[19]N. Austern, Phys. Rev. 100, 1522 (1955).
[20]B. T. Feld, Nuovo Cimento Suppl. 2, 145 (1955).
[21]W. Rarita and J. Schwinger, Phys. Rev. 60, 61 (1941).
[22]These wave functions have been used in practical calculations by many authors. See, for example, S. Barshay and G. E. Brown, Phys. Letters 16, 165 (1965).
[23]R. Blankenbecler, M. L. Goldberger, and F. R. Halpern, Nucl. Phys. 12, 629 (1959).
[24]These approximations reproduce the model of Austern (Ref. 19). Armed with the complete matrix element, one has the basis of a more systematic theory of deuteron photodisintegration in the enhancement region, but this is not the prime concern of this note.
[25]In the calculations we take κ, the center-of-mass photon momentum, as ~290 MeV/c. This is where the real part of the resonance-energy denominator in Eq. (3b) vanishes, for $Q \cong 0$. The experimental cross section peaks nearer to ~290 MeV/c laboratory photon momentum. This shift toward lower energy is possibly due, in part, to the fact that the very significant cross-section contribution from other than the isobar mechanism is decreasing as the isobar amplitude increases in magnitude, and is familiar from pion photoproduction on hydrogen. It can also be due, in part, to a somewhat lower "effective" isobar mass in this process.
[26]R. R. Wilson, Phys. Rev. 86, 125 (1952); 104, 218 (1956).
[27]N. Austern, Phys. Rev. 85, 283 (1952); 108, 973 (1957).
[28]Note that in the approximation (Ref. 24) of zero orbital angular momentum for the intermediate-state N^* and N, the amplitude a_0 does not contain the time-reversal noninvariant vertex of Eq. (2), because a spin-$\frac{3}{2}$ isobar and a spin-$\frac{1}{2}$ nucleon cannot then be an intermediate state in a transition to a final state with total angular momentum zero.
[29]Experiment also indicates something of a fore-aft asymmetry. Small amplitudes for transitions $E(2) \to {}^3D$, in interference with the $E(1) \to {}^3P$ amplitudes can produce this; again the $E(2)$ amplitudes to final triplet states will not interfere with the isobar amplitude. See L. I. Schiff, Phys. Rev. 78, 733 (1950); J. F. Marshall and E. Guth, Phys. Rev. 78, 738 (1950).
[30]D. Goulianos and D. Bartlett, private communication.
[31]We make the perhaps obvious remark that if the experiment gives a null result, it will not, in itself, resolve the question of time-reversal invariance.

RHO-MESON DIFFERENTIAL PRODUCTION CROSS SECTION BY 1.7-BeV/c π^-p INTERACTIONS*

D. D. Allen, G. P. Fisher, G. Godden, J. B. Kopelman, L. Marshall, and R. Sears
University of Colorado, Boulder, Colorado
(Received 13 May 1966)

In a study of two-pronged events from 1.7-BeV/c π^- interactions in the 20-inch Brookhaven National Laboratory hydrogen bubble chamber, we find that the three-body final state is dominated by ρ-meson formation in a quasi-two-body channel, i.e., $\pi+N \to \rho+N \to \pi+\pi+N$. We have analyzed the data from 11 000 two-pronged events in such a way as to isolate the resonant state as freely as possible from nonresonant background. In this way comparisons of rho production with predictions of various models are simplified.

A visual ionization check was made on each two-pronged event for compatibility with the mass fit as computed by the GUTS kinematics program. A given hypothesis was rejected if the probability was less than 0.04 for obtaining a χ^2 greater than the fitted one. If two final-state hypotheses (A, B) were ambiguous on the basis of ionization, but

$$\left[\frac{\chi^2}{C}\right]_A - \left[\frac{\chi^2}{C}\right]_B \geq 3,$$

where C is the number of constraints for that hypothesis, then the event was classified as hypothesis B. After this condition was imposed, problems of events remaining ambiguous between any two of the possible channels, e.g. π^-p, $\pi^-p\pi^0$, $\pi^-\pi^+n$, etc., were resolved by methods of the kind discussed by Allen et al.[1] We estimate a maximum of 5% of events in any one final state remaining ambiguous between two or more of these final states.

The cross section we obtain for $\pi^-+p \to \pi^- + p + \pi^0$ at 1.7 BeV/c is 5.4 ± 0.5 mb. Of this, we find 2.1 ± 0.2 mb goes via the channel $\pi^- + p \to \rho^- + p$. The absolute value of the $\pi+p \to \rho + p$ production cross section is obtained by normalizing to total and to zero-degree differential elastic-scattering cross sections measured by counters, as discussed in Ref. 1. The greater part of the quoted uncertainties arise from these counter measurements. Data on the ρ^0 channel at this energy have been previously published by Fickinger, Robinson, and Salant.[2]

The dependence of the production cross section of the $\pi^-\pi^0$ system on the four-momentum transfer squared (t) to the nucleon is given in

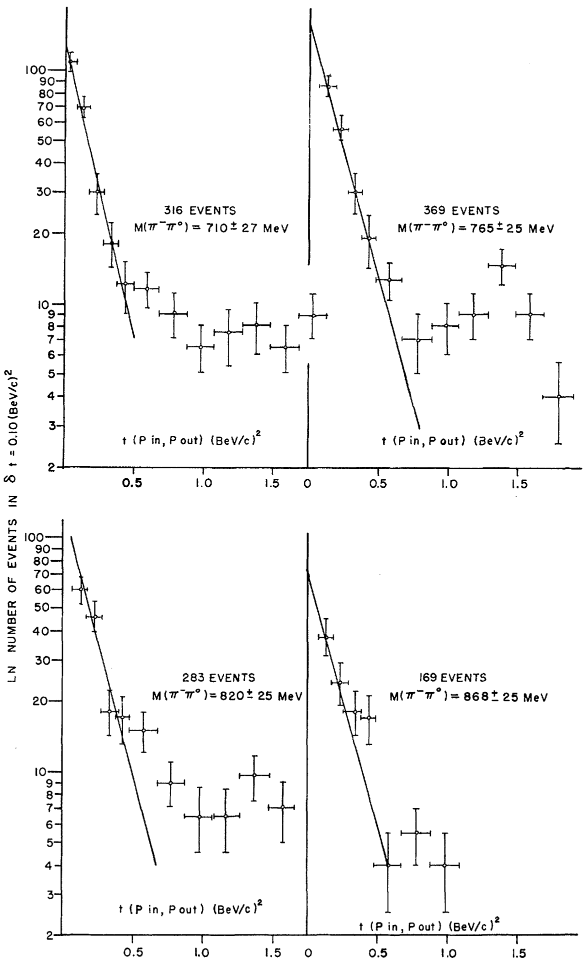

FIG. 1. For 1.7-BeV/c $\pi^- + p \to \pi^- + p + \pi^0$, number of events in intervals of 0.10 (BeV/c)^2 four-momentum transfer versus four-momentum transfer between incoming and outgoing proton, for di-pion mass cuts of total width ~50 MeV. For the mass cut centered on the rho mass, the effect of three-body background is relatively least, and the diffraction pattern becomes apparent.

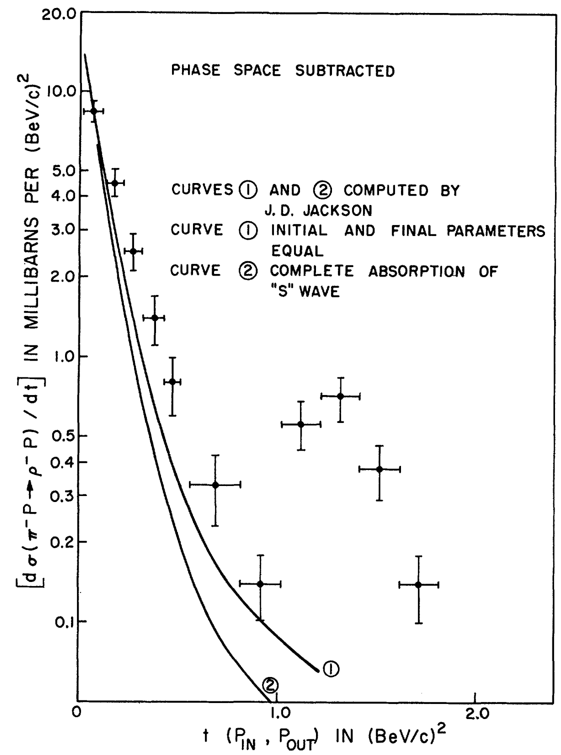

FIG. 2. The differential rho-production cross section $d\sigma(1.7\text{-BeV}/c\ \pi^- + p \to \rho^- + p)/dt$ versus four-momentum transfer between incoming and outgoing proton.

Fig. 1 for various mass cuts in the region of the ρ resonance, and has been corrected for nonresonant background. We found it necessary to make narrow mass cuts in order to eliminate major concealing effects of background. Even at the peak of the ρ resonance (765 MeV), it is a 20% effect, and away from this peak the percentage of background rapidly increases. Distributions in t selected by wider cuts in the mass of di-pion or cuts centered at other masses show loss of detail both for ρ^- and for ρ^0 production. In particular, in these cases the relatively large background tends to obscure the minimum between first and second diffraction peaks. The well-known exponential behavior of the cross section[3] as a function of t is clearly indicated in the region $t \leq 0.6$ (BeV/c)^2 and is found to have the form

$$d\sigma/dt = A_\rho \exp(-\gamma_\rho t),$$

where t is the invariant square of the four-momentum transfer from the incoming proton to the outgoing nucleon.

We find that the differential production cross sections for ρ^- and ρ^0 have the same shape: a diffraction pattern showing the forward diffraction cone, a first minimum, second maximum, and second minimum. For ρ^0, $A_\rho = 2.0 \pm 0.2$ mb/(BeV/c)^2 and $\gamma_\rho = 5.8 \pm 0.5$ (BeV/c)$^{-2}$; and for the ρ^-, $A_\rho = 2.5 \pm 0.2$ mb/(BeV/c)^2, and $\gamma_\rho = 5.3 \pm 0.5$ (BeV/c)$^{-2}$. This may be compared with $\gamma_\pi = 7.1 \pm 0.5$ (BeV/c)$^{-2}$ for $\pi^- p$ elastic scattering at 1.7 BeV/c [1] and with very similar values[3] for πp elastic scattering at higher energies. This is also comparable with values of γ for other kinds of two-body reactions, e.g., pp, $\bar{p}p$, and Kp collisions.[3,9] Recently, Byers and Yang[4] have suggested that the constancy

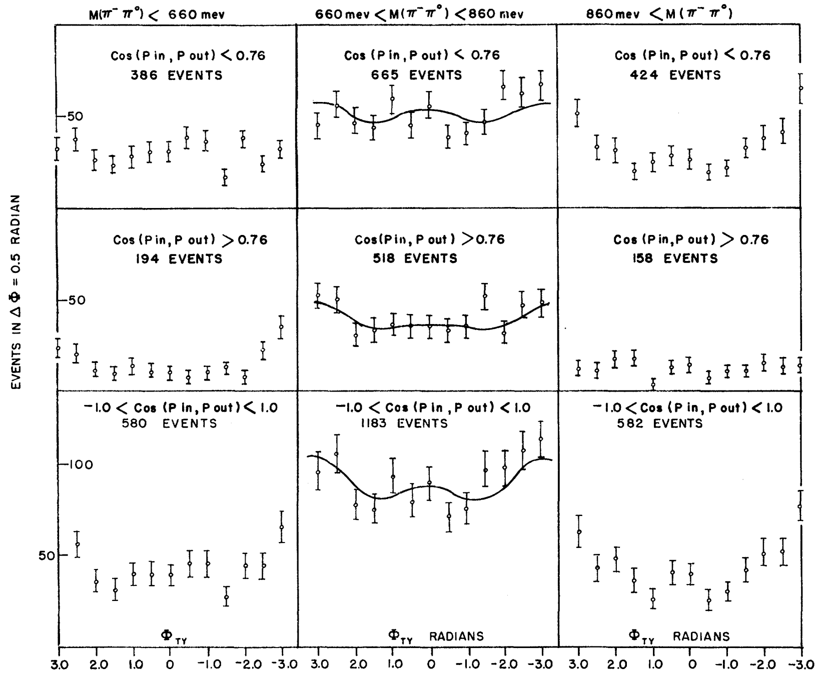

FIG. 3. Distribution of 1.7-BeV/c $\pi^- + p \rightarrow \pi^- + p + \pi^0$ events versus Treiman-Yang angle for di-pion mass below, in, and above the region of the rho. The solid lines represent a best fit to the data and show a dependence of the form $A + B\cos\varphi + C\cos^3\varphi$ instead of the predicted dependence $A + B\cos^2\varphi$.

of γ is related to a characteristic volume of strong interaction roughly independent of the nature of the exchange, be it spin, isotopic spin, strangeness, or momentum. In the present case, comparison of γ_ρ with γ_π at 1.7 BeV/c indicates inequality of the respective interaction volumes of the rho and pi meson with the nucleon.[5]

Jackson[6] computed the differential cross sections for π^- production at 1.7 BeV/c shown in Fig. 2, on the basis of the absorption-model modification of the one-pion exchange mechanism[7] and using the 1.7-BeV/c π^-p elastic cross section of Ref. 1. For curve 1, Fig. 2, the Gottfried-Jackson pion and rho parameters have been taken to be equal; for curve 2, complete absorption of s wave has been assumed. Neither of these curves fits the data, both overestimating the slope of the forward peak. The absorptive one-particle-exchange model is in difficulties in other ways at this energy. Examination of Fig. 3 indicates that, assuming the di-pion to have spin 1, the predictions of the one-particle-exchange model for the dependence on the Treiman-Yang angle do not agree with the dependence found experimentally. The absorptive model predicts[7] a dependence of the form $A + B\sin^2\varphi$, whereas the events in a mass cut 760 ± 100 (MeV/c)^2 show a dependence of the form $A + B\cos\varphi + C\cos^3\varphi$, where φ is the Treiman-Yang angle. This disagreement is in some contrast to the experimental observation[8] at 4.0 BeV/c where qualitatively the dependence is as predicted by the absorption model, but quantitatively the coefficients A and B do not agree with those for one-pion exchange.

In Fig. 4 the differential ρ^--production cross section has been fitted by a bright-ring diffraction pattern given by

$$d\sigma/d\Omega = \{kRJ_0(2kR\sin\tfrac{1}{2}\theta)\Delta R\}^2,$$

where $R = 0.83 \pm 0.05$ F, $\Delta R = 0.20 \pm 0.01$ F, and k is the center-of-mass system momentum. The coherent bright-ring model has previously given good representation for other two-body reactions, for example in K^-p charge exchange[9] at 2.0 BeV/c where $R = 0.68$ F and $\Delta R = 0.09$ F; and in elastic K^-p scattering[10] at 1.45 BeV/c where $R = 0.60$ F and $\Delta R = 0.15$ F.

For comparison, we note that the optical model for a black disk has been found to fit the 1.7-BeV/c π^-p elastic-scattering differential cross section[1] for a radius $R_\pi = 0.80 \pm 0.02$ F.

The optical model, with its considerable recent successes in representing high-energy

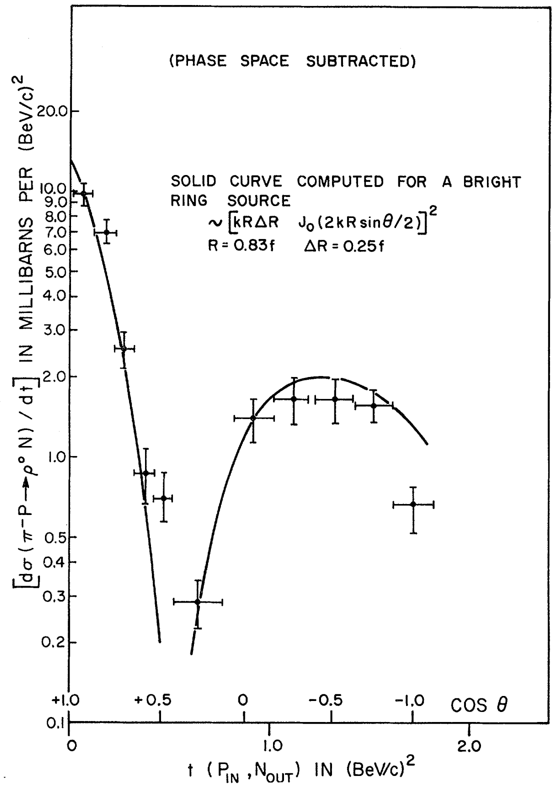

FIG. 4. The differential rho-production cross section $d\sigma(1.7\text{-BeV}/c\ \pi^- + p \rightarrow \rho^0 + n)/dt$ versus four-momentum transfer between incoming proton and outgoing neutron. This curve is taken from data supplied by Fickinger, Robinson, and Salant.[2]

differential cross sections, offers useful parametrization of the data in terms of radii and volumes of interaction.

We are greatly indebted to Dr. E. O. Salant for lending the film, and to Serge Paul-Emile and to our technicians for their indispensable help.

*Research supported by the U. S. Atomic Energy Commission Contract No (11-1)-1537.

[1]D. D. Allen, G. Godden, G. P. Fisher, J. B. Kopelman, L. Marshall, and R. Sears, Phys. Letters 21, 468 (1966). (We estimate that at most 4% of the events in the $\pi^- p\pi^0$ final state have been designated as elastics, and less than 1% of the $\pi^- p\pi^0$ events used in this report are due to contamination from other final states.)

[2]W. J. Fickinger, D. K. Robinson, and E. O. Salant, Phys. Rev. Letters 10, 457 (1963); and Proceedings of the Athens Conference on Resonant Particles, Ohio University, Athens, Ohio, 1965 (to be published).

[3]Aachen-Berlin-CERN Collaboration, CERN Report No. CERN/TC/P-65-23, 1965, to be published; K. J. Foley, S. J. Lindenbaum, W. A. Love, S. Ozaki, J. J. Russell, and L. C. L. Yuan, Phys. Rev. Letters 10, 376 (1963); 11, 425 (1963). S. Brandt, V. T. Cocconi, D. R. O. Morrison, A. Wroblewski, P. Fleury, G. Kayas, F. Muller, and C. Pelletier, Phys. Rev. Letters 10, 413 (1963); I. Mannelli, A. Bigi, R. Carrara, M. Wahlig, and L. Sodickson, Phys. Rev. Letters 14, 408 (1965); A. V. Stirling, P. Sonderegger, J. Kirz, P. Falk-Vairant, O. Guisan, C. Bruneton, P. Borgeaud, M. Yvert, J. P. Guillaud, C. Caverzasio, and B. Amblard, Phys. Rev. Letters, 14, 763 (1965).

[4]N. Byers and C. N. Yang, Phys. Rev. 142, 976 (1966), and see references quoted in this paper.

[5]C. Iddings and L. Marshall, to be published.

[6]J. D. Jackson, private communication to G. P. Fisher.

[7]K. Gottfried and J. D. Jackson, Nuovo Cimento 33, 309 (1964); 34, 735 (1964).

[8]I. Derado, V. P. Kenney, and W. D. Shephard, Phys. Rev. Letters 13, 505 (1964).

[9]D. Barge, W. Chu, L. Leipuner, R. Crittenden, H. J. Martin, F. Ayer, L. Marshall, A. C. Li, W. Kernan, and M. L. Stevenson, Phys. Rev. Letters 13, 69 (1964).

[10]A. Fridman, O. Renary, A. Michalon, B. Schiby, R. Strub, and G. Zech, "Elastic K^- Proton Interaction at 1.45 GeV/c and Comparison with Absorptive Models" (to be published).

CURRENT COMMUTATORS, REPRESENTATION MIXING, AND MAGNETIC MOMENTS*

Haim Harari†
Stanford Linear Accelerator Center, Stanford University, Stanford, California
(Received 22 April 1966)

A detailed analysis of the various sum rules which have been recently derived from the chiral U(3)⊗U(3) algebra of currents[1] indicates that the exact sum rules may be approximated by sums over a few intermediate states which fall into a relatively simple <u>reducible</u> representation of the current algebra. In a previous paper[2] (hereafter denoted by I) we have shown that the positive-helicity state of the nucleon can be properly described as having components in the $\{(\underline{6},\underline{3})L_z=0\}$, $\{(\underline{3}^*,\underline{3})L_z=0\}$, and $\{(\underline{3},\underline{3})L_z=1\}$ representations of U(3)⊗U(3), and that by adjusting one free mixing angle one can then correctly predict the experimental values of G_A, the axial vector coupling constant in β decay; G^*, the strength of the axial-vector tran-