PROPOSAL TO TEST TIME-REVERSAL INVARIANCE IN THE REACTIONS $\gamma+d \rightleftarrows n+p$

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It is important to perform feasible experimental tests of the hypothesis[1,2] that time-reversal invariance may fail in an electromagnetic interaction. If this hypothesis can be laid to rest, undivided attention can be turned to the problem of the nature of weak[3] or superweak[4,5] violations of CP invariance demanded by experiment.[6-8] Unfortunately, a number of a priori feasible experimental tests in the domain of elementary-particle physics appear, on closer analysis, to be expected to yield only relatively small effects, even if CP and T invariances are significantly violated.[9] We refer, in particular, to (a) $\pi^0 \to 3\gamma$,[1,2,10] (b) $\eta \to \pi^0+e^++e^-$,[2] (c) the asymmetry between positive and negative pions in $\eta \to \pi^+ + \pi^- + \pi^0$,[2,11-13] (d) the Λ polarization normal to the decay plane in $\Sigma^0 \to \Lambda + e^+ + e^-$,[2] (e) $\varphi \to \rho^0+\gamma$, $\omega \to \rho^0+\gamma$,[2] and (f) the spin-correlation effects in $e^-+p \to e^- + N^*$[14,15] and $\bar{p}+p \to \bar{\Lambda}+\Lambda$.[1]

We wish to suggest that a significant test of time-reversal invariance can be carried out by comparing the differential cross sections for the reactions

$$\gamma+d \to n+p, \tag{1a}$$

$$n+p \to \gamma+d, \tag{1b}$$

at, or above, about 290-MeV/c photon laboratory momentum, or about 590-MeV neutron laboratory kinetic energy, respectively, with unpolarized initial and final particles. These energies are in the region of a known[16-18] bump in the total cross section for Reaction (1a), whose peak height is about 65-75 μb. Reaction (1b) has not been studied yet. This bump is undoubtedly associated with the influence of the first nucleon isobar, N^*, upon the process.[19,20] We present below a theoretical estimate of the possible failure of the reciprocity relationship between the differential cross sections for Reactions (1a) and (1b) caused by the possible

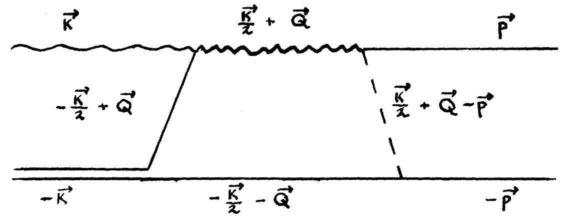

FIG. 1. Time-ordered Feynman graph for computing the influence of the isobar, N^*, on the process $\gamma+d \to n+p$. Nucleons are denoted by solid lines; the isobar, by a jagged line; the photon, by a wavy line; and the pion, by a dashed line. Three-momenta carried by the particles are indicated.

failure of time-reversal invariance in the vertex for

$$\gamma+N \rightleftarrows N^*, \tag{2}$$

where N denotes a nucleon. The estimate is based upon a systematic version of the moderately successful heuristic model for deuteron photodisintegration given some time ago by Austern.[19] We find that the differential cross sections for Reactions (1a) and (1b) may differ by as much as ~40%, if time-reversal invariance fails. A comparison of these differential cross sections is thus likely to be a good experimental test of the hypothesis.

The model of Austern[19] estimates, by means of a heuristic (and consequently simple) perturbation-theoretic calculation, the matrix element for deuteron photodisintegration via an intermediate state in which one of the nucleons from the deuteron has been excited to the $J=T=\frac{3}{2}$ isobar at a mass of about 1.24 BeV, by absorption of a magnetic-dipole photon. A systematic field-theoretic calculation of this matrix element, in ordinary perturbation theory, involves the evaluation of the matrix element for the time-ordered Feynman graph shown in Fig. 1. The expression for this matrix element, in the center-of-mass system, is given by

$$M = iG\rho\left\{\frac{m^2}{4\kappa D_0 E^2}\right\}^{1/2}\int\frac{d\vec{Q}}{(2\pi)^3}\left\{\frac{m^2 m^*}{2\omega E^* e_1 e_2}\right\}\sum_{r,s,\alpha}\left[\frac{\bar{u}(N_2)\gamma_5 u^s(n_2)\bar{u}(N_1)q_\lambda Z_\lambda{}^\alpha(N^*)}{(E-e_2-\omega)} \times \frac{\bar{Z}_\nu{}^\alpha(N^*)\gamma_5\gamma_\mu u^r(n_1)\bar{u}^s(n_2)\Gamma\bar{u}^r(n_1)f_{\mu\nu}}{(\kappa+D_0-E^*-e_2+i\gamma^*/2)(D_0-e_1-e_2)}\right] \tag{3a}$$

$$M=iG\rho\left\{\frac{m^2}{4\kappa D_0E^2}\right\}^{1/2}\int\frac{d\vec{Q}}{(2\pi)^3}\left\{\frac{1}{16\omega E^*e_1e_2}\right\}\left[\frac{\bar{u}(N_1)O_{\lambda\nu}(\not{N}^*+m^*)\gamma_\mu}{(2m_D)}\right.$$
$$\left.\times\frac{(\not{n}_1-m)\not{t}(\not{D}+m_d)(\not{n}_2+m)C\bar{u}^{\mathrm{T}}(N_2)q_\lambda f_{\mu\nu}}{(E-e_2-\omega)(\kappa+D_0-E^*-e_2+i\gamma^*/2)(D_0-e_1-e_2)}\right],\quad(3b)$$

with

$$\Gamma=(\not{D}+m_d)\not{t}C/2m_d,$$
$$O_{\lambda\nu}=\frac{2}{3}\left\{3g_{\lambda\nu}-\gamma_\lambda\gamma_\nu-\frac{4N_\lambda^*N_\nu^*}{(m^*)^2}+\frac{(\gamma_\lambda\not{N}^*N_\nu^*+N_\lambda^*\not{N}^*\gamma_\nu)}{(m^*)^2}\right\},$$
$$f_{\mu\nu}=\kappa_\mu\epsilon_\nu-\kappa_\nu\epsilon_\mu,$$
$$C=\gamma_0\gamma_2.\quad(4)$$

In these equations, Dirac spinors for a particle with four-momentum N_ν are denoted by $u(N)$ [$u^{\mathrm{T}}(N)$ denotes the transposed spinor]; the Rarita-Schwinger[21,22] wave functions for a spin-$\frac{3}{2}$ isobar are denoted by $Z_\nu(N^*)$; t_ν denotes the polarization pseudovector for the deuteron[23]; ϵ_ν denotes the photon polarization vector; and C denotes the charge-conjugation matrix. In Eq. (3a) we indicate explicit sums over intermediate nucleon spin states, denoted by r and s, and intermediate isobar spin states, denoted by α; Eq. (3b) results from performing these sums. The masses of deuteron, nucleon, isobar, and pion are denoted by m_d, m, m^*, and μ, respectively. The four-momenta of the particles participating in Fig. 1 are defined by

$$\kappa_\nu=(\kappa,\vec{\kappa}),\quad D_\nu=(D_0,-\vec{\kappa}),$$
$$(N_1)_\nu=(E,\vec{p}),\quad (N_2)_\nu=(E,-\vec{p}),$$
$$(n_1)_\nu=(e_1,\vec{Q}-\vec{\kappa}/2),\quad (n_2)_\nu=(e_2,-\vec{Q}-\vec{\kappa}/2),$$
$$q_\nu=(\omega,\vec{Q}+\vec{\kappa}/2-\vec{p}),\quad N_\nu^*=(E^*,\vec{Q}+\vec{\kappa}/2),\quad(5)$$

where the energies are the usual total energies for a particle of given mass and three-momentum. The symbol ρ denotes a numerical isotopic-spin factor; the quantity G is a product of four known coupling parameters,

$$G=(a/\sqrt{2})gG_\pi^*G_\gamma^*.\quad(6)$$

We denote by g the pion-nucleon coupling constant; G_π^* denotes the isobar-nucleon-pion coupling constant determined by the isobar width, $\gamma^*\cong 120$ MeV; G_γ^* denotes the coupling constant of the magnetic-dipole photon to nucleon and isobar. Its magnitude is calculated by computing the total cross section for $\gamma+p\rightarrow\pi^0+p$ via the isobar and setting this equal to about 250 μb at a photon laboratory momentum of about 300 MeV/c. Finally, the quantity $a/\sqrt{2}$ denotes the deuteron–two-nucleon "coupling constant" and is estimated, in a standard manner,[23] from the asymptotic form of the deuteron S-state wave function (we neglect the deuteron D state). In our calculation we use the following values:

$$g^2/4\pi\cong 15,$$
$$(G_\pi^*)^2/4\pi\cong 0.179\mu^{-2},$$
$$(G_\gamma^*)^2/4\pi\cong(6.8\text{ BeV}^{-2})\,(e^2/4\pi)$$
$$\text{with } e^2/4\pi=1/137,$$
$$a^2/4\pi\cong 0.466.\quad(7)$$

We carry out a straightforward, but lengthy, reduction of Eq. (3b) to an expression in terms of Pauli spinors. In this reduction we neglect, everywhere in the numerator, quantities of order κ/m, $\kappa/2p$, Q/m, and Q/p with respect to the leading terms. In the energy denominators we consistently neglect[19] the relative velocity of the isobar-nucleon intermediate state.[24] The result, after accounting for isotopic spin factors from both the proton and neutron absorption of the photon, is then the following remarkably simple matrix element, which represents a field-theoretic derivation of Austern's model:

$$M=G\left\{\frac{1}{3}\left(\frac{2}{3}\right)^{1/2}\right\}\left\{\frac{m^2}{m_d\kappa E^2}\right\}^{1/2}\frac{p^2\kappa/\sqrt{2}\,\omega}{(\kappa+m-m^*+i\gamma^*/2)(m+\omega-E)}\int\frac{d\vec{Q}}{(2\pi)^3(mB+Q^2)}$$
$$\times\{3\hat{p}\cdot\hat{\epsilon}\hat{p}\cdot\hat{\kappa}\times\hat{\epsilon}-\hat{\epsilon}\cdot\hat{\kappa}\times\hat{\epsilon}\}\chi_2^{\mathrm{T}}\sigma_y\chi_1. \quad (8)$$

In Eq. (8), $\omega=(p^2+\mu^2)^{1/2}$; B denotes the deuteron binding energy; χ_1 and χ_2 are Pauli spinors for the nucleons labeled 1 and 2, respectively; σ_y is the Pauli spin operator; and the vector symbols denote unit vectors. The last two factors constitute a projection operator for the ${}^1D_2$ state with $T=1$ of the final two-nucleon system—this is the dominant transition induced by the mechanism in Fig. 1.[19] The magnitude of this amplitude's contribution to the total cross section is fixed (since G is known) but for the need to cut off the integral over the deuteron internal momentum, $\vec{Q}$. Our procedure is to determine the cutoff momentum, $Q_{\max}$, by equating the total cross section computed from this amplitude to about 27 μb, which is roughly the height of the bump in the experimental total cross section,[16-18] above the preceding valley.[25] We find $Q_{\max}\cong 2.53(mB)^{1/2}$, which we consider a reasonable value, in support of the use of the asymptotic properties of the deuteron-two-nucleon vertex in computing the matrix element for the process in Fig. 1.

The hypothetical failure of time-reversal invariance is introduced simply by giving a phase Δ, different from 0 or π, to the vertex represented by G_γ^*. The matrix element M is then proportional to $e^{i\Delta}$. Upon computing, in the same approximation, the matrix element M_R for the inverse process, Reaction (1b), we see that it will be given by Eq. (8) with $e^{i\Delta}$ changed to $e^{-i\Delta}$. We can now point to the essential effect that can give rise to a sizable failure of reciprocity. For illustration, let $|\Delta|\cong\pi/2$, and let us be at an energy such that the real part of the resonance-energy denominator in Eq. (8) vanishes. Then, with both amplitudes approximately real (neglecting the phase of $\chi_2^{\mathrm{T}}\sigma_y\chi_1$), we have

$$M\cong -M_R. \quad (9)$$

The isobar amplitude changes sign in going from Reaction (1a) to its inverse. If there is another largely relatively real amplitude (even if quite small in absolute square) with which the isobar amplitude interferes, reciprocity between the differential cross sections can be grossly violated, as we show numerically below.

We have made a concrete estimate. Experiment[16-18] indicates that there is a dominant isotropic component in the differential cross section for Reaction (1a) in the region of the enhancement. We therefore add amplitudes for two transitions that are strongly felt[18,26,27] to play a role at lower energies, $M(1)\to{}^1S_0$, denoted by a_0, and $E(1)\to{}^3P_0$, denoted by b_0. We take these amplitudes as real, assuming, in particular, that the real parts of the n-p ${}^1S_0$ and ${}^3P_0$ phase shifts are small at these energies.[28] The complete matrix element is then

$$\mathfrak{M}=M+a_0\hat{\epsilon}\cdot\vec{\kappa}\times\hat{\epsilon}\chi_2^{\mathrm{T}}\sigma_y\chi_1+ib_0\hat{\epsilon}\cdot\hat{\epsilon}\chi_2^{\mathrm{T}}\vec{\sigma}\cdot p\vec{\sigma}_y\chi_1. \quad (10)$$

At $\kappa\cong 290$ MeV/c, we take a total cross section, σ, for deuteron photodisintegration of about 75 μb, with about 27 μb from $M(1)\to{}^1D_2$, about 45 μb from $E(1)\to{}^3P_0$, and only about 3 μb (or about 4% of σ), from $M(1)\to{}^1S_0$.[27] If we write

$$a_0=[a(8\pi)^{-1/2}][e(4\pi)^{-1/2}](m_S)^{-2}\times\{m^2/m_d\kappa E^2\}^{1/2},$$
$$b_0=[a(8\pi)^{-1/2}][e(4\pi)^{-1/2}](m_P)^{-2}\times\{m^2/m_d\kappa E^2\}^{1/2}, \quad (11)$$

the latter two partial cross sections correspond to reasonable masses of the order of the pion mass:

$$m_S\cong 195 \text{ MeV},$$
$$m_P\cong 138 \text{ MeV}. \quad (12)$$

Only the singlet states interfere in the differential cross sections; therefore the small amplitude, a_0, is essential to the failure of reciprocity. Removing the statistical and phase-space factors, we find, for Reaction (1a), with

θ the center-of-mass angle between incident photon and produced neutron,

$$\frac{6\kappa}{p}\frac{d\sigma}{d\Omega}=(1.66\ \mu\text{b})\{8.66+3\sin^2\theta\}\times\left\{1+R(\theta)\cos(\delta_\gamma+\Delta)\left[\frac{3\sin^2\theta-2}{3\sin^2\theta+2}\right]\right\}, \quad (13)$$

and for Reaction (1b), with θ the center-of-mass angle between incident neutron and produced photon,

$$\frac{4p}{\kappa}\frac{d\sigma_R}{d\Omega}=(1.66\ \mu\text{b})\{8.66+3\sin^2\theta\}\times\left\{1+R(\theta)\cos(\delta_\gamma-\Delta)\left[\frac{3\sin^2\theta-2}{3\sin^2\theta+2}\right]\right\}, \quad (14)$$

where

$$|R(\theta)|=\frac{0.94}{\{1+6.66/(3\sin^2\theta+2)\}}. \quad (15)$$

The phase $-\delta_\gamma$ is the phase of the resonance denominator in M; we do not take this as $\pi/2$, but rather, because of the effect of nonzero deuteron internal momenta, we conservatively estimate $|\tan\delta_\gamma|\cong 3$. Defining the difference between Eqs. (13) and (14), divided by the sum, as A, we compute for $\theta=0$ or π,

$$|A|\cong 14.6\%\ \text{for}\ |\Delta|=\pi/4$$
$$\cong 20.6\%\ \text{for}\ |\Delta|=\pi/2; \quad (16a)$$

for $\theta=\pi/2$,

$$|A|\cong 5.4\%\ \text{for}\ |\Delta|=\pi/4$$
$$\cong 7.6\%\ \text{for}\ |\Delta|=\pi/2. \quad (16b)$$

The sign of D changes in going from the poles to $\theta=\pi/2$. Within the approximation scheme of this calculation, the total cross sections are equal. (They are, of course, in general, unequal, if time-reversal invariance fails.)

We want to emphasize that the above significant violations of reciprocity are estimated solely by invoking a theoretically consistent amount of isobar amplitude to account for the experimental bump (accounting for about one-third of σ) in Reaction (1a), together with a small interfering amplitude, known to be present at lower energies (accounting for only about 4% of σ). Further, the ratio of the isotropic term to the coefficient of $\sin^2\theta$ in Eq. (13) is completely consistent with the experimental data.[17,29]

A study of Reaction (1b) with neutrons of well-defined energies appears to be feasible at the Princeton-Pennsylvania accelerator, using time-of-flight techniques and the particular bunching characteristics of this machine's proton beam.[30] It is not absolutely essential to know the neutron flux well—the shape of the differential cross sections can be compared. Improved measurements on Reaction (1a) in the region of the enhancement can be done at a number of electron accelerators. The estimates presented here indicate that a test of reciprocity for the differential cross sections to about 5% would be a significant test of the hypothesis that time-reversal invariance fails in a high-energy electromagnetic interaction.[31]

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[25]In the calculations we take κ, the center-of-mass photon momentum, as ~290 MeV/c. This is where the real part of the resonance-energy denominator in Eq. (3b) vanishes, for $Q \cong 0$. The experimental cross section peaks nearer to ~290 MeV/c laboratory photon momentum. This shift toward lower energy is possibly due, in part, to the fact that the very significant cross-section contribution from other than the isobar mechanism is decreasing as the isobar amplitude increases in magnitude, and is familiar from pion photoproduction on hydrogen. It can also be due, in part, to a somewhat lower "effective" isobar mass in this process.

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[29]Experiment also indicates something of a fore-aft asymmetry. Small amplitudes for transitions $E(2) \rightarrow {}^3D$, in interference with the $E(1) \rightarrow {}^3P$ amplitudes can produce this; again the $E(2)$ amplitudes to final triplet states will not interfere with the isobar amplitude. See L. I. Schiff, Phys. Rev. 78, 733 (1950); J. F. Marshall and E. Guth, Phys. Rev. 78, 738 (1950).

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RHO-MESON DIFFERENTIAL PRODUCTION CROSS SECTION BY 1.7-BeV/c π^-p INTERACTIONS*

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In a study of two-pronged events from 1.7-BeV/c π^- interactions in the 20-inch Brookhaven National Laboratory hydrogen bubble chamber, we find that the three-body final state is dominated by ρ-meson formation in a quasi-two-body channel, i.e., $\pi+N \rightarrow \rho+N \rightarrow \pi+\pi+N$. We have analyzed the data from 11 000 two-pronged events in such a way as to isolate the resonant state as freely as possible from nonresonant background. In this way comparisons of rho production with predictions of various models are simplified.

A visual ionization check was made on each two-pronged event for compatibility with the mass fit as computed by the GUTS kinematics program. A given hypothesis was rejected if the probability was less than 0.04 for obtaining a χ^2 greater than the fitted one. If two final-state hypotheses (A, B) were ambiguous on the basis of ionization, but

$$\left[\frac{\chi^2}{C}\right]_A - \left[\frac{\chi^2}{C}\right]_B \geqslant 3,$$

where C is the number of constraints for that hypothesis, then the event was classified as hypothesis B. After this condition was imposed, problems of events remaining ambiguous between any two of the possible channels, e.g. π^-p, $\pi^-p\pi^0$, $\pi^-\pi^+n$, etc., were resolved by methods of the kind discussed by Allen et al.[1] We estimate a maximum of 5% of events in any one final state remaining ambiguous between two or more of these final states.

The cross section we obtain for $\pi^-+p \rightarrow \pi^- + p + \pi^0$ at 1.7 BeV/c is 5.4 ± 0.5 mb. Of this, we find 2.1 ± 0.2 mb goes via the channel $\pi^- + p \rightarrow \rho^- + p$. The absolute value of the $\pi+p \rightarrow \rho + p$ production cross section is obtained by normalizing to total and to zero-degree differential elastic-scattering cross sections measured by counters, as discussed in Ref. 1. The greater part of the quoted uncertainties arise from these counter measurements. Data on the ρ^0 channel at this energy have been previously published by Fickinger, Robinson, and Salant.[2]

The dependence of the production cross section of the $\pi^-\pi^0$ system on the four-momentum transfer squared (t) to the nucleon is given in