SUPERFLUID DENSITY AND SCALING LAWS FOR LIQUID HELIUM NEAR T_λ

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The superfluid density near T_λ is found to vary as $(T_\lambda - T)^\zeta$, where $\zeta = 0.666 \pm 0.006$, and the Josephson relation between the specific heat and the superfluid density is tested experimentally. It is shown that, assuming the Widom-Kadanoff scaling laws, the superfluid coherence length varies as $(T_\lambda - T)^{-\zeta}$.

Although progress has been made in the understanding of higher order phase transitions, many crucial aspects remain unsolved. In particular, the λ transition in liquid He^4 is of interest because of the possible role of quantum effects. A particularly fruitful approach to the general problem has been the method of critical exponents: by assuming that the physical quantities of interest can be approximated by a simple power-law behavior $X_i = (T_c - T)^{y_i}$ in the limit $T \to T_c$. Several inequalities involving these critical exponents have been derived using thermodynamic arguments.[1] Using plausible assumptions, Widom[2] and Kadanoff[3] by separate arguments obtain these "scaling laws" as equalities. Josephson[4] has discussed the Widom-Kadanoff scaling laws with regard to He II near the λ point. In particular, he relates the superfluid density $\rho_s \sim (T_\lambda - T)^\zeta$ to the coherence length ξ and the departure of the superfluid correlation function from the Ornstein-Zernicke theory. Among the measurable quantities for He II, data exist for the superfluid density, expansion coefficient, compressibility, and specific heat. Of these, the superfluid density and the specific heat are used in Josephson's discussion of the λ transition. By assuming $C_p \sim (\Delta T)^{-\alpha'}$ with $\alpha' \equiv 0$, Josephson shows that the Widom-Kadanoff scaling laws are consistent with $\zeta = \frac{2}{3}$. Clow and Reppy[5] have measured the superfluid exponent ζ by a gyroscopic technique; they obtain the value 0.67 ± 0.03. In this Letter we report a more accurate measurement of $\rho_s(T)$ near T_λ, which we use together with an analysis of the specific-heat data of Fairbank and Kellers[6] to test the Widom-Kadanoff scaling laws. In addition, we obtain the temperature dependence of the short-range coherence length and discuss the allowable solutions for the remaining undetermined critical exponents.

Our experimental method is that of the oscillating disk pile[7] as indicated in Fig. 1. The superfluid density is related to the measured period of oscillation τ by the relation

$$\frac{\rho_s}{\rho} = \frac{2}{1-(\tau_0/\tau_\lambda)^2}\left(\frac{\tau_\lambda - \tau}{\tau_\lambda}\right)\left(1 - \frac{\tau_\lambda - \tau}{2\tau_\lambda}\right), \tag{1}$$

where ρ is the total fluid density, τ_0 is the vacuum period, and τ_λ is the period at the λ point. For temperatures near T_λ, $\rho_s/\rho \ll 1$ and $(\tau_\lambda - \tau)/\tau_\lambda \ll 1$, whence

$$\rho_s \propto (\tau_\lambda - \tau). \tag{2}$$

Thus the determination of the temperature dependence of ρ_s near T_λ reduces to the precise measurement of the period τ as a function of temperature.

In previous experiments in which this tech-

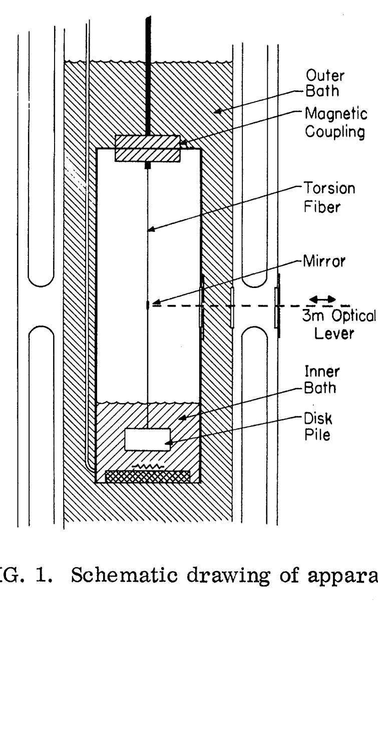

FIG. 1. Schematic drawing of apparatus.

nique was used, severe difficulties in period measurement were encountered due to changes in environmental conditions. In order to eliminate the effects of convection currents and small shifts in room temperature, the entire torsion-pendulum assembly is surrounded by an outer He II bath. The inner bath is separately temperature regulated to a few μdeg K. The interdisk spacing is small compared with the viscoelastic penetration depth at T_λ, and the viscosity correction to the superfluid-density critical exponent ζ is negligible.

The experimental procedure consists of first condensing purified liquid He^4 in the inner bath space and then cooling the outer bath below T_λ. After remaining below T_λ for several hours, the system is warmed slowly to a new temperature near T_λ and the disk pile set into oscillation ($\tau \sim 20$ sec) by an external magnetic coupling. At least 50 observations of the period are made at each temperature. Near T_λ the error in τ never exceeds 0.2 msec, and the inner bath temperature is stable to a few μdeg K. This process is continued at increasing temperatures, but T_λ is never exceeded. T_λ is found by observing the point of discontinuous warming rate of the resistance thermometer located in the inner bath near the disk pile. The resistance R_λ corresponding to T_λ is reproducible to within an equivalent ± 2 μdeg K. Very low values of thermometer and light-beam power were used, and changes in these and the inner bath level had no effect on the measurements. The period of oscillation τ_λ at the λ point is found from the experimental data. Period measurements for $T \geq T_\lambda$ cannot be made because of boiling.

In the analysis of the period-temperature data, we first obtain a least-squares fit to the relation

$$(T_\lambda - T)^\zeta = A + B\tau \tag{3}$$

for many values of ζ. We then observe the behavior of the standard deviation squared, $\sigma_\tau^2 = \sum(\Delta\tau)^2/N-1$, as a function of ζ. The best value of ζ is taken to be that value corresponding to the minimum in $\sigma_\tau^2(\zeta)$. The corresponding values of A and B give the intercept τ_λ for $T_\lambda - T = 0$. Each run yields a best value for ζ, and the standard deviation σ_ζ of ζ can be obtained from the set of ζ values for all runs. The same values of ζ and σ_ζ are obtained by geometrical construction from a graph of $\sigma^2 = \sum_{\text{runs}} \sigma_\tau^2$ vs ζ. Figure 2 shows σ^2 vs ζ for data between 60 μdeg and 50 mdeg K from T_λ. Using the probable error, we obtain $\zeta = 0.666 \pm 0.006$ for the ρ_s critical exponent. In Fig. 3 the data from the same runs are presented as $\tau_\lambda - \tau$ vs $T_\lambda - T$. Finally, using the vacuum period measurements at T_λ, we obtain[8]

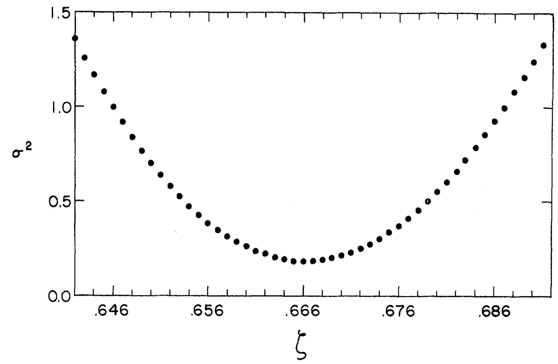

FIG. 2. Sum of squared deviations of data from $(T_\lambda - T)^\zeta = A + B\tau$ as a function of the ρ_s critical exponent ζ.

$$\rho_s/\rho = 1.43(T_\lambda - T)^\zeta, \tag{4}$$

with $\zeta = 0.666 \pm 0.006$.

The assumption that $\rho_s \sim |\Psi|^2$ is equivalent[4] to assuming that the order-parameter correlation function $C(r)$ at T_λ falls off with distance as $1/r$. Classically, the asymptotic form of the fluid two-particle correlation function is $1/r$. The failure of the classical theory in the case of the critical point has led many authors to propose a deviation from the classical dependence on distance[9] $\sim r^{-1-\eta}$. Widom[2] and Kadanoff[3] have independently arrived at the

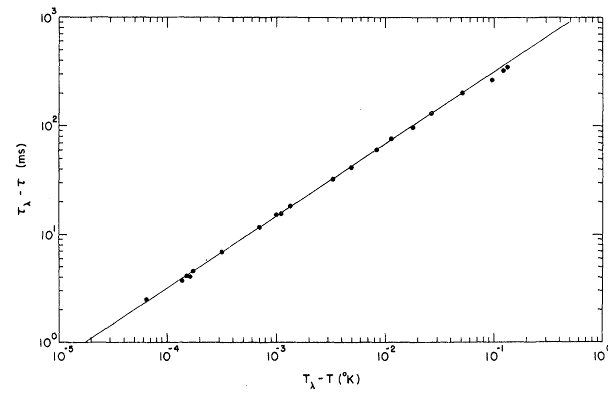

FIG. 3. Period-temperature data. Slope of solid line is $\frac{2}{3}$. ρ_s deviates from the $(T_\lambda - T)^{2/3}$ behavior above $T_\lambda - T \approx 60$ mdeg.

following scaling laws for the critical exponents:

$$2-\alpha' = 3\nu', \tag{5a}$$

$$\gamma' + 2\beta + \alpha' = 2, \tag{5b}$$

$$\gamma' = (2-\eta)\nu', \tag{5c}$$

where these exponents are defined by the limiting behavior at small $T_\lambda - T$ of various physical quantities[10]: $C_v \sim (\Delta T)^{-\alpha'}$, $\Psi \sim (\Delta T)^{\beta}$, $\xi \sim (\Delta T)^{-\nu'}$, $\chi \sim (\Delta T)^{-\gamma'}$, $C(r) \sim r^{-1-\eta}$, where χ is the effective susceptibility and $\Delta T = T_\lambda - T$. By dealing separately with the superfluid phase susceptibility "$\chi_\perp$," Josephson[4] obtains the remarkable result

$$\rho_s \sim (\Delta T)^{2\beta-\eta\nu'}. \tag{6}$$

Josephson then utilizes the Widom-Kadanoff scaling laws (5a), (5b), and (5c) to obtain the relation

$$2\beta - \eta\nu' = (2-\alpha')/3. \tag{7}$$

We have made a least-squares fit of the specific-heat data of Fairbank and Kellers[6] in the range $\Delta T < 50$ mdeg to the function

$$C_p = (U/\alpha')[(\Delta T)^{-\alpha'} - 1] + V. \tag{8}$$

Using the same techniques of analysis that were used for our ρ_s data, we obtain the specific-heat critical exponent and its probable error: $\alpha' = -0.014 \pm 0.016$. With this value for α', the right-hand side of Eq. (7) becomes 0.671 ± 0.005. Similarly, our experiment yields 0.666 ± 0.006 for the left-hand side. Thus Eq. (7) is proven experimentally to this accuracy, and the Widom-Kadanoff scaling rules (5a), (5b), and (5c) are likewise shown to be a valid (though not necessarily unique) set of relations for He II near T_λ.

Rearranging (5a), (5b), and (5c) and using $\zeta = 2\beta - \eta\nu'$, we obtain an equivalent set of scaling laws involving the ρ_s critical exponent ζ:

$$2\beta + \gamma' = 3\zeta, \tag{9a}$$

$$\eta\nu' + \gamma' = 2\zeta, \tag{9b}$$

$$\nu' = \zeta. \tag{9c}$$

The last of these relations indicates that the Widom-Kadanoff scaling laws, together with our value for ζ, give the result $\nu' = 0.666 \pm 0.006$ for the coherence-length critical exponent. Approximating ξ_0 in the relation $\xi = \xi_0(\Delta T)^{-\nu'}$ by setting ξ equal to the interatomic spacing in the $T = 0$ limit then yields an estimate for the characteristic distance over which the phase of Ψ may change: For $T_\lambda - T = 10^{-9}$°K, $\xi \sim \frac{1}{10}$ mm. Thus we can expect the behavior of He II to remain qualitatively similar over the range $10^{-7} \lesssim T_\lambda - T < 10^{-1}$°K. If we assume $\zeta \equiv \frac{2}{3}$ in (9a), (9b), and (9c), the problem reduces to two equations, (9a) and (9b), in three unknowns: β, η, γ'. Thus the values of η and γ' associated with the most commonly discussed[9] values of β can be determined: $\beta = \frac{1}{2}$ (molecular field theory), $\eta = \frac{1}{2}$, $\gamma' = 1$; $\beta = \frac{1}{3}$, $\eta = 0$, $\gamma' = \frac{4}{3}$; $\beta = \frac{5}{16}$ (three-dimensional lattice-gas Ising model), $\eta = -\frac{1}{16}$, $\gamma' = \frac{11}{8}$.

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[10]The scaling laws have been derived using the specific heat at constant volume. However, for the λ transition in liquid He^4 it can be shown on thermodynamic grounds that α' is given by C_p.