ratios $R = \Delta\sigma^+/\Delta\sigma^-$ where the $\Delta\sigma$'s are the partial cross sections for identical increments of solid angles or q^2. Values of R calculated from data taken in two separate runs agree within the statistical errors. The combined values are reported here. The results of the second run indicated a 0.5% shift of the positron energy relative to the electron energy, which was verified by magnetic analysis of the beams at the target position. We have calculated a correction to R of about 1.5% on the basis of the measured shift. A radiative correction has been calculated[3] using Eq. (3.23) of the paper of Yennie, Frautschi, and Suura.[4] For our kinematic limits this correction is made by subtracting from the measured R a quantity Δ given by

$$\Delta = 0.011 - 0.0026 \ln q^2,$$

where q^2 is given in $(\mathrm{GeV}/c)^2$.

Values of R are determined from plots such as Fig. 3(a) which shows the distribution in q^2 of the elastic events from the first run. From the combined data from both runs we obtain, after making the radiative correction,

$$R = 0.996 \pm 0.020$$

for $0.35 \leq q^2 \leq 0.93$, where the error is the statistical error which is dominant.

In Fig. 3(b) we plot R for four intervals of q^2 together with the results of Yount and Pine[5] and Browman, Liu, and Schaerf.[6] These earlier Stanford results suggested that R may be increasing slowly with q^2, but our results do not support this. We conclude that the two scattering cross sections are the same within our experimental error after correcting for the radiation of real photons by the scattering and recoil particles.

The results given here come from about 80% of the 40 000 pictures taken at 1200 MeV. We have also taken 25 000 pictures at 810 MeV. A more detailed description of the experiment and final results will be given in a future paper.

We wish to thank Professor R. R. Wilson, who suggested this experiment and helped us design the positron beam, and Professor Martin Feldman for his contribution to our preliminary studies. We are grateful for the efficient and patient work of our scanners, Mrs. Mary Johnson, Miss Geraldine Jackson, and Mr. Richard Johnson. Several discussions with Mr. Anil Rae and Professor Donald Yennie are gratefully acknowledged. We also thank the operating crew of the Cornell synchrotron for their generous cooperation.

*Work supported in part by the U. S. Office of Naval Research and the National Science Foundation.

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MESON-BARYON COUPLING CONSTANTS IN BROKEN SU(3) AND THE ALGEBRA OF CURRENTS

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Using the algebra of currents, we study the renormalization effects on the meson-baryon coupling constants, due to SU(3) breaking.

Using the algebra of currents[1] and the hypothesis of partial conservation of axial-vector currents (PCAC), we obtain sum rules for meson-baryon coupling constants in broken SU(3) with symmetry-breaking effects taken into account to first order. Our sum rules are much stronger than those obtained by earlier workers using pure group-theory methods.[2] The most important aspect of our results is the following: Those sum rules which involve only pion-baryon couplings or only kaon-baryon couplings are exactly the same as the corresponding sum rules obtained in the limit of SU(3) symmetry.[3] Likewise for η couplings. To first order, renormalization effects due to symmetry breaking are thus entirely absent in these sum rules. Renormalization effects appear only when pion-baryon couplings are compared to kaon-baryon (or η-baryon) couplings. One consequence of this situation has immediate experimental relevance. The ratios of decay widths in the observed processes $N^* \to N+\pi$, $Y_1^* \to \Lambda+\pi$, $Y_1^* \to \Sigma+\pi$, and $\Xi^* \to \Xi+\pi$ are unrenormalized and are given by their SU(3)-symmetric values. This last result is well supported by present experimental data.[4] For the general case our sum rules may be used to check the applicability of current-algebra methods to purely strong-interaction processes.[5]

Let us consider meson-baryon vertex $B' \to B + P$, where P is a pseudoscalar particle. Let $f_0(B'BP)$ denote this vertex in the limit of exact SU(3). Now the symmetry-breaking interaction which produces the mass differences can be considered to be proportional to the operator S_8—the space integral of the eighth component of quark scalar density.[6] Hence the first-order broken-symmetry correction to this vertex may be written as

$$f_1(B_i'B_jP_k) = \lambda\langle B_jP_k|S_8|B_i'\rangle. \tag{1}$$

In (1) λ is a constant and i, j, and k are SU(3) indices. Reducing P_k and applying PCAC we get

$$(2q_0^{\,k})^{1/2}\langle B_jP_k|S_8|B_i'\rangle = \frac{-i}{C_k}\int d^4x \exp(-iq^kx)(\Box^2 - m_k^{\,2}) \times\langle B_j|[\partial_\mu J_{5\mu}^{\,k}, S_8]|B_i'\rangle\theta(-x_0). \tag{2}$$

In (2) C_k is the PCAC constant $\partial_\mu J_{5\mu}^{\,k} = C_kP_k(x)$ and m_k the mass of the kth meson. From (2) we obtain in the standard manner[7]

$$\lim_{q^k\to 0}(2q_0^{\,k})^{1/2}\langle B_jP_k|S_8|B_i'\rangle = \frac{-m_k^{\,2}}{C_k}\langle B_j|[F_k^{\,5}, S_8]|B_i'\rangle. \tag{3}$$

Using the commutation relation

$$[F_i^{\,5}, S_j] = -id_{ijk}S_k^{\,5}, \tag{4}$$

we obtain finally the vertex $f(B'BP)$ with symmetry-breaking effect taken into account to first order[9]:

$$f(B_i'B_jP_k) = f_0(B_i'B_jP_k) + id_{k8k}\lambda C\langle B_j|S_k^{\,5}|B_i'\rangle. \tag{5}$$

In (5), the constant C stands for $m_k^{\,2}/C_k$ and the matrix element $\langle B_j|S_k^{\,5}|B_i'\rangle$ is to be evaluated in SU(3) limit. We now consider applications of Eq. (5) to various cases.

(1) Baryon-decuplet–baryon-octet–pseudoscalar-octet coupling.—B_i' belongs to the $\frac{3}{2}^+$ baryon decuplet and B_j to the $\frac{1}{2}^+$ baryon octet. Applying the Wigner-Eckart theorem to $f_0(B_i'B_jP_k)$ and $\langle B_j|S_k^{\,5}|B_i'\rangle$ we obtain a two-parameter formula for each vertex:

$$-\sqrt{2}f(N^*N\pi) = \sqrt{6}f(Y_1^*\Sigma\pi) = 2f(Y_1^*\Lambda\pi) = 2f(\Xi^*\Xi\pi) = G_0 + (\lambda C/\sqrt{3})G_1, \tag{6}$$

$$\sqrt{2}f(N^*\Sigma K) = \sqrt{6}f(Y_1^*\Sigma K) = -\sqrt{6}f(Y_1^*N\bar{K}) = 2f(\Xi^*\Sigma\bar{K}) = -2f(\Xi^*\Lambda\bar{K}) = f(\Omega\Xi\bar{K}) = G_0 - (\lambda C/2\sqrt{3})G_1, \tag{7}$$

$$2f(Y_1^*\Sigma\eta) = 2f(\Xi^*\Xi\eta) = G_0 - (\lambda C/\sqrt{3})G_1. \tag{8}$$

In the above, G_0 and G_1 are reduced matrix elements coming from $f_0(B_i'B_jP_k)$ and $\langle B_j|\times S_k^{\,5}|B_i'\rangle$, respectively. Eq. (6) shows that the ratio of any two pion coupling constants is the same as its value in SU(3) limit. This ratio is unaffected by symmetry breaking. A similar statement about kaon and η couplings is made by Eqs. (7) and (8), respectively. However, the pion and kaon couplings are no longer related in simple numerical ratios and broken-symmetry effects now appear. One sum rule exists connecting the π, K, and η couplings. This is

$$2\sqrt{6}f(Y_1^*N\bar{K}) - (1/\sqrt{2})f(N^*N\pi) + 3f(\Xi^*\Xi\eta) = 0. \tag{9}$$

At present not enough is known about these coupling constants to subject Eq. (9) to direct test. However, it should be emphasized that the lack of renormalization in the decays $N^* \to N+\pi$, $Y_1^* \to \Lambda+\pi$, $Y_1^* \to \Sigma+\pi$, and $\Xi^* \to \Xi+\pi$ is fully supported by present experiments.[4] In fact, this observation was the starting point of the present investigation.

(2) Baryon-octet–pseudoscalar-octet–baryon-octet coupling.—In this case B_i' and B_j are both $\frac{1}{2}^+$ baryons. Proceeding exactly as above

we obtain a four-parameter formula for each coupling constant. Thus we may have eight sum rules connecting the twelve meson-baryon coupling constants.[10]

These sum rules are

$$\sqrt{3}g_{\Sigma\Lambda\pi}+g_{\Sigma\Sigma\pi}=2g_{NN\pi}, \tag{10}$$

$$g_{NN\pi}+g_{\Xi\Xi\pi}=g_{\Sigma\Sigma\pi}, \tag{11}$$

$$g_{\Sigma\Sigma\eta}=-g_{\Lambda\Lambda\eta}, \tag{12}$$

$$g_{NN\eta}+g_{\Sigma\Sigma\eta}+g_{\Xi\Xi\eta}=0, \tag{13}$$

$$\sqrt{3}g_{N\Lambda K}-g_{N\Sigma K}=2g_{\Xi\Xi K}, \tag{14}$$

$$2g_{N\Sigma K}+\sqrt{3}g_{\Xi\Lambda K}+g_{\Xi\Sigma K}=0, \tag{15}$$

$$g_{NN\pi}+2\sqrt{3}g_{N\Lambda K}-2g_{N\Sigma K}+\sqrt{3}g_{NN\eta}+2\sqrt{3}g_{\Sigma\Sigma\eta}=0, \tag{16}$$

$$g_{\Xi\Xi\pi}+\tfrac{1}{2}\sqrt{3}g_{\Sigma\Lambda\pi}+g_{N\Sigma K}+\sqrt{3}g_{N\Lambda K}+\sqrt{3}g_{NN\eta}+\tfrac{1}{2}\sqrt{3}g_{\Sigma\Sigma\eta}=0. \tag{17}$$

Eqs. (10)-(15) are exactly the same as those obtained in SU(3) limit. Renormalization effects are present only in Eqs. (16) and (17). Using forward dispersion relations, Lusignoli et al.[8] have recently estimated $g_{N\Lambda K}$ and found substantial deviation from the SU(3)-invariant prediction. This may imply the presence of large renormalization effects. Needless to say, such a situation is entirely consistent with the above sum rules although, once again, present knowledge of the coupling constant does not permit us to check sum rules (10)-(17). Finally, we should mention the following: In the limit of zero meson momentum the pion-baryon vertex functions would vanish provided we use momentum conservation. This causes no difficulty, however, since the analytic structure of the right-hand side of Eq. (2) enables one to define a new function which is free from both momentum-conservation and mass-shell constraints. In any case, the pion-baryon coupling constants, which are defined through appropriate Lorentz-scalar functions, are perfectly well defined in the above limit. Thus, our sum rules are not subject to any ambiguity, except for possible off-mass-shell effects.

(3) Baryon-singlet–baryon-octet–pseudoscalar-octet coupling.—In this case B_j belongs to the $\frac{1}{2}^+$ baryon octet and B_i is the $Y_0^*(1405)$ singlet. We have a two-parameter expression for each vertex and obtain the following sum rules:

$$f(Y_0^*\bar{K}N)=f(Y_0^*\Xi K), \tag{18}$$

$$f(Y_0^*\Sigma\pi)+3f(Y_0^*\Lambda\eta)=4f(Y_0^*\bar{K}N). \tag{19}$$

It will be interesting to check Eqs. (18) and (19) directly.

We thank Professor S. Okubo for much helpful discussion and encouragement. We thank Professor Abdus Salam and Professor P. Budini and the International Atomic Energy Agency (IAEA) for hospitality at the International Centre for Theoretical Physics. One of us (S.K.B.) thanks UNESCO for financial support.

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[10]In this case, the assumption of Ref. 9 yields stronger results than Eqs. (10)-(14) of the text: $g_{\Sigma\Lambda\pi}/g_{NN\pi}=2(1-\alpha)/\sqrt{3}$, $g_{\Xi\Xi\pi}/g_{NN\pi}=2\alpha-1$, $g_{\Sigma\Sigma\pi}/g_{NN\pi}=2\alpha$, $g_{\Xi\Xi\eta}/g_{NN\eta}=2(1-\alpha)/(4\alpha-1)$, $g_{\Sigma\Sigma\eta}=-g_{\Lambda\Lambda\eta}$, $g_{\Xi\Xi\eta}/g_{NN\eta}=(1+2\alpha)/(1-4\alpha)$, $g_{N\Sigma K}/g_{N\Lambda K}=\sqrt{3}(2\alpha-1)/(1+2\alpha)$ $g_{\Xi\Sigma K}/g_{N\Lambda K}=\sqrt{3}/(1+2\alpha)$, $g_{\Xi\Lambda K}/g_{N\Lambda K}=(1-4\alpha)/(1+2\alpha)$, where $\alpha=F/(D+F)$. Thus, all pion couplings, estimated with $g_{NN\pi}$ and α (which may be taken to be the same as that for weak axial-vector current) as input, are exactly the same as the SU(3)-symmetric values. If we accept the estimate of Ref. 8, $g_{N\Lambda K}{}^2\simeq 4.8$, we then obtain (with $\alpha\simeq 0.35$[11]) $g_{N\Sigma K}{}^2\simeq 0.5$, $g_{\Xi\Lambda K}{}^2\simeq 0.3$, and $g_{\Xi\Sigma K}{}^2\simeq 5.0$.

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NEUTRAL-MESON PRODUCTION CROSS SECTIONS AND MIXING ANGLES IN A QUARK MODEL

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The recent success of an extremely simplified quark model[1,2] for forward elastic scattering may be due to the particular simplicity of zero-momentum-transfer processes. In this Letter we present predictions for neutral-meson production processes at finite momentum transfer[3] which may provide a more sensitive test of the model and which also give values for the mixing angles in the meson nonets. These predictions and mixing angles depend only upon the validity of the quark model[1,2] and are independent of SU(3)-symmetry-breaking effects in the transition amplitudes. Reasonable agreement with experiment has been found for those cases where adequate data are available except in one case which will be discussed.

We assume that a meson is a quark-antiquark pair but require no assumptions regarding the baryon structure. We further assume that the transition amplitude for any meson-baryon reaction is expressible as the sum of the constituent quark-baryon and antiquark-baryon scattering amplitudes.[1,2] Thus the transition between the quark-antiquark-baryon states $|(q_a\bar{q}_{a'})B_a\rangle$ and $|(q_b\bar{q}_{b'})B_b\rangle$ is given by the expression[4]

$$\langle (q_a\bar{q}_{a'})B_a|(q_b\bar{q}_{b'})B_b\rangle = \langle q_aB_a|q_bB_b\rangle\delta_{a'b'} + \langle\bar{q}_{a'}B_a|\bar{q}_{b'}B_b\rangle\delta_{ab}. \quad (1)$$

The assumption (1) is assumed to hold separately for each possible set of polarization states of the initial and final quark-antiquark-baryon systems. We consider only relations between reactions involving the same baryon states and different members of the same initial and the same final meson nonets. The explicit calculations show that the relations obtained are independent of the initial and final meson and baryon spin states. The same relations thus hold for all polarization amplitudes separately.

From the fundamental assumption (1) it follows that all reactions which require changes in the states of both the quark and the antiquark in the meson are forbidden in this model. Obvious examples of such transitions are those involving a double charge exchange or a double strangeness exchange. We also obtain the following selection rules:

$$\langle\pi^- p|M_\lambda n\rangle = \langle\pi^+ n|M_\lambda p\rangle = \langle\pi^+ p|M_\lambda N^{*++}\rangle = \langle\pi^- p|M_\lambda N^{*0}\rangle = 0, \quad (2a)$$

where M_λ denotes the particular linear combinations of neutral meson states which contain only strange quarks. This state is denoted by $(\lambda\bar{\lambda})$ in Ref. 2.

The M_λ state is given in terms of the corresponding unitary octet states M_1 and M_8 by

$$M_\lambda = (1/\sqrt{3})(\sqrt{2}M_8 - M_1) \quad (2b)$$

for both the vector and pseudoscalar cases. The corresponding orthogonal state which contains only nonstrange quarks coupled to isospin zero is given by

$$M_{n0} = (1/\sqrt{3})(M_8 + \sqrt{2}M_1). \quad (2c)$$