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[9]If we assume that the pseudoscalar currents of baryons $\langle B|S^5|B'\rangle$ are dominanted by P_8 contributions, Eq. (5) can be written as $f(B_i'B_jP_k)=(1+C'd_{k8k})\times f_0(B_i'B_jP_k)$, where C' is a constant independent of B and B'. Our assumption is justified if the W(3) model or σ model is a good approximation.

[10]In this case, the assumption of Ref. 9 yields stronger results than Eqs. (10)-(14) of the text: $g_{\Sigma\Lambda\pi}/g_{NN\pi}=2(1-\alpha)/\sqrt{3}$, $g_{\Xi\Xi\pi}/g_{NN\pi}=2\alpha-1$, $g_{\Sigma\Sigma\pi}/g_{NN\pi}=2\alpha$, $g_{\Xi\Xi\eta}/g_{NN\eta}=2(1-\alpha)/(4\alpha-1)$, $g_{\Sigma\Sigma\eta}=-g_{\Lambda\Lambda\eta}$, $g_{\Xi\Xi\eta}/g_{NN\eta}=(1+2\alpha)/(1-4\alpha)$, $g_{N\Sigma K}/g_{N\Lambda K}=\sqrt{3}(2\alpha-1)/(1+2\alpha)$, $g_{\Xi\Sigma K}/g_{N\Lambda K}=\sqrt{3}/(1+2\alpha)$, $g_{\Xi\Lambda K}/g_{N\Lambda K}=(1-4\alpha)/(1+2\alpha)$, where $\alpha=F/(D+F)$. Thus, all pion couplings, estimated with $g_{NN\pi}$ and α (which may be taken to be the same as that for weak axial-vector current) as input, are exactly the same as the SU(3)-symmetric values. If we accept the estimate of Ref. 8, $g_{N\Lambda K}^2\simeq 4.8$, we then obtain (with $\alpha\simeq 0.35$[11]) $g_{N\Sigma K}^2\simeq 0.5$, $g_{\Xi\Lambda K}^2\simeq 0.3$, and $g_{\Xi\Sigma K}^2\simeq 5.0$.

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NEUTRAL-MESON PRODUCTION CROSS SECTIONS AND MIXING ANGLES IN A QUARK MODEL

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The recent success of an extremely simplified quark model[1,2] for forward elastic scattering may be due to the particular simplicity of zero-momentum-transfer processes. In this Letter we present predictions for neutral-meson production processes at finite momentum transfer[3] which may provide a more sensitive test of the model and which also give values for the mixing angles in the meson nonets. These predictions and mixing angles depend only upon the validity of the quark model[1,2] and are independent of SU(3)-symmetry-breaking effects in the transition amplitudes. Reasonable agreement with experiment has been found for those cases where adequate data are available except in one case which will be discussed.

We assume that a meson is a quark-antiquark pair but require no assumptions regarding the baryon structure. We further assume that the transition amplitude for any meson-baryon reaction is expressible as the sum of the constituent quark-baryon and antiquark-baryon scattering amplitudes.[1,2] Thus the transition between the quark-antiquark-baryon states $|(q_a\bar{q}_{a'})B_a\rangle$ and $|(q_b\bar{q}_{b'})B_b\rangle$ is given by the expression[4]

$$\langle (q_a\bar{q}_{a'})B_a|(q_b\bar{q}_{b'})B_b\rangle = \langle q_aB_a|q_bB_b\rangle\delta_{a'b'} + \langle\bar{q}_{a'}B_a|\bar{q}_{b'}B_b\rangle\delta_{ab}. \quad (1)$$

The assumption (1) is assumed to hold separately for each possible set of polarization states of the initial and final quark-antiquark-baryon systems. We consider only relations between reactions involving the same baryon states and different members of the same initial and the same final meson nonets. The explicit calculations show that the relations obtained are independent of the initial and final meson and baryon spin states. The same relations thus hold for all polarization amplitudes separately.

From the fundamental assumption (1) it follows that all reactions which require changes in the states of both the quark and the antiquark in the meson are forbidden in this model. Obvious examples of such transitions are those involving a double charge exchange or a double strangeness exchange. We also obtain the following selection rules:

$$\langle\pi^-p|M_\lambda n\rangle = \langle\pi^+n|M_\lambda p\rangle = \langle\pi^+p|M_\lambda N^{*++}\rangle = \langle\pi^-p|M_\lambda N^{*0}\rangle = 0, \quad (2a)$$

where M_λ denotes the particular linear combinations of neutral meson states which contain only strange quarks. This state is denoted by $(\lambda\bar{\lambda})$ in Ref. 2.

The M_λ state is given in terms of the corresponding unitary octet states M_1 and M_8 by

$$M_\lambda = (1/\sqrt{3})(\sqrt{2}M_8 - M_1) \quad (2b)$$

for both the vector and pseudoscalar cases. The corresponding orthogonal state which contains only nonstrange quarks coupled to isospin zero is given by

$$M_{n0} = (1/\sqrt{3})(M_8 + \sqrt{2}M_1). \quad (2c)$$

The vector-meson states φ and ω are now believed to be very close to the states (2b) and (2c). This corresponds to a mixing angle of 35.3°, and is the classification given by the nonstrange SU(4) subgroup of SU(6).[5] The selection rules[6] (2a) hold for all spin orientations of the strange quarks; i.e., for the pseudoscalar and for all three vector polarization states of the meson M_λ.

Relations between production processes for physical meson states are obtained from the relation (2) by expressing M_λ as the appropriate linear combination of physical pseudoscalar or vector mesons:

$$\frac{\langle\pi^- p\,|\,\varphi n\rangle}{\langle\pi^- p\,|\,\omega n\rangle}=\frac{\langle\pi^+ p\,|\,\varphi N^{*++}\rangle}{\langle\pi^+ p\,|\,\omega N^{*++}\rangle}=\frac{\cos\alpha-\sqrt{2}\sin\alpha}{\sin\alpha+\sqrt{2}\cos\alpha}, \tag{3a}$$

$$\frac{\langle\pi^- p\,|\,\eta n\rangle}{\langle\pi^- p\,|\,X^0 n\rangle}=\frac{\langle\pi^+ p\,|\,\eta N^{*++}\rangle}{\langle\pi^+ p\,|\,X^0 N^{*++}\rangle}=\frac{\cos\beta-\sqrt{2}\sin\beta}{\sin\beta+\sqrt{2}\cos\beta}, \tag{3b}$$

where α and β are the usual mixing angles[7] expressing the physical meson states in terms of the unitary singlet and unitary octet states, ω_1, η_1, ω_8, and η_8:

$$\varphi=\omega_8\cos\alpha-\omega_1\sin\alpha, \tag{4a}$$

$$\omega=\omega_8\sin\alpha+\omega_1\cos\alpha, \tag{4b}$$

$$\eta=\eta_8\cos\beta-\eta_1\sin\beta, \tag{4c}$$

$$X^0=\eta_8\sin\beta+\eta_1\cos\beta. \tag{4d}$$

The relations (3) provide a test of the quark model and a value of the mixing parameter for each of the two cases.

Similar relations are obtainable for the production of any nonet of higher boson resonances described by a "kinetic supermultiplet"[8]; i.e., a single quark-antiquark pair in some state of orbital angular momentum. Thus if the 2^+ mesons are described in this way, a test of the quark model and a value for the mixing angle is given by the relation

$$\frac{\langle\pi^- p\,|f^0 n\rangle}{\langle\pi^- p\,|f' n\rangle}=\frac{\langle\pi^+ p\,|f^0 N^{*++}\rangle}{\langle\pi^+ p\,|f' N^{*++}\rangle}=\frac{\cos\gamma-\sqrt{2}\sin\gamma}{\sin\gamma+\sqrt{2}\cos\gamma}, \tag{5}$$

where the mixing angle γ is defined by analogy with Eqs. (4).

A straightforward application of the relations (1) leads to a number of relations, for example,

$$\langle K^- p\,|M_\lambda\Lambda\rangle=\langle\pi^- p\,|K^{*0}\Lambda\rangle, \tag{6a}$$

$$\langle K^- p\,|M_{n0}\Lambda\rangle=\langle K^- p\,|\rho^0\Lambda\rangle. \tag{6b}$$

These can be combined to obtain some rules for cross sections of processes which are independent of mixing angles and involve only physical particles. Some examples are the following:

$$\bar\sigma(K^-+p\to\eta+\Lambda)+\bar\sigma(K^-+p\to X^0+\Lambda)=\bar\sigma(\pi^-+p\to K^0+\Lambda)+\bar\sigma(K^-+p\to\pi^0+\Lambda), \tag{6c}$$

$$\bar\sigma(K^-+p\to\varphi+\Lambda)+\bar\sigma(K^-+p\to\omega+\Lambda)=\bar\sigma(\pi^-+p\to K^{*0}+\Lambda)+\bar\sigma(K^-+p\to\rho^0+\Lambda), \tag{6d}$$

$$\bar\sigma(\pi^-+p\to\pi^0+n)+\bar\sigma(\pi^-+p\to\eta+n)+\bar\sigma(\pi^-+p\to X^0+n)=\bar\sigma(K^++n\to K^0+p)+\bar\sigma(K^-+p\to\bar K^0+n), \tag{7a}$$

$$\bar\sigma(\pi^++p\to\pi^0+N^{*++})+\bar\sigma(\pi^++p\to\eta+N^{*++})+\bar\sigma(\pi^++p\to X^0+N^{*++})$$
$$=3\bar\sigma(K^-+p\to\bar K^0+N^{*0})+\bar\sigma(K^++p\to K^0+N^{*++}), \tag{7b}$$

$$\bar\sigma(\pi^-+p\to\rho^0+n)+\bar\sigma(\pi^-+p\to\varphi+n)+\bar\sigma(\pi^-+p\to\omega+n)=\bar\sigma(K^++n\to K^{*0}+p)+\bar\sigma(K^-+p\to\bar K^{*0}+n), \tag{8a}$$

$$\bar\sigma(\pi^++p\to\rho^0+N^{*++})+\bar\sigma(\pi^++p\to\varphi+N^{*++})+\bar\sigma(\pi^++p\to\omega+N^{*++})$$
$$=3\bar\sigma(K^-+p\to\bar K^{*0}+N^{*0})+\bar\sigma(K^++p\to K^{*0}+N^{*++}). \tag{8b}$$

Other relations are obtained by substituting corresponding decuplet baryons in the final states of (6) and (7) or by replacing the meson nonet in the final state of any sum rule by any nonet of bosons described as a quark-antiquark pair. The notation $\bar\sigma$ denotes a quantity proportional to the square of the transition matrix element which must be multiplied by the appropriate correction due to phase space and "structure factors."

Let us now consider the comparison of these relations with experiment. We follow the prescription of Meshkov, Snow, and Yodh[9] and compare related reactions at equal Q values, with corrections for phase space. However, an additional correction is required in the model to include the dependence of the transition matrix element on the momentum transfer which can be different for different reactions at the same Q. The need for such a structure factor has been pointed out[2] and will be discussed in detail elsewhere.[10] The structure factor actually used in this work has been taken directly from the t dependence of the experimental meson-baryon elastic scattering.[11] The correction involves no adjustable parameters, and turns out to be small for all cases considered in this Letter.

We consider first the vector mesons, where the mixing angle is well determined from other considerations and indicates that the physical φ and ω are very close to the SU(4) eigenstates M_λ and M_{n0}. The mixing angle obtained from the masses[7,12] is insensitive to the power of the mass used in the Gell-Mann–Okubo octet mass formula. Values of ± 37 and $\pm 40°$ are obtained, respectively, from the linear and quadratic mass formulas.

The right-hand side of Eq. (3a) is identically zero for $\alpha = 35.3°$ and very close to zero for $\alpha = 37$ or $40°$. This is in excellent agreement with experimental data which show that φ production is very small compared to ω production. For example, $\sigma(\pi^+ + p \to \varphi + N^{*++})$ is less than 10 μb at $p_\pi = 3.65$ GeV/c,[13] while $\sigma(\pi^+ + p \to \omega + N^{*++})$ is 700 ± 80 μb at 3.65 GeV/c[13] and 300 μb at 4.0 GeV/c.[14] No data are reported for the $\pi^- p$ reactions given in Eq. (3a). However, in the corresponding $\pi^+ n$ reactions related by isospin, appreciable ω production has been reported in $\pi^+ + n \to \omega + p$, whereas no evidence for the corresponding φ production has been given.[15]

Relations (6a) and (6b) can be compared directly with experiment since the physical φ and ω can to a good approximation be replaced by $\varphi \approx M_\lambda$ and $\omega \approx M_n$. Relation (6a) is found to be in reasonably good agreement with experiment. The experimental values for the same Q values of 0.46 GeV are[16,17]

$$\sigma(K^- p \,|\, \varphi\Lambda) = (40 \pm 8)\ \mu\text{b at } p_K = 3.0\ \text{GeV},$$

$$\sigma(\pi^- p \,|\, K^{*0}\Lambda) = (53 \pm 8)\ \mu\text{b at } p_{\pi^-} = 2.7\ \text{GeV}.$$

Strong disagreement with experiment is found in the case of the relation (6b). The ω-production cross section is very much larger than the ρ^0 production.[18] Furthermore, the ω has an isotropic angular distribution, in contrast to the ρ and φ angular distributions which are peaked forward.[18] This indicates that the peripheral production mechanism which dominates ρ and φ production is completely overshadowed by some other mechanism for the case of ω production. It is not surprising that this additional mechanism is not well described by the quark model. The fundamental assumption (1) is clearly not valid for a reaction where the dominant mechanism gives an isotropic angular distribution.[2] The exact nature of this mechanism and why it should occur in the case of the ω and not for the ρ and the φ has so far not been given any simple explanation. This remains a peculiar effect regardless of the validity of the quark model. Further investigations of this point may reveal interesting features of strong-interaction dynamics.

Predictions (8) have not been compared with experiment because of the lack of data for either reaction at the same Q value.

We now consider the pseudoscalar mesons, where the mixing angle is still uncertain. In contrast to the case of the vector mesons, the value of the η-X^0 mixing angle determined from experimental masses depends strongly upon whether masses or squares of masses are used in the Gell-Mann–Okubo octet mass formula.[12] The values of β obtained are

$$\beta \approx \pm 23° \text{ for the linear mass relation,}$$

$$\beta \approx \pm 10° \text{ for the quadratic mass relation.}$$

Tests proposed previously to distinguish between these two values[12] neglect SU(3)-symmetry breaking in transition amplitudes and make other dynamical assumptions. If the quark model is valid for the mesons and Eq. (1) is a good approximation for the reactions considered, the relations (3b) may give a more reliable measure of β. Furthermore, the validity of the model and the assumption (1) for these processes can be checked independently by use of the first equality of Eq. (3b) and the sum rules (6)-(8) which are independent of the mixing angle.

Unfortunately, sufficient experimental data are not yet available for the X^0-production processes. The only reaction in Eqs. (3b) for which a considerable amount of data is available is $\pi^- + p \to \eta + n$. Using these data,[19] the predicted values of the $\pi^- + p \to X^0 + n$ cross section are calculated from Eq. (3b) for both values of β and plotted in Fig. 1. Corrections for phase-space and structure factors are included. However, the structure-factor correction is small for this case, and does not appre-

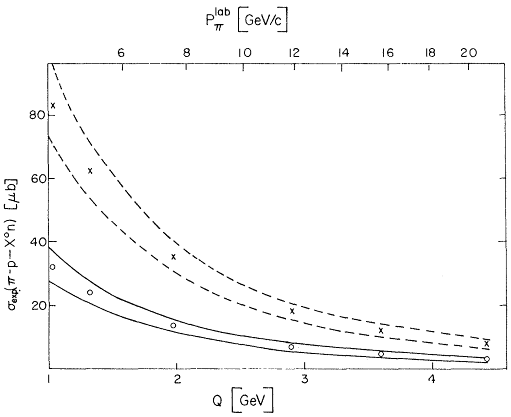

FIG. 1. Expected cross section for the reaction $\pi^- + p \to X^0 + n$ (or $\pi^+ + n \to X^0 + p$), calculated on the basis of the data on $\pi^- + p \to \eta + n$ of Ref. 19 and Eq. (3b). The crosses correspond to $\beta = -10°$ (squared masses), the circles to $\beta = -24°$ (linear masses). Relation (3b) is used for same Q values in the two reactions and the corresponding pion momentum for $\pi^- + p \to X^0 + n$ is given in the upper scale. The bands between the dashed and solid lines correspond to the experimental errors of the data in Ref. 19.

ciably affect the prediction. The values of the X^0 production cross sections are seen to be considerably different for the two mixing angles. Experimental values in good agreement with either curve should constitute good evidence for the model and for the particular value of the angle.

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as approximate symmetries, and which give a continuum of values for the mixing angle. Unless some other criterion is given for choosing the "right" algebra, there is no way of defining the proper mixing angle. This ambiguity is implicit in all attempts to define the mixing angle by strong or electromagnetic decay processes if the assumption is made either that strong interactions are invariant under SU(3) or that the electromagnetic interaction has octet transformation properties under SU(3). Each prediction of this type picks a particular SU(3) group out of the continuum of approximate symmetries, but there is no reason a priori to believe that this particular group and its associated mixing angle is better than any of the others. The quark model defines a unique SU(3) algebra by specifying that the quark triplet is classified in the fundamental representation. All SU(3) classifications and mixing angles are determined uniquely for any particle by specifying its wave function in the quark model.

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ELECTROMAGNETISM OF HADRONS AND HIGHER SYMMETRIES*

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Any physically meaningful attempt to find a simple Lie group of rank three, containing among its generators a part of the charge which commutes with $C_{\rm st}$ and containing SU(3) as a subgroup, is shown to imply the existence of fractionally charged particles. Conjugation with $C_{\rm st}$ is not an automorphism of such a group and its extension by the group made up of $C_{\rm st}$ and $C_{\rm st}^2 = E$ is shown to be impossible. The combination of the simple rank-three Lie group with $C_{\rm st}$ does not lead to a finite-dimensional semisimple Lie group in the neighborhood of the identity.

In an attempt to explain the apparent CP-invariance violation observed in K_2^0 decays,[1] Lee[2] has pointed out that the charge-conjugation operator (C_γ) which leaves invariant the electromagnetic interactions of the leptons may not be the same as the charge-conjugation operator ($C_{\rm st}$) of the strong interactions.[3] This leads, he has shown, in a natural way to the existence of a more general expression for the charge operator,

$$Q = Q_J + Q_K, \tag{1}$$

where Q_J is the part of the charge which anticommutes with $C_{\rm st}$, whereas Q_K is that part which commutes with $C_{\rm st}$. The part Q_K has gone undetected because all the particles observed to date have a Q_K of zero. Within his scheme (minimal electromagnetic coupling), Lee has found that Q_K commutes with Q_J and $H_{\rm st}$, and so it is another quantity conserved by the strong interactions.

We will show that any physically meaningful attempt to find a simple Lie group (G) of rank three which includes Q_K as a generator and