by fields. In de Broglie's point of view[1] it offers no answer yet to the spreading problem of external v-wave packets.

The observed baryon spectrum results in this model from quantized internal behavior corresponding to irreducible finite-dimensional representations of our groups of motions. Mass itself results from internal subquantum motion. Observed baryons just correspond to internal quantized behavior which yields a possible physical description of the "clocks" attached to material particles since the foundation of wave mechanics.[24] In this light it is remarkable that the known bosons can be considered as nonmassless quanta emitted when baryons undergo quantum jumps from on quantum state to another.[25]

The author is thankful for stimulating discussions with Professor Flato, Professor Rączka, and Professor Wataghin. He is specially indebted to Professor L. de Broglie with whom he has elaborated the essential physical ideas of the model, and to Professor Lichnerowicz for important contributions to its mathematical construction.

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ELECTROMAGNETIC MASS DIFFERENCES IN THE QUARK MODEL

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In a recent series of papers the possibility has been studied of known strongly interacting particles being bound states of quarks.[1] Until quarks are produced, if they exist, one looks for the influence they can have on the observed properties of elementary particles, and tries to see whether their existence is compatible with data. One looks for new predictions that can test their presence indirectly. Here we show that the quark model plus an extremely simple dynamical assumption are enough to give a remarkably good account of the electromagnetic mass differences of both baryons and mesons.

As usual, the mass within a given isospin multiplet is supposed to be the same for all members before electromagnetism is switched on. The shift is assumed to have the following form:

$$\delta M = M - M_0 = \sum_i \delta m_i + \sum_{i \neq j} D_{ij} + \cdots, \qquad (1)$$

where M is the physical mass of the baryon or meson in question, M_0 the corresponding unperturbed mass, and δm_i the shift in each individual quark forming the particle. D_{ij} accounts for the added two-body forces between quarks. Other terms are assumed small and neglected. No convincing argument can be produced for this simple additivity property in the face of the enormous binding of quarks within the given particle.[1]

We assume that mesons and baryons are composed of three kinds of quarks, denoted by α, β, γ, with the usual third-integral electric charge. The quantities m_i and D_{ij} are assumed to depend only upon the quantum numbers of the quarks involved, and to be the same for all states in the baryon octet and decuplet. The baryon mass differences are then expressed in terms of nine free parameters, $m_\alpha, m_\beta, m_\gamma, D_{\alpha\alpha}, D_{\beta\beta}, D_{\gamma\gamma}$, $D_{\alpha\gamma}, D_{\alpha\beta}, D_{\beta\gamma}$. A similar, different set of nine parameters is assumed to apply to the vector and pseudoscalar mesons. No relation is assumed between the values of the different parameters. Previous treatments, based either on SU(3), SU(6), or quark models, can be cast into the form (1), but with the parameters related by conditions which follow either from symmetry or from dynamical assumption. The assumptions used in this paper are therefore weaker than those used in previous treatments. The relations obtained here are, therefore, also obtained in the other treatments. We find no conflict with SU(3) or SU(6), and no new relations which are unobtainable by other methods.

The possibility of obtaining these relations under weaker assumptions is particularly significant in view of the strong disagreement found between many symmetry predictions and experiments. The suggestion has been made that quark-model assumptions without the explicit introduction SU(3) or SU(6) symmetry may give only the good predictions of the symmetry without the bad ones. This seems to be the case with the present work, as none of the bad predictions mentioned by Harari[2] follow from the assumptions used here. Let us now proceed to the calculation. The shifts can be written as follows:

$$\delta p = 2\delta m_\alpha + \delta m_\beta + D_{\alpha\alpha} + 2D_{\alpha\beta}, \tag{2}$$

$$\delta n = \delta m_\alpha + 2\delta m_\beta + D_{\beta\beta} + 2D_{\alpha\beta}, \tag{3}$$

$$\delta\Sigma^+ = 2\delta m_\alpha + \delta m_\gamma + D_{\alpha\alpha} + 2D_{\alpha\gamma}, \tag{4}$$

where δm_i is the mass shift of the ith quark, the letter representing a particle stands for its mass, and the indices on the D functions refer to the particles involved in the two-body interaction. Similar equations can be written immediately for all particles whose quark content is known. By taking differences within given isomultiplets and combining the resulting equations to eliminate the unknown parameters, one obtains several relations. To illustrate the method we derive one of them. Using Eqs. (2) and (3), we obtain

$$p - n = \delta m_\alpha - \delta m_\beta + D_{\alpha\alpha} - D_{\beta\beta} \tag{5}$$

and, by the same procudure,

$$\Sigma^+ - \Sigma^- = 2(\delta m_\alpha - \delta m_\beta) + D_{\alpha\alpha} - D_{\beta\beta} + 2(D_{\alpha\gamma} - D_{\beta\gamma}), \tag{6}$$

$$\Xi^- - \Xi^0 = -(\delta m_\alpha - \delta m_\beta) - 2(D_{\alpha\gamma} - D_{\beta\gamma}). \tag{7}$$

Combining Eqs. (5)-(7), we obtain

$$\Sigma^+ - \Sigma^- + \Xi^- - \Xi^0 = p - n, \tag{8}$$

which is the well-known Coleman-Glashow relation that holds to a high degree of accuracy. Note that this relation is obtained from Eq. (1) and from the quark content of baryons without any assumption about symmetries of the strong interactions higher than isospin, or about transformation properties of the photon.

The other relations read

$$N^{*+} - N^{*0} = (p - n), \tag{9}$$

$$N^{*++} - N^{*-} = 3(p - n), \tag{10}$$

$$Y^{*+} - Y^{*-} = \Sigma^+ - \Sigma^-, \tag{11}$$

$$\Xi^{*-} - \Xi^{*0} = \Xi^- - \Xi^0, \tag{12}$$

$$Y^{*+} - Y^{*0} = \Sigma^+ - \Sigma^0. \tag{12a}$$

Experimentally the situation is as follows[3]:

$N^{*++} - N^{*-} = -7.9 \pm 6.8$, $3(p-n) = -3.9$; (13)

$N^{*+} - N^{*0}$ not reported, $(p-n) = -1.3$; (14)

$Y^{*+} - Y^{*} = -17 \pm 7$ or -4.3 ± 2.2, $(\Sigma^+ - \Sigma^-) = -7.9$; (15)

$\Xi^{*-} - \Xi^{*0} = 6.3 \pm 4$,[4] $\Xi^- - \Xi^0 = 6.6$; (16)

$Y^{*+} - Y^{*0}$ not reported, $\Sigma^+ - \Sigma^0 = -3.1$. (16a)

All predictions are in agreement with experiment, though large errors make it difficult at the present time to decide between different theories. Strikingly enough, the prediction concerning Ξ^* reads, in SU(6),[5]

$$\Xi^{*-}-\Xi^{*0}=\Sigma^{+}+\Sigma^{-}-2\Sigma^{0}-(p-n)=2.8,$$

to be compared with Eq. (16).

One can now go to the mesons and deduce

$$\pi^{+}-\pi^{0}=\rho^{+}-\rho^{0}, \tag{17}$$

$$K^{+}-K^{0}=K^{*+}-K^{*0}. \tag{18}$$

Notice that Eqs. (17) and (18) do not relate strange to nonstrange mesons. Meson relations are a good test of the theory because most theories give either no relations[5] or the wrong ones.[2] No precise data are available yet, but there are indications that they might satisfy relations (17) and (18).[6]

Since the relations presented here are independent of the form of D, they should hold very well in all cases. Finally, let us emphasize once more that no symmetry other than isospin is implied in the Hamiltonian of the strong interactions, and that no transformation properties are assumed for the electromagnetic interaction. It is remarkable that all these relations can be written down without further assumptions than Eq. (1) and the quark content of particles.

It is a pleasure to acknowledge discussions with P. Federman, A. Katz, and A. Reiner. I thank especially H. J. Lipkin, whose comments greatly improved the paper.

Note added in proof.—Since the completion of this Letter several improvements have been made. A new relation was pointed out[7]:

$$2(p-n)+\Sigma^{+}+\Sigma^{-}-2\Sigma^{0}=N^{*++}-N^{*0}.$$

By introducing the spin and isospin dependence of the D functions one shows that all these relations for baryons are still valid except for (11) and (12) which are replaced by their sum. In the general case there are no relations for mesons. These results are studied in detail in a paper under preparation.[8]

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INABILITY OF THE QUARK TO BOOTSTRAP*

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Because of the success of the "eightfold way" version of SU(3),[1-3] the question that naturally arises is why the baryons take up the octet representation instead of the fundamental triplet. To shed light on this problem we studied the possibility of bootstrapping the $\underline{3}\otimes\underline{8}=\underline{3}\oplus\underline{6}^{*}\oplus\underline{15}$ system, which is a triplet of baryons scattered from a pseudoscalar octet of mesons. We report here the main results of this investigation. A complete description will appear later.

If the above system were not to bootstrap, then one might interpret this as suggesting that the "reason" why the triplet does not exist in nature is because it is dynamically unstable. On the other hand, if triplets were to remain undiscovered, a successful bootstrap would call into question the very idea of bootstraps, at least in the approximations in which this concept is now used.

The forces that we used were the exchange of vector mesons and the exchange of any $P_{1/2}$ bound states or $P_{3/2}$ resonances that emerge from the calculation. It should be noted that the triplet corresponds to the nucleon, and the