DRIFT INSTABILITY DUE TO IMPURITY IONS*

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(Received 11 July 1966)

It is well known that the two greatest barriers to controlled fusion are the stability problem and the impurity problem—the latter being of great importance since even a small quantity of impurity greatly enhances the radiation losses of the plasma.[1] Such impurities are likely to arise near the wall of a plasma container, especially in the absence of a divertor. In this note we point out that the presence of impurity ions, especially if their density gradient is different from the plasma density gradient, i.e., if they are peaked near the wall, can generate a new instability which is difficult to stabilize and which leads to diffusion of the impurities into the plasma.

Here we consider a one-dimensional model with density varying in the x direction and the main magnetic field along z. A situation with plasma pressure p much less than magnetic pressure $B^2/8\pi$ is assumed. Then, the electric field perturbations can be taken as potential: $\vec{E}=-\nabla\varphi$. Considering the wavelength of such perturbations to be greater than the typical Larmor radii of plasma particles, we make use of the Vlasov equation,

$$\frac{\partial f_i}{\partial t}+v_z\frac{\partial f_i}{\partial z}+c\frac{\vec{E}\times\vec{B}}{B}\cdot\frac{\partial f_i}{\partial\vec{r}}+Z_i\frac{eE_z}{M_i}\frac{\partial f_i}{\partial v_z}=0, \quad (1)$$

for the distribution functions $f=f(v_z,\vec{r},t)$, which describe the particle motion in terms of the drift of the guiding centers. The second term in Eq. (1) represents the motion of particles along magnetic field lines, which is influenced only by the electric-field component along $\vec{B}$ entering the fourth term. The third term represents the electric drift $c\vec{E}\times\vec{B}/B^2$ of the guiding centers. The equilibrium distribution functions for all species, which satisfy Eq. (1), have a form $f_0^i(x,v_z,v_\perp)$. The index $i=I,H,e$ labels the impurity, hydrogen ions, and electrons, respectively. The linear stability analysis is then reduced to solving the eigenvalue problem for the linearized equations,

$$i(\omega+k_zv_z)\delta f_i+\frac{cE_y}{B_0}\frac{\partial f_0^i}{\partial x}+Z_i\frac{eE_z}{Mi}\frac{\partial f_0^i}{\partial v_z}=0 \quad (2)$$

and

$$Z_I\int dv_z\,\delta f_I+\int dv_z\,\delta f_H=\int dv_z\,\delta f_e, \quad (3)$$

where

$$\delta f_i=\sum\delta f_i(x)\exp(ik_yy+ik_zz),$$

$$\vec{E}=\sum\vec{E}(x)\exp(ik_yy+ik_zz),$$

and Z_i represents the charge sign and number of each species. Equation (3) is the quasineutrality condition.

The substitution of δf_i from Eq. (2) into Eq. (3), since $E=-\nabla\varphi$, gives the final dispersion relation

$$k_y\frac{c}{B_0}\int\frac{dv_z}{\omega+k_zv_z}\left\{\frac{\partial f_0^H}{\partial x}-\frac{\partial f_0^e}{\partial x}+Z_I\frac{\partial f_0^i}{\partial x}\right\}+ek_z\int\frac{dv_z}{\omega+k_zv_z}\left\{\frac{1}{M_H}\frac{\partial f_0^H}{\partial v_z}+\frac{Z_I^2}{M_I}\frac{\partial f_0^I}{\partial v_z}+\frac{1}{m}\frac{\partial f_0^e}{\partial v_z}\right\}=0. \quad (4)$$

For the sake of definiteness, we choose $f_0^i=n_0^i(x)(M_i/2\pi T_i)^{1/2}\exp(-M_iv_z^2/2T_i)$. In this case the integrals may be performed to give

$$\sum_i Z_i^2\frac{n_i}{T_i}\left[1-(1+W_i)\left(1+\frac{\omega_i^*}{\omega}\right)\right]=0. \quad (5)$$

Here n_i represents the density; T_i the temperature; $\omega_i^*=k_yv_{di}$, v_{di} being the diamagnetic velocity $v_{di}=v_{thi}^2n_i'/(2n_i\Omega_i)$, $v_{thi}=(2T_i/M_i)^{1/2}$, $\Omega_i=Z_ieB_0/\mu_ic$ being the gyrofrequency; and

$$W_i=-\pi^{-1/2}\int_{-\infty}^{+\infty}\xi e^{-\xi^2}d\xi/(\xi+\omega/k_zv_{thi}).$$

In general, when terms of order k_yv_{thi}/Ω_i are taken into account and isotropic temperatures are assumed, the second term in brackets is

to be multiplied by $I_0(b_i)\exp(-b_i)$ with $b_i = k_y^2 T_i/M_i\Omega_i^2$ and I_0 the modified Bessel function of zero order.[2]

We first consider the case where the impurity concentration is small so that $n_I' \ll n_H'$. Then if the phase velocity is chosen, such that $\omega/k_z \gg v_{tH}$, the presence of a small concentration of impurity ions ($n_I' \ll n_H'$) is irrelevant, and we obtain the usual, stable drift oscillations for the case of zero Larmor radius and no temperature gradient. Then the more interesting case is that given by

$$v_{\mathrm{th}I} < \omega/k_z < v_{\mathrm{th}H} \ll v_{\mathrm{th}e}.$$

Note that this may come about even for $T_H = T_I$ if $M_I \gg M_H$. In this case we may use the proper asymptotic forms for the W functions occurring in Eq. (5),[2]

$$W_e = -1 + i\pi^{1/2}\omega/k_z v_{\mathrm{th}e},$$

$$W_H = -1 + i\pi^{1/2}\omega/k_z v_{\mathrm{th}H},$$

$$W_I = \tfrac{1}{2}k_z^2 v_{\mathrm{th}I}^2/\omega^2.$$

Noting that charge neutrality requires $\sum Z_i^2 n_i \omega_i^*/T_i = 0$, we may solve Eq. (5) to find

$$\omega = Z_I^2 \omega_I^* \frac{n_I}{T_I}\left(\frac{n_e}{T_e}+\frac{n_H}{T_H}-Z_I^2\frac{n_I}{T_I}W_I - i\pi^{1/2}\frac{(\omega+\omega_H^*)}{k_z v_{\mathrm{th}H}}\frac{n_H}{T_H}\right)^{-1}. \tag{6}$$

Here W_I is real since $\omega > k_z v_{\mathrm{th}I}$ and marginal stability obtains for $\omega = -\omega_H^*$. The condition for instability is then

$$\frac{n_H Z_I}{n_e T_H/T_e + n_H - n_I Z_I^2 W_I T_H/T_I} + \frac{n_H'}{n_I'} < 0. \tag{7}$$

Two types of instability can now occur. If the number of impurity ions is not too small, the impurity-ion sound wave can be made to go unstable if the denominator of the first term of Eq. (7) is negative. As the maximum value of W is about 0.2, the condition for this instability is

$$0.2 n_I Z_I^2 T_H/T_I > n_e + n_H. \tag{8}$$

Due to the possible large value of Z_I^2 this may occur at relatively modest impurity densities.

For the other mode we note that, if $n_I \ll n_H$, the instability condition is given by

$$\frac{n_H'}{n_I'} + \frac{n_H Z_I}{T_H/T_e n_e + n_H} < 0. \tag{9}$$

For a hydrogen plasma with $T_e/T_H \approx 1$, this reduces to

$$n_H'/n_I' Z_I + \tfrac{1}{2} < 0. \tag{10}$$

For this instability to occur, the density gradient of impurity ions must be in the opposite sense to that of the plasma ions and not too large. This condition would be likely to be met near the walls where the plasma density decreases and the impurity density increases towards the wall. In this case $\omega < \omega^*$ and the worst growth, obtained by adjusting k_z in Eq. (6), is

$$\mathrm{Im}(\omega) \approx \mathrm{Re}(\omega) \approx Z_I n_I'/n_H' \omega_H^*.$$

On the other hand, if the density gradient of the impurity species is of the order of that for the hydrogen species so that $Z_I n_I' + n_H' \approx 0$, the instability associated with Eq. (5) is of fluid type[3] in the sense that it does not involve wave-particle resonance and occurs in the limit $v_{\mathrm{th}e} > \omega/k_z > v_{\mathrm{th}H} > v_{\mathrm{th}I}$.

This instability is very reminiscent of the temperature-gradient instability[4] because it occurs for long wavelengths and involves ion rather than electron Landau damping. As in that case, it can be shown that shear is rather ineffective for stabilization.[3,4] We note that what is relevant for this instability is not the temperature gradient but the gradient of mean parallel velocity, which may be due to either a temperature gradient or a gradient of mean mass number as in our case. The quasilinear effect of this instability is of course to cause an outward diffusion of plasma ions and an inward diffusion of impurity ions.

Thanks are due to Professor Abdus Salam and the International Atomic Energy Agency for hospitality extended to us at the International Centre for Theoretical Physics, Trieste.

*This work has been supported in part by U. S. Atomic Energy Commission.

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INTERACTION OF 25-keV ELECTRONS WITH LATTICE VIBRATIONS IN LiF. EXPERIMENTAL EVIDENCE FOR SURFACE MODES OF LATTICE VIBRATION*

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(Received 29 June 1966)

In two former papers[1,2] it has been stated that monoenergetic[3] fast electrons interacting with molecules excite infrared active fundamental vibrations with high intensity. Consequently, strong interaction of fast electrons with optical lattice vibration should be expected in alkali-halide crystals. In these solids, however, as is well known, electromagnetic waves excite transverse vibrations ω_T of the lattice, whereas a charged particle excites the longitudinal modes ω_L. The maximum of light absorption and the most probable energy loss appear at different energies.

Figure 1 represents (a) the real and the imaginary part of the dielectric constant $\epsilon(\omega)=\epsilon_1(\omega)+i\epsilon_2(\omega)$, (b) the reflection spectrum (reststrahl band, and (c) the energy-loss function $-\mathrm{Im}[1/\epsilon(\omega)]$ according to the dielectric theory[4] of an infinite medium, in lithium fluoride. The absorption spectrum, which is essentially proportional to ϵ_2, shows a rather sharp peak at $\hbar\omega_T=0.039$ eV. The reststrahl band extends over a wide range from 0.03 to 0.08 eV. The energy-loss function calculated from the optical constants exhibits a peak at $\hbar\omega_L=0.081$ eV, where ϵ_1 vanishes, in accordance with the Lyddane-Sachs-Teller relation.[5]

The high-resolution technique recently developed[3] enables us now to separate those electrons which have lost such small amounts of energy from the unaffected electrons at 25-keV primary energy. In contrast to the energy-loss function, the measured energy-loss spectrum of a 400-Å LiF foil (Fig. 1) shows a broad band between 0.03 and 0.11 eV with a maximum at 0.047 eV. The spectrum is corrected by subtracting the slope of the no-loss line.[1] The excitation of lattice vibrations takes place with largest probability at an excitation energy between those of the transversal and longitudinal modes. The energy-loss spectra depend slightly on foil thickness (Fig. 2). The maximum shifts from 0.042 to 0.050 eV when the foil thickness increases from 240 to 700 Å.

In addition to these energy losses, energy gains of the primary electrons could also be observed in the spectrum (Fig. 2) changing their position in the same way. The energy gains disappear when the specimen is cooled down to the temperature of liquid air. Since processes leading to energy loss and energy gain have equal probabilities, the intensity ratio of energy gain and energy loss is proportional to the occupation probability of the state, which is a function of the temperature of the specimen.

The discrepancies between the computed and measured energy-loss spectra in Fig. 1 are caused by the fact that we are dealing with a finite medium. On the assumption that the excitation probability of lattice vibrations ω_L is

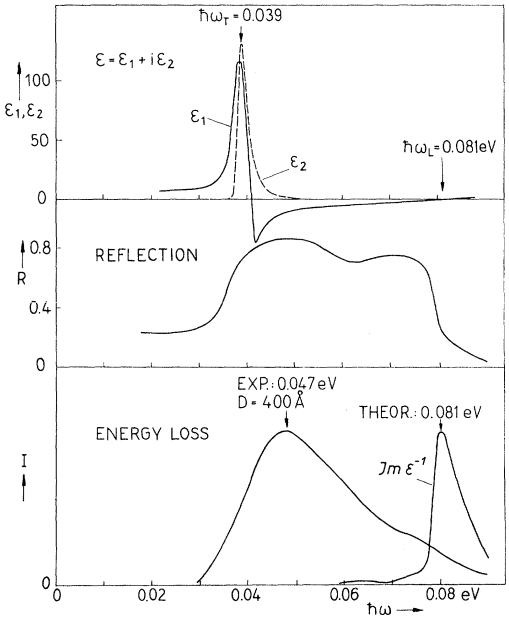

FIG. 1. Dielectric constant $\epsilon=\epsilon_1+i\epsilon_2$, reststrahl band R, energy-loss function $-\mathrm{Im}1/\epsilon$, and experimental energy-loss spectrum (normalized) of LiF on carbon substratum, $E_0=25$ keV, $\theta\leq 1\times 10^{-4}$. Optical data taken from M. Gottlieb, J. Opt. Soc. Am. 50, 343 (1960).