SPIN-2^+ MESON DECAYS IN THE QUARK MODEL*

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The spin-2 nonet is described by SU(6) $\underline{405}$ wave functions that are eigenstates of the number of strange quarks. The decay branching ratios are obtained by a rearrangement of the quarks and antiquarks into two quark-antiquark pairs without change of quantum numbers, after an appropriate spin recoupling.

The decays of the spin-2 nonet have been considered by Glashow and Socolow[1] on the basis of SU(3) symmetry, and by other investigators[2-5] using higher symmetries. Although reasonable agreement with experiment is obtained in these treatments, recent experimental data[6] indicate some discrepancies. Furthermore, the most striking features of these decays, namely the forbiddenness of certain modes, does not arise in any simple way in these treatments.

The absence of the decay $f' \to 2\pi$, in spite of its large phase space, is reminiscent of the forbiddenness of the decay $\varphi \to \rho + \pi$. Both are described in SU(3) by representation mixing, with the mixing angle and decay parameters chosen to give an accidental cancellation of the transition matrix element. This suggests the existence of a selection rule which might follow from some higher symmetry. The SU(6)-type[7] symmetries that include spin predict such selection rules but have the defect that spin conservation forbids many experimentally strong decays, such as the decay $\rho \to \pi + \pi$, as well as all of the dominant decay modes of the 2^+ mesons.[8] The hybrid group $SU(6)_W$ solves the problem of the decay of the vector mesons.[8] The use of W-spin conservation without requiring ordinary spin conservation allows the observed decays; conservation of W_λ, the "strange-quark W spin," forbids the decay $\varphi \to \rho + \pi$. However, W_λ conservation does not lead to useful selection rules for the decay of the spin-2^+ mesons. The state $S=2$, $S_z=0$ is a mixture of $W=2$ and $W=0$, and therefore has a component with $W=0$ even if it contains strange quarks. Thus in $SU(6)_W$ the decay $f' \to 2\pi$ can only be forbidden by accidental cancellation of matrix elements that appear as free parameters.[5]

The decay $f' \to 2\pi$ is trivially forbidden if f' is constructed from a $\lambda\bar{\lambda}$ pair, and if conservation of the total number n_λ (rather than the total W spin) of λ and $\bar{\lambda}$ quarks is postulated. However, the conservation of n_λ together with $SU(6)_W$ leads to difficulties. The algebra generated by all the commutators of n_λ and the $SU(6)_W$ generators is $U(6) \otimes U(6)$, which includes ordinary spin conservation and forbids the strong 2^+ decays. This suggests that one should look for a formulation in which n_λ rather than $SU(6)_W$ is conserved.

In a quark model with the 2^+ mesons described by two quarks and two antiquarks, their decay is qualitatively different from the vector-meson decays which involve the creation of a quark-antiquark pair. The spin-2^+ decays conserve the total number of quarks and antiquarks, and can therefore be treated as a simple rearrangement of the constituent quarks into two quark-antiquark pairs. In such a model, the requirement that the rearrangement takes place without change of internal quantum numbers automatically includes n_λ conservation. Such a model has been used successfully to treat proton-antiproton annihilation into mesons.[9]

In this paper we apply the rearrangement model to the 2^+ decays. The mesons are assumed to be described by wave functions that are eigenstates of n_λ and that correspond to the representation $(\underline{21}, \underline{21}^*)$ of $U(6) \otimes U(6)$, i.e., to symmetric coupling of both quarks and antiquarks.[5] Since the spins are all parallel, this means a symmetric coupling of both quarks and antiquarks[10] in $U(3) \otimes U(3)$; i.e., the representation $(\underline{6}, \underline{6}^*)$ of $U(3) \otimes U(3)$. The wave functions can be written explicitly:

$$f = (q_n q_n \bar{q}_n \bar{q}_n)_{I=0,\,S=2}, \tag{1a}$$

$$A_2 = (q_n q_n \bar{q}_n \bar{q}_n)_{I=1,\,S=2}, \tag{1b}$$

$$K^{**} = (q_n q_n \bar{q}_n \bar{\lambda})_{I=\frac{1}{2}, S=2}, \tag{1c}$$

$$f' = (q_n \lambda \bar{q}_n \bar{\lambda})_{I=0, S=2}, \tag{1d}$$

where q_n denotes either state of the nonstrange isodoublet quark. The wave functions[5] have a well-defined n_λ and are well-defined linear combinations of the SU(3) representations $\underline{1}$, $\underline{8}$, and $\underline{27}$.

The possible two-body decay modes are (a) into two pseudoscalar mesons (*PP*), and (b) into a vector and a pseudoscalar meson (*VP*). Both decays must be *D*-wave to conserve angular momentum and parity. Thus the final states are ($PP{:}S=0, L=2, J=2$) and ($VP{:}S=1, L=2, J=2$). Because the spin S must change from 2 to either 0 or 1, the decay cannot be described by a simple quark rearrangement, without change of quantum numbers, as in the case of proton-antiproton annihilation.[8] Instead we assume that there is a "quark-spin recoupling" that changes the total spin S from 2 to 0 or 1 and that the decay proceeds by rearrangement from the transformed state.

The recoupling is described as follows: In the meson wave function, consider a typical term T which is described as two quark-antiquark pairs ($q_a\bar{q}_b$) and ($q_c\bar{q}_d$) each coupled to spin 1 and the two pairs coupled to total spin 2, so that

$$T = \sum_{m_a, m_b, m_c, m_d, M', M''} (\tfrac{1}{2}m_a \tfrac{1}{2}m_b | 1M')(\tfrac{1}{2}m_c \tfrac{1}{2}m_d | 1M'') \times (1M'1M'' | S,M) | q_a, m_a; \bar{q}_b, m_b; q_c, m_c; \bar{q}_d, m_d\rangle, \tag{2}$$

where $|q_a, m_a; \bar{q}_b, m_b; q_c, m_c; \bar{q}_d, m_d\rangle$ is a state in which the quark q_a is in the spin state $s_z = m_a$, etc., and $(j_a m_a j_b m_b | JM)$ are Clebsch-Gordan coefficients. For the 2^+ mesons, $S=2$. The recoupling is performed formally by simply changing the value of S in the expression (2) for all terms. For the *PP* decays, we choose $S=0$, $M=0$; for the *VP* decays we choose $S=1$, $M=1$. The z axis is chosen in the direction of the decay momenta; thus there is no contribution[8] from $S=1$, $M=0$. This procedure is unique if ambiguities regarding relative phases of different terms are removed by requiring the transformation to conserve isospin in all cases and G parity for all eigenstates of G.

Table I gives the results and compares them with the previous SU(3) predictions.[1] We have allowed for singlet-octet mixing in the η by defining the physical η and mixing angle α by the relation

$$|\eta\rangle = \cos\alpha(\lambda\bar{\lambda}) + \sin\alpha(\eta\bar{\eta} + p\bar{p})/\sqrt{2}. \tag{3}$$

The SU(3) results quoted assume that the η is pure octet. It is not easy to introduce π-X^0 mixing into such SU(3) calculations, because the singlet amplitudes introduce new free parameters in addition to the mixing angle.[11] The agreement is seen to be good, particularly if α is very small. Small α corresponds to the case in which the η is nearly pure ($\lambda\bar{\lambda}$). Although this choice is not conventional, it is reasonably close to a value obtained from the Gell-Mann–Okubo mass formula with linear masses.[11]

The nature of the differences between the quark-model results and SU(3) is illuminated by the following discussion. The rearrangement process itself maintains symmetry under U(6)⊗U(6) and its subgroups U(3)⊗U(3) and SU(3). The recoupling transformation must break U(6)⊗U(6), since it changes the total spin which is included among the generators of U(6)⊗U(6). The recoupling to spin zero preserves as symmetries the subgroups U(3)⊗U(3) and SU(3), since the Clebsch-Gordan coefficients appearing in Eq. (2) have the same permutation symmetry for $S=0$ and $S=2$. However, the recoupling to spin 1 changes the permutation symmetry and breaks U(3)⊗U(3) and SU(3). That our model must break SU(3) in the *VP* decays is evident. The requirement of simultaneous conservation of SU(3) symmetry and n_λ would lead to invariance under U(3)⊗U(3), and that would be incompatible with charge-conjugation invariance in the *VP* sector.[12]

Since the *PP* decays preserve symmetry under SU(3), the only differences between the present results and previous calculations are (a) the use of the eigenfunctions of n_λ, with the implied mixture of $\underline{27}$ into the meson wave

Table I. Comparison between theoretical predictions and experiment.

Decay mode	SU(3) prediction with phase space	Square of Clebsch-Gordan factor	Quark-model prediction with phase space	Phase space	Experiment[b]
$A_2 \to \rho + \pi$	100[a]	$\frac{1}{4}$	100[a]	13.2	80 ± 14
$K^{**} \to K^* + \pi$	38	$\frac{1}{8}$	50.3	13.3	43 ± 15
$K^{**} \to \rho + K$	11	$\frac{1}{8}$	14.8	3.9	14 ± 5
$K^{**} \to \omega + K$	4	1/24	3.9	3.1	7 ± 4
$f' \to K + K^*$	24	$\frac{1}{2}$	42.5	2.8	34 ± 15
$f \to \pi + \pi$	100[a]	$\frac{1}{4}$	100[a]	53.6	118 ± 20
$f \to K + \bar{K}$	2.4	0	0	5.3	2 ± 2
$f \to \eta + \eta$	0.2	$(\frac{3}{8}) \sin^4\alpha$	$4.2 \sin^4\alpha$	1.5	small
$A_2 \to \eta + \pi$	11	$(\frac{1}{2}) \sin^2\alpha$	$94 \sin^2\alpha$	25.2	3 ± 2
$A_2 \to K + \bar{K}$	6[a]	0	0	9.2	4.5 ± 1.5
$K^{**} \to K + \pi$	42	$\frac{1}{8}$	42	45.1	39 ± 20
$K^{**} \to K + \eta$	1.4	$(\frac{3}{8}) \sin^2\alpha$	$38.4 \sin^2\alpha$	13.7	2 ± 2
$f' \to \pi + \pi$	1.7	0	0	101.8	0 ± 6
$f' \to K + \bar{K}$	31	$\frac{1}{4}$	53.0	28.4	51 ± 25
$f' \to \eta + \eta$	9	$(\frac{1}{2}) \cos^2\alpha \sin^2\alpha$	$67 \cos^2/\alpha \sin^2\alpha$	17.9	small

[a]The underlined predicted values are taken as input.
[b]The experimental values are taken from Ref. 6.

functions, and (b) the unique relation between the singlet, octet, and 27-plet decay amplitudes given by the rearrangement procedure.

Better experimental data on the seemingly forbidden decays would be of interest. These include $f \to K + \bar{K}$, $A_2 \to K + \bar{K}$, and $f' \to \pi + \pi$, which are forbidden by n_λ conservation, and also all the decay modes involving the η.

Better data on the K^{**} branching ratios are also of particular interest.[13]

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[5]M. Elitzur and H. J. Lipkin, to be published, consider the decays of 2^+ mesons using the n_λ eigenstates (1) for the mesons, under the assumption of $SU(6)_W$ invariance.

[6]M. Goldberg, J. Leitner, R. Musto, and L. O'Raifeartaigh, to be published. The SU(3) predictions given here differ from those of Ref. 1 only because different values for meson masses change the phase space.

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[8]H. J. Lipkin and S. Meshkov, Phys. Rev. 143, 1269 (1966).

[9]H. R. Rubinstein and H. Stern, Phys. Letters 21, 447 (1966).

[10]The groups $U(n) \otimes U(n)$ discussed in this paper always refer to $U(n)_q \otimes U(n)_{\bar{q}}$, i.e., to separate $U(n)$ groups for quarks and antiquarks.

[11]A. J. MacFarlane and R. H. Socolow, Phys. Rev. 144, 1194 (1966), consider η-X^0 mixing in 2^+ decays, but arbitrarily set values of several free parameters equal to zero.

[12]Both the vector and pseudoscalar mesons are in $(\underline{3}, \underline{3}^*)$ representations in $U(3) \otimes U(3)$, but they have opposite behavior under C. Thus, the final PP state is even under permutations in $U(3) \otimes U(3)$ and can be either $(\underline{6}, \underline{6}^*)$ or $(\underline{3}^*, \underline{3})$. The VP state must be odd and is a mixture of $(\underline{6}, \underline{3})$ and $(\underline{3}^*, \underline{6}^*)$. Since the initial 2^+ meson is $(\underline{6}, \underline{6}^*)$, preservation of $U(3) \otimes U(3)$ symmetry is possible for the PP decays and impossible for the VP decays.

[13]The K^{**} branching ratio predictions are critical tests of SU(3), since they are valid for arbitrary mixing of singlet, octet and 27-plet in the 2^+ wave function.[5] The results recently quoted by J. M. Bishop, A. T. Goshaw, A. R. Erwin, M. A. Thompson, W. D. Walker, and A. Weinberg, Phys. Rev. Letters 16, 1069 (1966), are (56 ± 10):(10 ± 5):(0.7 ± 0.8) for the $K^*\pi$:ρK:ωK branching ratios. The low value for the ωK mode is in serious disagreement with both SU(3) and the present work. If this low value is correct (this mode was not seen in the experiment quoted and the value is only an upper limit), then agreement with experiment cannot be obtained in any higher symmetry treatment of 2^+ decays that does not include SU(3)-symmetry breaking.