thesis of Ref. 5). Note, however, that (5) is not quite the same as the SU(6) result[9] which in our notation reads

$$(f_\rho^2/4\pi)=(G_{\pi NN}^2/4\pi)(g_V/g_A)^2(m_M^2/m_B^2)$$
$$=(9/25)(G_{\pi NN}^2/4\pi)(m_M^2/m_B^2), \qquad (6)$$

where m_M and m_B, respectively, stand for the mean masses of the meson 35-plet and the baryon 56-plet of SU(6).

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DIVERGENCE CONDITIONS AND SUM RULES*

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Recently several sum rules have been derived employing current commutation[1] rules and divergence conditions for those currents. As is well known, the application of commutation rules involves the manipulation of the so-called Schwinger terms,[2] and where some of these calculations avoid such complications, others may be criticized in this respect. An alternative derivation of these sum rules, based on assumptions other than current commutation relations may, therefore, be of help in understanding the mechanism involved.

Consider the vector current of hadrons that is coupled to leptons and photons. Neglecting higher-order electromagnetic (em) and weak interactions one customarily assumes, following Feynman and Gell-Mann,[3]

$$\partial_\mu \vec{J}_\mu^{\,V}=0. \qquad (1)$$

As is well known, em interactions break this law for the charged components of $\vec{J}_\mu$. Similarly the weak interactions break (1) for the neutral component because they carry off a nonzero charge. (Remember that $\vec{J}_\mu$ is the hadron current only.) We will try to find the first-order em and weak effects on (1).

According to the principle of minimality, we find the em effect on (1) by substituting $\partial_\mu - ieA_\mu$ for ∂_μ applying to a (negative) charged field. Thus, neglecting here the case that $\vec{J}_\mu^{\,V}$ itself contains derivatives, we find

$$\partial_\mu \vec{J}_\mu^{\,V}=+ie\vec{A}_\mu\times\vec{J}_\mu^{\,V},$$
$$(\vec{A}_\mu\times\vec{J}_\mu)^i=i\epsilon_{ijk}A_\mu^{\,j}J_\mu^{\,k}, \qquad (2)$$

where $\vec{A}_\mu$ is an isotopic vector whose first two components are zero. Equation (2) is already sufficient to derive the Cabibbo-Radicati[4] sum rule.

In accordance with the observations made above, we generalize (2) to include also first-order weak interaction effects:

$$\partial_\mu \vec{J}_\mu^{\,V}=ie\vec{A}_\mu\times\vec{J}_\mu^{\,V}+ig\vec{l}_\mu\times\vec{J}_\mu^{\,V}. \qquad (3)$$

Here l_μ represents the lepton current.[5] Equation (3) is valid if no axial currents are present. The generalization to include axial currents also requires some care. Let us introduce two charged vector-boson fields $\vec{W}_\mu{}^A$ and $\vec{W}_\mu{}^V$ that couple to axial and vector hadron currents, respectively. Both couple to leptons. The correct form of (3), accounting also for charge carried away by axial bosons, as in the process $\gamma+N\to N+W^A$ is

$$\partial_\mu \vec{J}_\mu{}^V = +ie\vec{A}_\mu\times\vec{J}_\mu{}^V + ig\vec{W}_\mu{}^V\times\vec{J}_\mu{}^V + ig\vec{W}_\mu{}^A\times\vec{J}_\mu{}^A, \qquad (4)$$

where g is the coupling constant figuring in W-hadron interactions.

In analogy with (4) we generalize the well-known partially conserved axial-vector current relation[6] for the axial current to

$$\partial_\mu \vec{J}_\mu{}^A = ia\vec{\pi}(x) + ie\vec{A}_\mu\times\vec{J}_\mu{}^A + ig\vec{W}_\mu{}^V\times\vec{J}_\mu{}^A + ig\vec{W}_\mu{}^A\times\vec{J}_\mu{}^V, \qquad (5)$$

where

$$a = M_N M_\pi{}^2 g_A/g_\gamma K^{NN\pi}(0).$$

Equation (5) gives rise to the well-known Adler-Weisberger[7,8] sum rule. We wish to emphasize that the introduction of two bosons is purely a matter of technical expedience, and one may, in fact, replace both W^V and W^A by the lepton current.

Vector current sum rules.–Consider the process

$$\gamma+N\to N'+W^V,$$

where q, p, p', q' are photon, initial nucleon, final nucleon, and vector-boson (or lepton pair) four-momenta. The S-matrix element is given by

$$S = \langle N', W_{\text{out}}{}^V|\gamma N_{\text{in}}\rangle.$$

By standard techniques (we extract only the photon part),

$$S = ie\frac{e_\mu{}^i}{(2q_0)^{1/2}}\int d^4x\, e^{-iqx}\langle N', W_{\text{out}}{}^V|J_\mu{}^{Vi}(x)|N\rangle$$

$$\equiv ie\frac{e_\mu{}^i}{(2q_0)^{1/2}}M_\mu{}^i. \qquad (6)$$

In here the upper indices refer to isospin. $e_\mu{}^i$ describes isospin and spatial polarization of the photon.

From translation invariance we have, for $M_\mu{}^i$ defined by (6),

$$q_\mu M_\mu{}^i = (p'+q'-p)_\mu M_\mu{}^i = -i\int d^3x\, e^{-iqx} \times\langle N', W_{\text{out}}{}^V|\partial_\mu J_\mu{}^{Vi}(x)|N\rangle. \qquad (7)$$

Using (4) [or (3)] we have

$$q_\mu M_\mu{}^i = +ig\epsilon_{ijk}\int d^4x\, e^{-iqx} \times\langle N', W_{\text{out}}{}^V|W_\nu{}^{Vj}(x)J_\nu{}^{Vk}(x)|N\rangle. \qquad (8)$$

In lowest order of weak interactions the W^V field equals the "out" field, and we may absorb the W^V from the "out" state by

$$q_\mu M_\mu{}^i = +ig\epsilon_{ijk}\frac{w_\nu{}^j}{(2q_0')^{1/2}}\int d^4x\, e^{i(q'-q)x} \times\langle N'|J_\nu{}^{Vk}(x)|N\rangle. \qquad (9)$$

In here $w_\nu{}^j$ describes W^V polarizations. The right-hand side of (9) is, of course, known. We get

$$q_\mu M_\mu{}^i = +ig\epsilon_{ijk}\frac{w_\nu{}^j}{(2q_0')^{1/2}}(2\pi)^4\delta_4(p+q-p'-q') \times\bar{U}(p')\Gamma_\nu{}^{Vk}(p',p)U(p),$$

where $\Gamma_\nu{}^{Vk}(p',p)$ is the em vertex of the nucleon

$$\Gamma_\nu{}^{Vk}(p',p) = -\tfrac{1}{2}i\tau^k\{iF_1[(p-p')^2]\gamma^\nu + (\kappa/2M)\sigma^{\nu\lambda}(p-p')_\lambda\},$$

$$\sigma^{\nu\lambda} = \tfrac{1}{2}(\gamma^\nu\gamma^\lambda - \gamma^\lambda\gamma^\nu). \qquad (10)$$

From here on the calculation parallels the calculations of Low,[9] Gell-Mann and Goldberger,[10] and Beg.[11] Going back to the original S-matrix element, we write

$$S = ieg\frac{e_\mu^i w_\nu^j}{(4q_0q_0')^{1/2}}(2\pi)^4\delta_4(p+q-p'-q') \times M_{\mu\nu}^{ij}(q',q). \tag{11}$$

The above calculation shows that

$$q_\mu M_\mu^{ij}(q',q) = +i\epsilon_{ijk}\bar{U}(p')\Gamma_\nu^{Vk}(p',p)U(p). \tag{12}$$

$M_{\mu\nu}^{ij}$ may be separated in a Born term and a "nonpole" contribution[12]

$$M_{\mu\nu}^{ij} = M_{\mu\nu B}^{ij} + U_{\mu\nu}^{ij}, \tag{13}$$

where $M_{\mu\nu B}^{ij}$ is known in terms of the vertex functions Γ_μ^{Vi} and Γ_ν^{Vj} evaluated at the pole. $U_{\mu\nu}^{ij}$ is an unknown function. $M_{\mu\nu}^{ij}$ satisfies crossing symmetry

$$M_{\mu\nu}^{ij}(q',q) = M_{\nu\mu}^{ji}(-q,-q'). \tag{14}$$

Expanding both sides of (12) for small q and q' and employing (14) gives $U_{\mu\nu}^{ij}$ up to first order in q and q':

$$U_{\mu\nu}^{ij} = -\bar{U}(p')^i\tfrac{1}{4}[\tau^i,\tau^j] \times[\{-2iq_\mu'\gamma^\nu - 2iq_\nu\gamma^\mu + i\delta_{\mu\nu}(\gamma q + \gamma q')\}F_j'(0) + (\kappa/2m)\sigma^{\nu\mu}]U(p), \tag{15}$$

where

$$F_1'(0) = \partial F_1(x)/\partial x|_{X=0}. \tag{16}$$

Thus the matrix element $M_{\mu\nu}^{ij}$ describing the process $\gamma + N \to N' + W^V$ is determined by (4) up to first order in photon and W^V energies. This may be tested experimentally by actually measuring this reaction in the low-energy limit. Alternatively, following Drell and Hearn[13] and Beg,[11] one may write down dispersion relations for the various invariant amplitudes, and assuming no subtractions, sum rules emerge. We refer to Beg's paper[11] for further details.

Axial current sum rules.—The Adler-Weisberger[7] sum rule may be obtained by considering the process

$$W^A + N \to W^A + N'$$

for bosons with zero three-momenta in the nucleon rest system. Ignoring the no-scattering part of the S matrix we have

$$S = ige_\mu^i \int d^4x\, e^{-iqx}\langle NW_{\text{out}}^A|J_\mu^{Ai}(x)|N\rangle \equiv ige_\mu^i M_\mu^i. \tag{17}$$

Employing (5),

$$q_\mu M_\mu^i = a\int d^4x\, e^{-iqx}\langle NW_{\text{out}}^A|\pi^i(x)|N_{\text{in}}\rangle + iq\epsilon_{ijk}\int d^4x\, e^{-iqx} \times\langle NW_{\text{out}}^A|W_\nu^{Aj}(x)J_\nu^{Vk}(x)|N\rangle. \tag{18}$$

Again, the last term may be explicitly calculated to give

$$+\frac{iq\epsilon_{ijk}e_\nu^{ij}}{(2q_0')^{1/2}}\bar{U}(p')\Gamma_\nu^{Vj}(p',p)U(p)(2\pi)^4 \times\delta_4(p+q-p'-q'). \tag{19}$$

The rest of the calculation is totally identical to Adler's calculation, Ref. 8, from formula (37) on in that paper, with (19) above for the matrix element of the equal-time commutator of the axial charges.

It appears that the divergence conditions (4) and (5) are, for practical purposes, equivalent to the equal-time commutation rules given by Gell-Mann.[1] Of course, many other very interesting properties of current algebra are not to be found in (4) and (5), but on the other hand, the Schwinger ambiguities[2] do not play any role if one employs the divergence equations for practical applications.

It may finally be remarked that (4) and (5) may be applied, in the manner invented by Low[9] and Gell-Mann and Goldberger[10] to all kinds of processes involving lepton pairs, photons, and pions to determine low-energy behavior. Examples are to be found, for instance, in the work of Callan-Treiman,[14] Weinberg,[15] and others.

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STUDY OF Y_1^* RESONANT AMPLITUDES BETWEEN 1660 AND 1900 MeV*

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A partial-wave analysis of the reaction $K^- + n \to \Lambda + \pi^-$ has confirmed the spin-parity assignments for $Y_1^*(1765)$ and $Y_1^*(2030)$ and measured the mass, width, and $\Lambda\pi$ branching ratio of $Y_1^*(1765)$ as 1776 ± 6 MeV, 129 ± 16 MeV and 0.14 ± 0.02, respectively. A tentative spin-parity assignment for $Y_1^*(1660)$ and $Y_1^*(1915)$ is also made. The resonant amplitudes $Y_1^*(1765)$ and $Y_1^*(1915)$ are in phase at the resonant energy and are 180° out of phase with $Y_1^*(1660)$ and $Y_1^*(2030)$.

The cross section for the pure isospin $I = 1$ channel $K^- + p \to \Lambda + \pi^0$ in the c.m. energy interval 1660 to 1900 MeV shows a broad rise centered around 1780 MeV.[1] We have analyzed the angular distributions and polarizations in the reaction $K^- + n \to \Lambda + \pi^-$ in this energy interval in order to study Y_1^* resonant amplitudes in the $\Lambda\pi$ channel.

The known $I = 1$ resonances between 1660 and 1900 MeV are $Y_1^*(1660)$ and $Y_1^*(1765)$. In addition, amplitudes due to $Y_1^*(1915)$ and $Y_1^*(2030)$ may be present in the energy interval under study.

$Y_1^*(1660)$.—This resonance has $J = \frac{3}{2}$, $x_{\bar{K}N} \cong 0.15$ and $x_{\Lambda\pi} \cong 0.05$, where x_R is the branching ratio in the channel R.[2] The parity is uncertain.

$Y_1^*(1765)$.—The assignment I, $J^P = 1$, $\frac{5}{2}^-$ has been deduced from a study of the reaction $K^- + \text{nucleon} \to Y_0^*(1520) + \pi$[3]; $x_{\bar{K}N} = 0.5$, and $x_{\Lambda\pi}$ is not known.

$Y_1^*(1915)$.—This resonance was recently discovered as a bump in the K^-n total cross section; $(J + \frac{1}{2})x_{\bar{K}N} = 0.31$, but J, P, and $x_{\Lambda\pi}$ are unknown.[4]

$Y_1^*(2030)$.—A study of the reactions $K^- + p \to \Lambda + \pi^0$ and $K^- + p \to \bar{K}^0 + n$ in the K^- momentum interval 1220 to 1700 MeV/c has given I, $J^P = 1$, $\frac{7}{2}^+$, with $x_{\bar{K}N} = 0.25$ and $x_{\Lambda\pi} = 0.16$.[5]

The analysis described below leads to the following results: (i) The bump at 1780 MeV in the cross section for $K^- + \text{nucleon} \to \Lambda + \pi$ is due to a Y_1^* resonance of mass 1776 ± 6 MeV, width 129 ± 16 MeV, $J^P = \frac{5}{2}^-$, and $x_{\Lambda\pi} = 0.14 \pm 0.02$. We identify this resonance with $Y_1^*(1765)$ and confirm the previous $I\,J^P$ assignment.[3] (ii) We verify that the parity of $Y_1^*(2030)$ is positive. (iii) The parity of $Y_1^*(1660)$ is probably negative; a conclusive parity determination is not possible because the $Y_1^*(1660)$ amplitude is relatively weak in the $\Lambda\pi$ channel and there is insufficient data around 1660 MeV in this experiment. (iv) There are some indications that $J^P = \frac{5}{2}^+$ and $x_{\Lambda\pi} = 0.12 \pm 0.08$ for $Y_1^*(1915)$. (v) We observe that the relative phase φ of $Y_1^*(1765)$ and $Y_1^*(2030)$, each at the resonant energy, is 162 ± 9 deg; this phase difference is always 0 deg in the elastic channel. It also seems probable that $Y_1^*(1765)$ is in phase with $Y_1^*(1915)$ at the resonant energy, but 180 deg out of phase with $Y_1^*(1660)$. These observations can be related to the rel-