measured amplitudes and their errors (indicated by dotted lines). The two directions for $\Sigma_0{}^+$ correspond to the ratio $|S/P|$ being greater than or less than 1. Because α_0 is very nearly -1, the orientation of the $\Sigma_0{}^+$ amplitude in the S-P plane is extremely dependent on the value of α_0, resulting in a large uncertainty in the $\Sigma_0{}^+$ direction. To reduce this uncertainty to a value comparable with that of $\Sigma_+{}^+$ and Σ^- would require, in our experiment, a precision $\Delta\alpha_0 = \pm 0.001$, if α_0 is -1.

The nonleptonic-Σ-decay amplitudes also enter into the prediction by Lee[9] that $\sqrt{3}\,\Sigma_0{}^+ + \Lambda = 2\Xi^-$. A recent compilation shows that this relation is well satisfied.[10] The more precise values of Σ decay parameters presented here are, if anything, in even better agreement with Lee's prediction.

We wish to express our appreciation to Mr. Glenn Eckman and the crew of the 25-inch bubble chamber, and to Mr. Henry Laney and the scanners on this experiment. We are especially grateful to Dr. Margaret Alston for her help on many phases of the analysis. Finally we gratefully acknowledge the continued support and encouragement of Professor Luis W. Alvarez.

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*Work done under the auspices of the U. S. Atomic Energy commission.

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EXCHANGE OF EVEN- AND ODD-PARITY BARYON-MESON RESONANCES AND BACKWARD ELASTIC SCATTERING*

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If there are two meson-baryon resonances or bound states with opposite parities but with the same remaining internal quantum numbers including signature, one at c.m. energy W_1 with $l = J - \frac{1}{2}$ and the other at W_2 with $l = J + \frac{1}{2}$, then the residue of the Regge trajectory on which they lie must change sign between W_1 and $-W_2$. Consequences of the exchange of such a trajectory to backward elastic scattering are discussed.

According to the MacDowell symmetry,[1] the πN amplitudes for orbital angular momenta $l = J \pm \frac{1}{2}$ for a given total angular momentum, J, and isotopic spin, I, are related as follows:

$$h^J_{J+\frac{1}{2}}(W+i\epsilon) = -h^J_{J-\frac{1}{2}}(-W-i\epsilon), \qquad (1)$$

where W is the total c.m. energy, and for $W \geqslant m + m_\pi$,

$$h^J_{J-\frac{1}{2}}(W+i\epsilon) = \frac{W}{E+m}\,\frac{\exp[i\delta_{J-\frac{1}{2}}(W)]\sin\delta_{J-\frac{1}{2}}(W)}{k}, \qquad (2)$$

E and k being the energy of the nucleon (of mass m) and c.m. momentum, respectively. Relation (1) can be extended to complex J plane where in addition to J and I, the signature, τ, of both the amplitudes must be the same.[2]

On can generalize the above relations also to the scattering of pseudoscalar mesons by baryons (e.g., KN scattering).[1] The interesting thing about the MacDowell symmetry, of course, is that it relates the amplitude at W for a given parity to that at $-W$ for the opposite parity. Consider the situation where, for positive W, $\alpha(W)$ is the leading pole of $h_{J-\frac{1}{2}}{}^J(W)$ in the complex J plane. If $\alpha(W)$ continues to dominate in the negative-W region then clearly, for given I and τ, resonances and bound states of both parities will lie on the same trajectory in the W plane.[3] Thus, for instance, $\Lambda(\frac{1}{2}^+;\ I=0,\ \tau=+)$ must lie on the same trajectory in the W plane as $Y_0^*(1405)$ $(\frac{1}{2}^-;\ I=0,\ \tau=+)$. Similarly $\Sigma(\frac{1}{2}^+;\ I=1,\ \tau=+)$ and $Y_1^*(1765)$ $(\frac{5}{2}^-;\ I=1,\ \tau=+)$, and $Y_1^*(1385)$ $(\frac{3}{2}^+;\ I=1,\ \tau=-)$ and $Y_1^*(1660)$ $[\frac{3}{2}^-(?);\ I=1,\ \tau=-]$ must lie on the same trajectories. The type of $\alpha(W)$ involved in such situations is described in Fig. 1. For the case where strangeness is zero, there are as yet no such well-established opposite-parity partners to the known resonances or bound states. However, the N and Δ trajectories do appear to be essentially linear in the $u(=W^2)$ plane and can be represented by[4-6]

$$\alpha_R(W)=A+BW+CW^2, \tag{3}$$

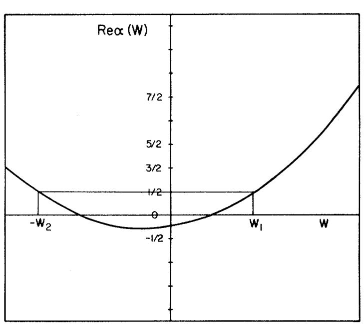

FIG. 1. A typical curve for $\mathrm{Re}\,\alpha(W)$ for pseudoscalar meson-baryon system. For given I, S, and τ, the trajectory describes states with both parities. As an example, the curve is assumed to represent positive-signature particles. On the basis of the MacDowell symmetry (1), it describes a $\frac{1}{2}^-$ particle of mass W_1 and a $\frac{1}{2}^+$ particle of mass W_2. The region of interest to the backward scattering in s channel is given by $0\leq W \leq u_B^{1/2}$.

where R denotes the real part and $C>0$. The presence of the quadratic term in W ensures that the trajectory will have a shape similar to the one given in Fig. 1. If B is small (for straight-line trajectories in the u plane, B vanishes identically), then resonances of opposite parity may well exist.[7]

Under the above conditions, particularly (1), (2), and (3), consider the case where $\alpha(W)$ dominates $h_{J-\frac{1}{2}}{}^J(W)$:

$$h_{J-\frac{1}{2}}{}^J(W)=\beta(W)/[J-\alpha(W)], \tag{4}$$

where $\beta(W)$ is the residue and $\alpha(W)$ can be expanded in a power series around an arbitrary point $W=W_0$,

$$\alpha(W)=\alpha(W_0)+d(W)|_{W=W_0}(W-W_0)+\cdots,$$

where

$$d(W)=d\alpha(W)/dW. \tag{5}$$

Near a resonance at $W=W_1$, taking $W_0=W_1$ in the above expressions we get

$$h_{J-\frac{1}{2}}{}^J(W)\simeq\frac{\beta_R(W_1)/d_R(W_1)}{W_1-W-i\alpha_I(W_1)/d_R(W_1)}, \tag{6}$$

where $d_R(W_1)$ and $\alpha_I(W_1)$ are positive (the subscript I denotes imaginary part). Now if $W=W_2$ is the resonance position for $h_{J+\frac{1}{2}}{}^J(W)$ described by the same $\alpha(W)$, then using (1) and (4) and taking $W_0=-W_2$, $d_R(-W_2)=-|d_R(-W_2)|$ (see Fig. 1), and noting that the imaginary parts of $\alpha(W_2+i\epsilon)$ and $\alpha(-W_2-i\epsilon)$ have the same (positive) sign, we obtain

$$h_{J+\frac{1}{2}}{}^J(W)\simeq\frac{-\beta_R(-W_2)/|d_R(-W_2)|}{W_2-W-i|\alpha_I(-W_2)/d_R(-W_2)|}. \tag{7}$$

Now the residue of the Breit-Wigner pole for elastic scattering must be positive. Therefore, while $\beta_R(W_1)$ is positive, $\beta_R(-W_2)$ must be negative. Hence, $\beta_R(W)$ must change sign between W_1 and $-W_2$.[8] This sign change should occur even in the absence of any resonances or bound states as long as $\alpha(W)$ reaches the right-half J planes corresponding to both the amplitudes.[9] This is so because in the latter event, the restrictions on the signs of $d_R(W)$ and $\alpha_I(W)$ remain the same as in the resonance case.

Note that all the results about $\beta(W)$ mentioned above also apply to the reduced residue[10]

$$g(W)=(s_0/k^2)^{\alpha(W)-\frac{1}{2}}\beta(W), \tag{8}$$

where s_0 is some convenient scaling factor. Therefore, $g(W)$ must also change sign if $\beta(W)$ does. We will assume that $g(W)$ changes sign by going through zero at some point between W_1 and $-W_2$.[11] Because of unitarity, this point should be between the two thresholds [for πN scattering this would be between $\pm(m+m_\pi)$]. In what follows we shall propose different mechanisms by which a zero in $g(W)$ can come about and then discuss the consequences for backward meson-baryon scattering in the s channel.

From the usual N/D formalism, it follows that

$$g(W)=\frac{N(W)}{\partial D(W)/\partial J}\bigg|_{J=\alpha(W)},$$
$$=G(W)\,|_{J=\alpha}\,d(W), \tag{9}$$

where $G(W)$ is the residue of the pole in the energy ($=W$) plane given by

$$G(W)=-\frac{N(W)}{\partial D(W)/\partial W}\bigg|_{J=\alpha(W)},$$

and $d(W)$ has already been defined in (5). It should be emphasized here that for α in the right-half J plane and W below threshold, $G(W)$ is proportional to the normalization of the bound-state wave function and is, therefore, positive and nonzero.[12] Another point we would like to emphasize is that the type of trajectories that we are considering (see Fig. 1) all have a minimum at some value of W between the two thresholds, i.e.,

$$d(W)=0 \text{ for } \alpha(W)=\alpha(W_{\min}). \tag{10}$$

Let us now mention two simplest alternatives by which $g(W)$ can go through zero.[13] (i) One possibility is that $g(W)$ is zero where $d(W)$ is. This means that $g(W)$ vanishes at the point where $\alpha(W)$ is minimum. This would also mean that $G(W)\,|_{J=\alpha}$ remains of the same (positive) sign but that $\partial D/\partial J\,|_{J=\alpha}$ becomes infinite at $W=W_{\min}$ and, therefore, in the J plane D has a singularity at $J=\alpha(W_{\min})$.

(ii) The other alternative is to assume that D does not have any singularities in the J plane and, therefore, $\partial D/\partial W$ is zero precisely at the same point where $d(W)$ is. The zero in $g(W)$ should then come about from the vanishing of $N\,|_{J=\alpha}$ at some other value of W ($=W_c$, say). Thus N should have a zero at $W=W_c$. This would mean that $G\,|_{J=\alpha}$ should be infinite at $W=W_{\min}$ and zero at $W=W_c$. If our previous statement regarding the relation between G and the normalization of the bound-state wave function is correct, then $\alpha(W_c)$ cannot be in the right-half J plane.[14]

If s is taken as the direct-channel variable and $u(=W^2)$, the crossed-channel variable, then for $s\gg m^2$ the differential cross section for the meson-baryon scattering near the backward direction is dominated by the u-channel poles and is given by[4-6,10]

$$\frac{d\sigma}{d\Omega}\approx\tfrac{1}{4}\,|A_1^{\,u}(W)+A_1^{\,u}(-W)|^2 + \frac{(u_B-u)}{4}\left|\frac{A_1^{\,u}(W)-A_1^{\,u}(-W)}{W}\right|^2, \tag{11}$$

where

$$A_1^{\,u}(W)=g(W)(\alpha+\tfrac{1}{2})\frac{\pi^{1/2}\Gamma(1+\alpha)}{\Gamma(\frac{3}{2}+\alpha)} \times\left[\frac{1\pm\exp[-i\pi(\alpha-\frac{1}{2})]}{\sin\pi(\alpha-\frac{1}{2})}\right]\left(\frac{s}{s_0}\right)^{\alpha(W)-\frac{1}{2}}. \tag{12}$$

u_B is the value of u in the backward direction ($\cos\theta_s=-1$), and s_0 is the same scaling factor as in (8). The entire physical scattering region is given by $-u_L<u<u_B$, where u_L is positive and increases linearly as s increases. Since we are here concerned with real values of W, the region of interest in the W plane is $0\leq W\leq u_B^{1/2}$ (note that u_B is +).

It should be clear from expressions (11) and (12) that a zero in $g(W)$ will have an important effect on the backward scattering, particularly if this zero occurs inside or near the region $0\leq W\leq u_B^{1/2}$. To our knowledge, the possibility of such zeros has not been considered in the previous Regge phenomenology. Our considerations here have shown that, in the W plane, zeros should be present in the residues of Σ, Λ, and N trajectories and perhaps in the residues of other meson-baryon systems as well. As far as the alternatives (i) and (ii) mentioned above are concerned, it is perhaps best to consider both the alternatives in any phenomenology and decide on the basis of experiments which is the better one.

*Work supported in part by U. S. Atomic Energy Commission Contract No. AEC AT(11-1)34 P107A.

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[3]It is possible that there are two trajectories, $\alpha_1(W)$ which dominates the positive-parity amplitude and $\alpha_2(W)$ which dominates the negative-parity amplitude. However, this picture would lead to crossing of the trajectories and will not be within the conventional Regge framework.

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[5]V. Barger and D. Cline, Phys. Rev. Letters 16, 913 (1966).

[6]C. Chiu (University of California Lawrence Radiation Laboratory, Berkeley, private communication).

[7]For instance, it has been suggested by Chiu (Ref. 6) that a $\frac{5}{2}^-$ πN resonance (for which there is some experimental evidence) may lie on the same trajectory on which the nucleon lies. As far as Δ_δ and N_γ trajectories are concerned, even though the analysis of Barger and Cline (Ref. 5) suggests that they are almost straight lines, there are as yet no opposite-parity partners observed experimentally.

[8]At this point it is interesting to compare our results with those of (spin-zero) meson-meson amplitudes where parity and signature signify the same thing. These amplitudes are functions of W^2. Therefore, the corresponding $\alpha(W)$ is a symmetric function of W and gives rise to the same particles on both sides of the W plane. This is in contrast to the meson-baryon case where the two sides correspond to states with opposite parities. The meson-baryon amplitudes have an additional factor $(E+m)^{-1}$ [see (1)] which is just the factor which insures proper threshold behavior in the negative-W plane. It is this factor which is responsible for the negative sign in (1), so that while the meson-meson residues do not change sign, the meson-baryon residues do.

[9]What constitutes a right-half J plane in the relativistic case is, of course, unclear, but one should certainly expect all the bound states and resonances to lie in it.

[10]The notations $g(W)$ and $A_1^u(W)$ given below are the same as in Ref. 6.

[11]The other possibility is that the function $\beta_R(W)$ or $g(W)$ develops an imaginary part, so that instead of going through zero, it goes along a contour around it. This would imply singularities in addition to the normal ones. Such possibilities have not been considered in the conventional Regge framework.

[12]See, for instance, B. R. Desai, Phys. Rev. 138, B1174 (1965).

[13]Also see comments in Ref. 3 and 11.

[14]In the (spin-zero) meson-meson case mentioned above (Ref. 8) both $d(W)$ and $\partial D/\partial W$ vanish at $W=W_{\rm min}$. It is conceivable that this particular property would be true also for the meson-baryon case. If this is so, then alternative (ii) would be preferred.

NEW DETERMINATION OF THE φ SPIN AND G PARITY

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The present knowledge of the φ quantum numbers ($J^{PG}=1^{--}, I=0$) has been obtained from an interpretation of its decay modes into $\bar{K}K$ and the search for charged φ's in the final states $\Sigma K\bar{K}$ produced in K^-p collisons.[1] The data overwhelmingly favor the 1^{--} assignment; however, the spin determination is based on the assumption that the φ meson has a "reasonable" structure to which neutral and charged kaons are equally coupled. In fact, the structure assumed is an approximation by a square potential well of "reasonable" radius. Although this assumption is very plausible it has not been tested yet, and doubts could be raised about its validity.

Due to the important role that the φ meson plays in SU(3) symmetry, an independent method yielding the φ spin has been desirable. In this Letter a new determination of the φ spin which is independent of any assumption on the structure of the φ is reported; at the same time we obtain confirmation that $G_\varphi=-1$.

In a systematic analysis, undertaken by our groups, of annihilations of antiprotons brought