as an example of a space-time where it is impossible to find a connected spacelike surface cutting every timelike line. We will call a connected closed spacelike surface without boundary a slice and a slice which does not intersect any timelike or null line more than once a partial Cauchy surface. It is easy to show that if space-time admits a slice then either that slice is a partial Cauchy surface or space-time has a covering space in which each connected component of the image of the slice is a partial Cauchy surface. We may apply the following theorem to space-time or to the covering space since a singularity in the covering space implies one in the space covered.

Theorem 3. – Space time is not singularity free if condition (1) holds and (6) there exists a compact partial Cauchy surface H whose unit normals v^a are diverging, (7) every point q has a neighborhood W such that every timelike and null line from q leaves W and does not re-enter.

Condition (7) is really a statement about causality.[13] It would seem a very reasonable requirement.

Finally, we make a brief mention of the technique used to prove these results. A point p is said to be conjugate to a point q along a timelike geodesic γ if p lies on the caustic of timelike geodesics through q. The point p is said to be conjugate to a spacelike surface H if it lies on the caustic of the geodesics normal to H. By the formula for variation of arc length it can be shown that a geodesic γ normal to H from H to a point r cannot be the longest timelike line from H to r if there is a point conjugate to H on γ between H and r. By the conditions of the theorems it is possible to show that there is a point conjugate to H within a bounded distance b on every geodesic normal to H. On the other hand, it is possible to show that there are points from which there is a longest timelike line to H of finite length greater than b. This establishes a contradiction which shows that the assumed conditions are incompatible with space-time being singularity free.

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SINGULARITIES IN CLOSED UNIVERSES

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In recent years, several theorems[1,2] have appeared which show that the singularities observed in certain exact solutions of the Einstein equations cannot be avoided by small perturbations in these solutions. These theorems are not sufficiently strong, however, to settle completely the question whether any nonsingular solutions of Einstein's equations can provide a reasonable model of our own universe. In particular, should the trapped surface of Penrose[1] or the expanding Cauchy surface of Hawking[2] occur in our own immediate vicinity, we could, by suitably distributing masses and thus affecting the local geometry, destroy these symptons of prospective collapse. Would we thus save the entire universe from evolving into a singular state? This seems unlikely, and one would like to express this feeling as a theorem.

We should like here to present a result which takes a further step toward the more limited goal of establishing that, with some suitable definition of "closed," all closed solutions, with the exception of the exception of the trivial flat geometries,[3] evolve into a singular state.

It is convenient to introduce some definitions. The first is closely related to the visual horizon introduced by Rindler[4] for the treatment of space times with special symmetries. Let V_4 be a four-manifold carrying a metric with signature $(-,+,+,+)$. Let S be a compact spacelike surface (without boundary) of V_4. There is said to be a horizon at a point P of S if there exists a timelike curve which intersects S, but which either fails to enter the forward or fails to enter the backward light cone of P. (For example, if the geometry can be sliced into compact spacelike sections, a point P without a horizon has the property that every point of some spacelike section can be reached by a forward timelike curve from P, and every point of another spacelike section can be reached by a backward timelike curve from P.) Consider the family of forward-pointing timelike geodesics emanating normally from S. The unit tangent vector, ξ^μ, to this congruence may be written as the gradient of a scalar field φ which takes the value zero on S. Here ξ^μ and φ will be called, respectively, the vector and scalar fields generated by S. The covariant divergence, $\xi^\mu{}_{;\mu}$, of ξ^μ on S defines the expansion of S.

We shall now establish the following theorem: Let V_4 be a four-manifold carrying a metric $g_{\mu\nu}$ with signature $(-,+,+,+)$. Suppose the following conditions are satisfied.

(1) The Ricci tensor everywhere obeys[1] $R_{\mu\nu}\xi^\mu\xi^\nu \geq 0$ for every timelike ξ^μ, equality holding only if $R_{\mu\nu}=0$. (From the Einstein equations without cosmological constant, this is a restriction on the stress energy of the matter under consideration. For example, for the case of a perfect fluid, this condition requires $\rho \geq -p$ and $\rho > -p$.)

(2) A continuous choice of the forward light cone can be made everywhere on V_4.

(3) V_4 contains a compact spacelike Cauchy[2] surface S.

(4) There is at least one point P of S without a horizon.

Then V_4 either is a flat solution, or else has a singularity. (A space is said to be singular[5] if it contains an inextendable timelike or null geodesic of finite affine length.)

We shall outline the argument here: Condition (4) and the construction of Avez[6] suffice to establish that there is a spacelike surface S' through P having the properties that its area is greater than or equal to that of every other spacelike surface through P, and it is twice differentiable everywhere except possibly at P. From these two properties it follows that the expansion of S' is zero except possibly at P (where it need not be defined). By "rounding off the corner" at P, we may construct a compact spacelike surface S'', everywhere twice differentiable, whose expansion is zero except possibly in a neighborhood of P, where if nonzero, it is of one sign only (say, for definiteness, negative). Let θ be some scalar field on S''. Then on translating each point of S'' a distance $\epsilon\theta$ $(\epsilon \ll 1)$ along the congruence generated by S'', we obtain a new spacelike surface S''' whose expansion differs from that of S'' by

$$-\epsilon\theta[R_{\mu\nu}\xi^\mu\xi^\nu+\xi^\mu{}_{;\nu}\xi^\nu{}_{;\mu}]+\epsilon h^{ij}\theta_{|ij}+O(\epsilon^2). \quad (1)$$

Here h^{ij} $(i,j=1,2,3)$ is the positive definite induced metric on S'', and a slash denotes the covariant derivative with this metric. Since the coefficient of θ in the first term of expression (1) is nonpositive on S'', it follows by a theorem of Hodge[7] that we may choose θ so that the expansion of S''' is everywhere negative [case (1)], or everywhere zero [case (2)].

In case (1), the hypothesis is satisfied for the theorem of Hawking,[2] establishing that a singularity is inevitable. In case (2), we consider the one-parameter family of surfaces $\varphi=\text{constant}$, where φ is the scalar field generated by S'''. From expression (1), the expansion of these surfaces must be everywhere nonpositive. Should the expansion at any point of any of these surfaces differ from zero, we could choose θ as before and establish the presence of a singularity. Therefore, the expansion must be zero on each of these surfaces if a singularity is to be avoided. This is possible[8] only if the Riemann tensor vanishes.

This and other related results will be discussed in more detail elsewhere. An accompanying note by Hawking[9] states the theorem on which this one is based, along with several other results which replace our "absence of horizon" fourth condition with various other assumptions. It is hoped that eventually all such mathematically convenient "fourth conditions" will be eliminated, leaving only the first three "physically reasonable" assumptions, and thus establishing that all "closed universes" develop a singularity.

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EXTENDED SOURCE OF ENERGETIC COSMIC X RAYS

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We report here on evidence for an extended source of x rays in a region near the direction of the north galactic pole. This result is based upon data collected during two recent balloon flights launched from Holloman Air Force Base in New Mexico.

The x-ray telescope was suspended vertically so that the detector axis had celestial coordinates given by a right ascension equal to the sidereal time at the local meridian and a declination given by the latitude of the balloon. The stability of the attitude was established to within $\frac{1}{2}$ deg by a camera that continuously photographed the horizon on infrared sensitive film. During the major portion of the flight of 6 December 1965 the detector axis was well out of the galactic plane, riding at a declination of +31 deg, and passed near the north galactic pole. The flight of 13 January 1966 carried the telescope across the galactic plane at a declination of +34 deg, intercepting the x-ray source Cygnus XR-1. The residual atmosphere during the ceiling coverage of these flights was 2.6-2.8 g/cm^2.

Figure 1 shows the main elements of the x-ray telescope. The central x-ray detection element is a cesium-iodide (thallium activated) scintillation crystal that is used to measure x rays in the interval 20-100 keV partitioned by 64 channels. The plastic scintillator (CH) serves as an anticoincidence detector of charged particles and also as the innermost section of a three-element graded x-ray shield consisting of tin on the outside and copper sandwiched between. The light from the cesium iodide is distinguished from that of the plastic scintillator by its longer decay time. The entrance port is a krypton gas proportional counter that serves as an anticoincidence detector and is also used to measure x rays in the approximate interval 10-30 keV, again partitioned by 64 channels. The transmission of the gas counter exceeds the 50% level at 25 keV (K x ray for Sn^{119m}). At this energy, the proportional counter exhibits a resolution of about 20% full width at half-maximum (FWHM) for both the primary and escape peaks. The resolution for the cesium-iodide crystal is about 30% FWHM at 74 keV (K x ray of Bi^{207}).

A structured response pattern was achieved

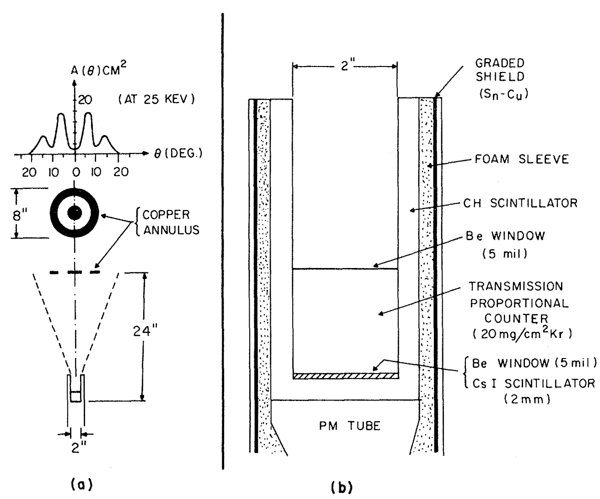

FIG. 1. (a) The x-ray spectrometer telescope utilizes the occultation scheme outlined; the detector appears at the bottom. (b) The essential features of the detector.