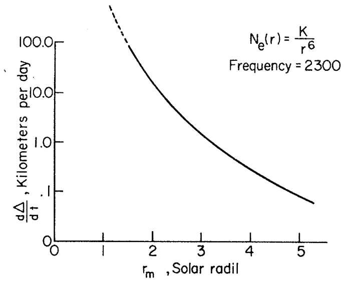

FIG. 2. Differences of group and phase delays for one-day Doppler counts against the distance of closest approach.

electron-density profile is obtained directly by this method even in regions where the G.R. effect is large compared with the coronal delay. Exact calculations would require the consideration of the path curvature; and $N_e(r_m)$ could be obtained by an iterative procedure on the equation equivalent to Eq. (13). The behavior of Eq. (13) with $n = 6$ using the above assumptions and employing a frequency counting interval of one day is shown in Fig. 2. It can be shown that measurements with sufficient accuracy to demonstrate these effects are feasible utilizing the currently scheduled National Aeronautics and Space Administration deep-space probes.[7]

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π^+p ELASTIC DIFFERENTIAL CROSS SECTIONS FROM 2.3 to 4.0 GeV/c†

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It is now known from the results of many experiments that πp scattering distributions for laboratory energies above 2 GeV have several general structural features in addition to the forward diffraction peak. In π^-p elastic scattering there is a secondary maximum or shoulder near $-t = 1.2$ for all laboratory momenta between 1.6 and 12 GeV/c.[1-4] A similar secondary maximum is found in π^-p charge-exchange scattering between 2.5 and 18 GeV/c.[5,6] Finally, sharp peaks corresponding to scattering in the backward direction have been observed in both π^+- and π^--p scattering for laboratory momenta between 3.5 and 8 GeV/c.[7-9] We report here differential elastic-scattering cross sections for π^+p scattering between 2.3 and 4 GeV/c that give further information on these effects.

Our data were obtained in a spark-chamber experiment carried out at the Argonne zero-gradient synchrotron (ZGS). The apparatus and method of analysis have been described in some detail in Ref. 1, where we presented π^-p data from the same experiment. This apparatus consisted of a coplanar array of spark chambers and controlling counters surrounding a liquid-hydrogen target in such a way that all elastic events with a center-of-mass scattering angle in the interval $-0.98 \leq \cos\theta \leq 0.98$ were detected with roughly equal probability. Identification of elastic events is based on a complete reconstruction of events in three dimensions so that the constraints available are scattering angle and coplanarity. These are sufficient to reduce background from inelastic events to below 1 μb/sr in the angular region where the cross section is small.

Our results for π^+p scattering at 2.3, 2.5, 2.7, 3.0, 3.5, and 4.0 GeV/c are presented in Fig. 1 together with data near the backward direction from other experiments. The errors given for our data are purely statistical. We

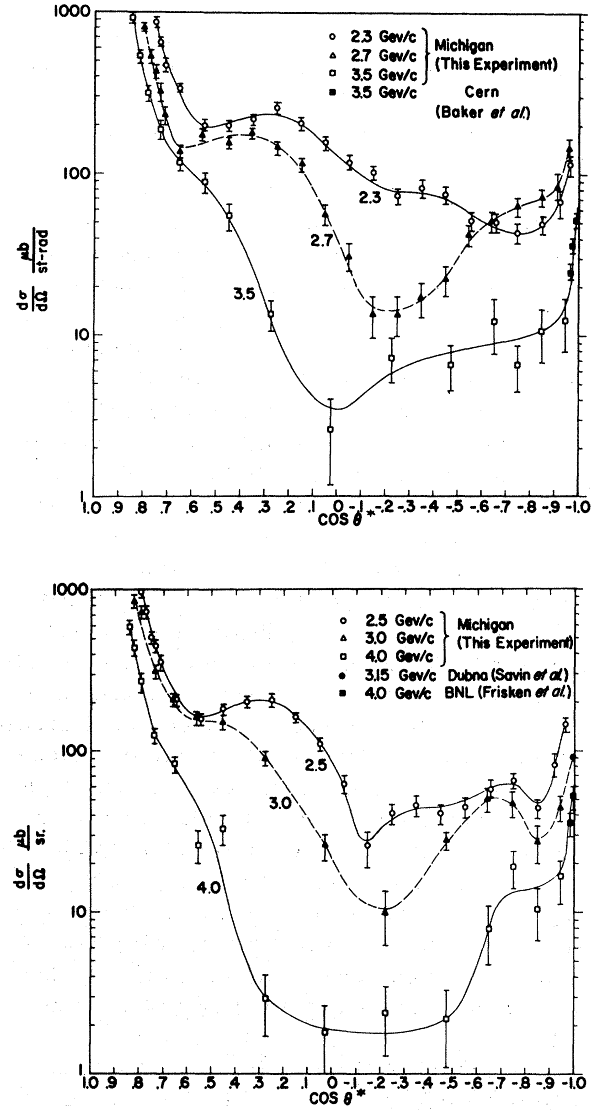

FIG. 1. Differential cross section for π^+p elastic scattering at laboratory momenta of 2.3, 2.5, 2.7, 3.0, 3.5, and 4.0 GeV/c. The data are presented on two graphs to minimize overlap of points from different energies and are accompanied by free-hand fits to organize the data for each energy. The CERN data are from Ref. 8, the Dubna data from Ref. 9, and BNL data from Ref. 7.

believe that there is an additional uncertainty in the absolute normalization of about 10%, because exponential extrapolations of the diffraction peak data in the momentum transfer region $0.05 \leq -t \leq 0.6$ $(\text{GeV}/c)^2$ to $-t = 0$ are systematically lower than the value calculated from the optical theorem and forward dispersion relations by about 10%.[10] It is not possible to determine whether this difference is

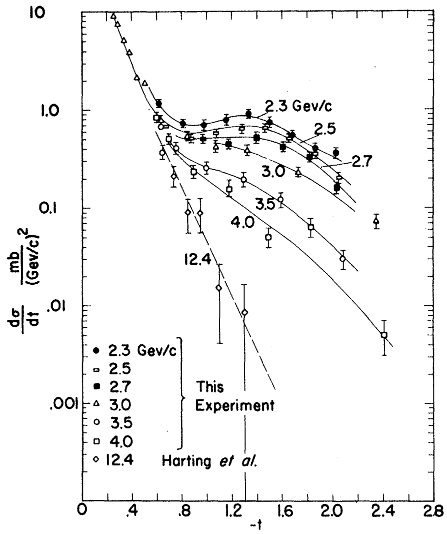

FIG. 2. Energy dependence of the secondary maximum for π^+p elastic scattering. The abscissa, $-t$, is in units of $(\text{GeV}/c)^2$. The data of Harting et al. are from Ref. 3.

due to a systematic normalization error or to a deviation of the diffraction peak from an exponential shape in the region $-t \leq 0.1$.

It is evident that all of the distributions of Fig. 1 have a secondary maximum or shoulder in addition to the forward-diffraction maximum. From Fig. 2 it can be seen that $d\sigma/dt$ for a given value of t decreases monotonically with increasing energy over the region of the second maximum. In particular, for $-t = 1.2$ $(\text{GeV}/c)^2$ we find that the dependence of our data on laboratory momentum, P_π, is well represented by $(d\sigma/dt)(-t=1.2) = \text{const.} \times P_\pi^{-2.6}$. This same expression works well for π^-p elastic scattering[1-4] between 2 and 8 GeV/c and for π^-p charge-exchange scattering[5,6] between 2 and 18 GeV/c (the constant for charge-exchange scattering is about $\frac{1}{6}$ of that for $\pi^\pm p$ elastic scattering).

It is perhaps worth emphasizing that the data of Fig. 2 suggest that resonances play a minor role in determining the energy dependence of the second maximum. In particular, the $N^*(2360)$ which is centered at 2.5 GeV/c[11] does not seriously distort the smooth energy dependence of the second maximum between 2.3 and 3.0 GeV/c. The absence of resonance effects does not seem unreasonable since the partial cross section that would result from the known $I = \frac{3}{2}$ N^* resonances[11] is less than 15% of the observed cross section in the second maximum

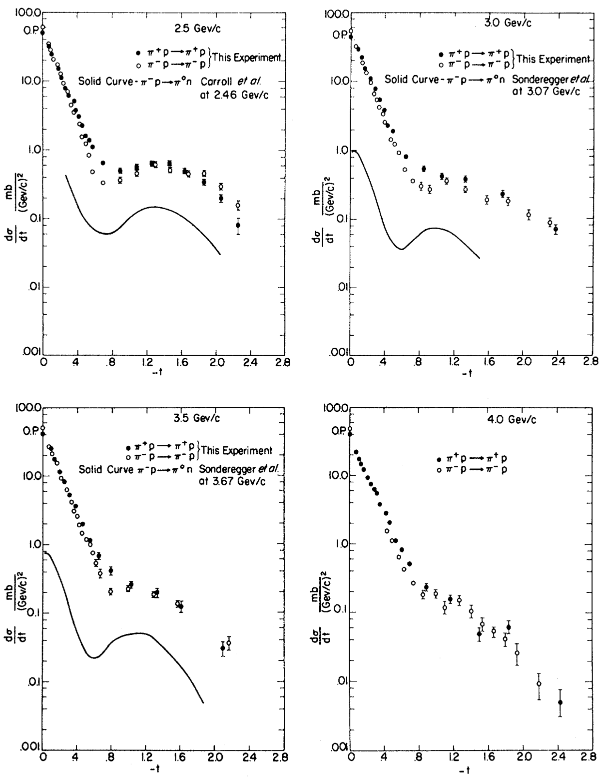

FIG. 3. Comparison of π^+p elastic-scattering data of this Letter with π^-p elastic-scattering data from Ref. 1 at laboratory momenta where both are available. The abscissas, $-t$, are in $(\mathrm{GeV}/c)^2$. The smooth curves are free-hand fits to charge-exchange data taken from Ref. 5 (2.46 GeV/c) and Ref. 6 (3.07 and 3.67 GeV/c).

for all of the data of Fig. 2. In making this estimate we have used the Breit-Wigner forms and parameters employed by Barger and Cline[12] in their calculation of backward π^-p-scattering cross sections.

In Fig. 3 we compare the secondary maxima observed in π^-p[1] and π^+p elastic scattering at lab momenta where both are available. The most striking feature of the data of Fig. 3 is that the small differences between the π^-p and π^+p cross sections are almost energy independent. A fit of the diffraction-peak data to $d\sigma/dt = Ae^{Bt}$ in the region $0.05 \leq -t \leq 0.6$ $(\mathrm{GeV}/c)^2$ gives a value of B of 7.5 ± 0.2 for all of the π^-p data and 6.7 ± 0.2 for all of the π^+p data.[10] The other obvious difference between the two charge states is that the π^+p cross section is in all cases larger than the π^-p cross section near $-t = 0.8$ by a factor of 1.5 to 2. The two sets of data are consistent with equality over most of the second maximum for the four momenta of Fig. 3. It is interesting that the second maximum in π^-p charge-exchange scattering[5,6] occurs at almost the same value of t as the second maximum in πp elastic scattering, and that the variation with energy of this effect for $0.8 \leq -t \leq 1.6$ is similar for all three charge states.

All of the angular distributions of Fig. 1 have, or strongly suggest, sharp backward peaks similar to those observed at higher energies.[7,8] It is interesting to note that the distributions at 2.5 and 3.0 GeV/c have a dip and a backward secondary maximum similar to the structure present in the combined data of the Cornell-BNL[7] and Pennsylvania[13] groups at 8.0 GeV/c. However, it seems unlikely that the Regge-pole interpretation[14] given to the 8-GeV/c data has much relevance at our low energies because of the obviously important role of resonances that is implied by the irregular energy variation of the backward data. Heinz and Ross[15] have shown that a dip and second maximum in the backward direction can also result from the interference between a resonant amplitude and a background amplitude that decreases smoothly going away from the backward direction. However, there is no detailed correspondence between the results of their calculation and the data of Fig. 1. It appears that the interpretation of πp elastic scattering in this energy region is extremely complex. One must simultaneously discover both the resonant and nonresonant amplitudes, unless a reliable theory for the nonresonant amplitudes can be developed.

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