ^2 for each fit and the corresponding probability are also listed.

Fit	Constant amplitudes	Resonant amplitudes	χ^2	Degrees of freedom	Probability
1a	S_1,P_1,P_3,D_3	D_5	359	200	4×10^{-12}
1b	S_1,P_1,P_3,D_3	F_5	724	200	$\ll10^{-20}$
2a	S_1,P_1,P_3,D_3	D_5,F_7	240	198	0.02
2b	S_1,P_1,P_3,D_3	D_5,G_7	353	198	10^{-11}
2c	S_1,P_1,P_3,D_3	F_5,F_7	717	198	$\ll10^{-20}$
2d	S_1,P_1,P_3,D_3	F_5,G_7	581	198	$\ll10^{-20}$
3	S_1,P_1,P_3,D_3	D_5,F_5,F_7	226	196	0.07
4a	S_1,P_1,P_3	D_3,D_5,F_7	148	120	0.04
4b	S_1,P_1,D_3	P_3,D_5,F_7	172	120	10^{-3}
4c	S_1,P_1,P_3,D_3	D_5,F_7	150	120	0.03

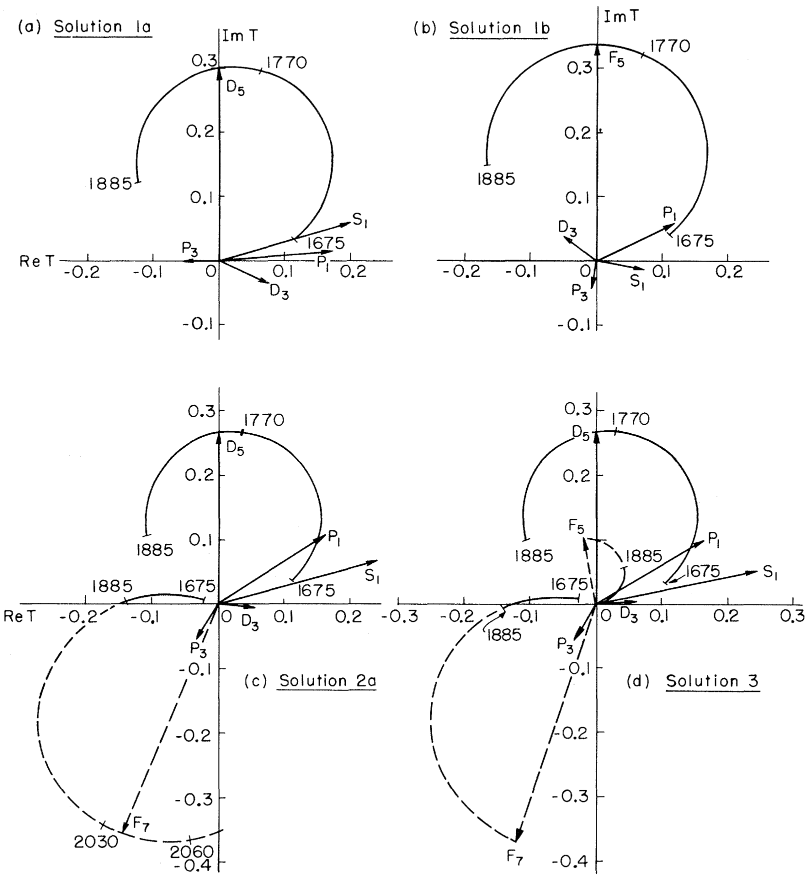

FIG. 2. Magnitude and phases of the amplitudes which best fit the experimental data for the assumption of constant S_1, P_1, P_3, D_3 amplitudes and (a) a resonant D_5 amplitude, (b) a resonant F_5 amplitude, (c) resonant D_5, $Y_1^*(2030)$ with $J^P = \frac{7}{2}^+$, or (d) resonant D_5, $Y_1^*(2030)$ with $J^P = \frac{7}{2}^+$, and $Y_1^*(1915)$ with $J^P = \frac{5}{2}^+$. The resonant amplitude traces a circle counterclockwise as the energy increases; c.m. energies are indicated on the periphery of the circle.

and width are in agreement with previous values.[3] This parity solution for $Y_1^*(2030)$ agrees with the recent measurement of Wohl, Solmitz, and Stevenson.[5]

Solution 2a is shown in Fig. 2(c). This set of amplitudes cannot generate the negative A_6 coefficient observed at 1855 MeV. The interference terms D_5, G_7 and F_5, F_7 are responsible for a negative A_6 coefficient. Since the single $J = \frac{7}{2}$ amplitude present has been identified as F_7, an F_5 amplitude is indicated. The negative A_6 coefficient is most marked at 1855 MeV; A_4 is negative at this energy, showing that $|F_5|^2$ is small. The fact that F_5 is relatively weak makes it impossible to determine whether or not this amplitude is resonant. However, we speculate that this amplitude may correspond to the recently discovered $Y_1^*(1915)$.[4] In fit 3 an F_5 amplitude of mass 1915 MeV and width 65 MeV was added to the D_5 and F_7 resonant amplitudes. The solution is shown in Fig. 2(d).

Table II summarizes the parameters of $Y_1^*(1765)$, $Y_1^*(2030)$, and $Y_1^*(1915)$ determined with varying degrees of certainty in fits 2 and 3. Fit 3 gives 1776 and 129 MeV, respectively for the mass and the width of $Y_1^*(1765)$, together with the value of $x_{\overline{K}N} x_{\Lambda\pi}$ for the three states. Using the published values of $x_{\overline{K}N}$, we determine the branching ratios into the $\Lambda\pi$ channel.

The errors quoted in Table II are the statistical errors calculated in our fitting program, increased by a factor of 2. The statistical errors have been doubled in an attempt to include uncertainties arising from the assumptions (a) that there are no nonresonant amplitudes present with the same spin and parity as the

Table II. Parameters and quantum numbers of $Y_1^*(1660)$, $Y_1^*(1765)$, $Y_1^*(1915)$, and $Y_1^*(2030)$. The quantities measured or verified in this experiment are underlined with a solid line; quantities suggested by this experiment are indicated by a broken line. The errors are statistical errors calculated in the fitting program, increased by a factor of 2 (see text).

Mass E_R (MeV)	Width Γ (MeV)	Spin J	Parity P	$x_{\bar{K}N}x_{\Lambda\pi}$	$x_{\bar{K}N}$	$x_{\Lambda\pi}$	φ (deg)
1660	44	$\frac{3}{2}$	−	0.009 ± 0.010	0.15	0.06 ± 0.06	207 ± 23
1776 ± 6	129 ± 16	$\frac{5}{2}$	−	0.071 ± 0.008	0.5	0.14 ± 0.02	0
1915	65	$\frac{5}{2}$	+	0.012 ± 0.008	0.10	0.12 ± 0.08	6 ± 18
2035	160	$\frac{7}{2}$	+	0.137 ± 0.050	0.25	0.55 ± 0.20	162 ± 9

resonances, and (b) that the background amplitudes are constant. Also the energy dependence used for Γ may not be exactly correct.

Until now we have neglected $Y_1^*(1660)$ because its amplitude in the $\Lambda\pi$ channel is weak.[2] In fits 4a and 4b we took the experimental data below 1800 MeV, where the $Y_1^*(1660)$ amplitude is more important, and we made the assumption that one of the $J=\frac{3}{2}$ amplitudes was due to a resonance of mass 1660 MeV and width 44 MeV. The magnitude and phase of the $J=\frac{3}{2}$ resonance were allowed to vary. Only the $\frac{3}{2}^-$ resonant hypothesis led to a satisfactory fit; the corresponding $x_{\bar{K}N}x_{\Lambda\pi}$ value is given in Table II. However, the data below 1800 MeV is almost equally well described by constant $\frac{3}{2}^-$ and $\frac{3}{2}^+$ amplitudes as shown by fit 4c, so that the $J^P=\frac{3}{2}^-$ assignment is not conclusive.[11]

Coupling constants $g_{\bar{K}NY^*}$ and $g_{\Lambda\pi Y^*}$. – The resonant D_5 amplitude was defined to be purely imaginary at $E=E_R$. The phase angles φ of the other resonant amplitudes at the resonant energies, relative to D_5, are shown in Table II. In the elastic channel, φ is always zero; in the inelastic channel it may be zero or 180 deg because the sign of the off-diagonal T matrix elements is undefined. The resonant amplitude in the elastic channel is proportional to $g_{\bar{K}NY^*}{}^2/(E_R-E-\frac{1}{2}i\Gamma)$ and in the $\Lambda\pi$ channel, to $g_{\bar{K}NY^*}g_{\Lambda\pi Y^*}/(E_R-E-\frac{1}{2}i\Gamma)$. For the elastic amplitude the numerator is always positive; in the inelastic channel the sign of the numerator depends on the relative sign of the coupling constants $g_{\bar{K}NY^*}$ and $g_{\Lambda\pi Y^*}$. The values of φ in Table II are consistent with $\varphi=180$ deg for $Y_1^*(2035)$ and $Y_1^*(1660)$, and $\varphi=0$ for $Y_1^*(1915)$. This shows that the product of the coupling constants $g_{\bar{K}NY^*}g_{\Lambda\pi Y^*}$ is of one sign for $Y_1^*(1765)$ and $Y_1^*(1915)$ and of the opposite sign for $Y_1^*(1660)$ and $Y_1^*(2030)$. The ambiguity arises because the over-all orientation of the amplitudes in the $\Lambda\pi$ channel, relative to the $\bar{K}N$ channel, cannot be determined by this analysis.

We note that the phase of the conjectured $Y_1^*(1915)$, $J^P=\frac{5}{2}^+$, amplitude in fit 3 is 6 ± 18 deg, in agreement with the requirement that the phase φ be 0 or 180 deg. The resonant nature of the F_5 amplitude is supported by this observation.

We thank Professor J. S. Ball, Professor G. L. Shaw, and Professor R. D. Tripp for discussions regarding the properties of resonant amplitudes. Eric Beals of the Lawrence Radiation Laboratory computing center advised on mathematics and programming.

*Work sponsored by the U. S. Atomic Energy Commission and the National Science Foundation.

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CHARGE INDEPENDENCE IN HIGH-ENERGY SCATTERING FROM DEUTERONS*

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(Received 20 July 1966)

It is shown that the usual shadow correction for high-energy scattering of particles by deuterons violates charge independence in the case of pions. Alternative formulas are derived which are applicable to this case.

Many years ago, Glauber, using an eikonal method, showed that the total cross section for scattering off deuterons could be approximated as the sum of the cross sections off protons and neutrons minus a so-called shadow term. In the original derivation[1] of the shadow term, as well as in subsequent refinements,[2,3] it is implicitly assumed that the incident particle has no internal degrees of freedom; in particular, no isospin. For pion-deuteron scattering this is not true and so, in this case, Glauber's arguments have to be generalized to include the effects of charge exchange scattering. Such a modification is clearly necessary because the use of the Glauber formula for pions would violate charge independence.

The consequences of charge independence are most easily seen if we represent the shadow term diagramatically[4,5] as in Fig. 1. There is an additional term with the protons and neutrons interchanged. The sum of these two diagrams may be evaluated[5] by taking only contributions from intermediate π^-, p, and n on their mass shells. In this way we obtain[6] an amplitude

$$F(\vec{q}) = \frac{i}{2\pi k}\int S(\vec{q}\,')f_{\pi^-p}^{\mathrm{el}}(\tfrac{1}{2}\vec{q}+\vec{q}\,') \times f_{\pi^-n}^{\mathrm{el}}(\tfrac{1}{2}\vec{q}-\vec{q}\,')d^2q', \quad (1)$$

where q is the three-momentum transfer to the deuteron, $S(q)$ the deuteron form factor, and $f_{\pi^-p}^{\mathrm{el}}(q)$ is the elastic π^-p amplitude for momentum transfer $t=-q^2$. By use of the optical theorem, this can be converted into an equation for a total cross section $\delta\sigma$

$$\delta\sigma = -\frac{2}{k^2}\mathrm{Re}\int S(\vec{q}\,')f_{\pi^-p}^{\mathrm{el}}(\vec{q}\,')f_{\pi^-n}^{\mathrm{el}}(-\vec{q}\,')d^2q'. \quad (2)$$

If it is further assumed that the amplitudes are purely imaginary and also that they are not rapidly varying functions of q near the forward direction, one can obtain a simplified formula for the π^-d total cross section.

$$\sigma_{\pi^-d} = \sigma_{\pi^-p} + \sigma_{\pi^-n} - (1/4\pi)\sigma_{\pi^-p}\sigma_{\pi^-n}\langle r^{-2}\rangle. \quad (3)$$

This equation, which is the one most commonly used in the analysis of experimental data, contains a parameter $\langle r^{-2}\rangle$ representing the

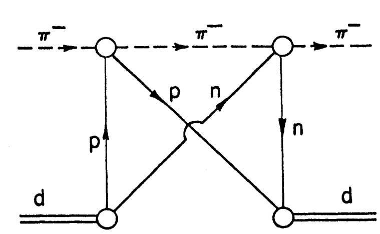

FIG. 1. Contribution to the Glauber shadow term.