PRESSURE DEPENDENCE OF THE RADIUS OF THE NEGATIVE ION IN HELIUM II†

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(Received 20 June 1966)

The pressure dependence of the radius of the negative ion in helium II is derived from measurements of trapping cross section in quantized vortex lines.

The negative ion in He II appears to be an electron localized in a region from which the atoms are essentially excluded. Such a "bubble" model of the ion has been the subject of continued theoretical discussion.[1-5] The underlying idea is that the repulsive interaction between an electron and a helium atom is so strong that the electron produces a density deficiency in the liquid over a radius of order 15 Å. The size of the cavity so formed should be sensitive to the hydrostatic pressure in the liquid. Mobility measurements in helium gas by Levine and Sanders provided the first clear-cut evidence for this model of the negative ion.[6] Parks and Donnelly[7] have shown that the radii of ions in He II can be estimated from their mean trapped lifetimes in quantized vortex lines and rings. Specifically, they showed that the lifetime of a negative ion trapped on a vortex line mainly involves $\exp(+\Delta i/kT)$, where Δi is essentially the kinetic energy of the volume of rotating superfluid excluded by the ion. From this it is obvious that the lifetime at a given temperature is a sensitive function of ion radius, which in turn is a function of pressure on the bubble model.

This Letter reports the pressure dependence of the radius of the negative ion as derived from trapping cross-section measurements in rotating He II. It is also shown that the pressure dependence of the negative-ion mobility can be explained in terms of the bubble model.

The measurements were performed in a revolving helium cryostat whose details will be published elsewhere. Pressure is applied to the helium in the experimental chamber via 18 in. of 30-mil capillary tubing, and pressures up to 20 atm were obtained. The trapping cross section, σ, is defined by

$$I(\Omega) = I_0 \exp(-2\Omega m h^{-1} \sigma x), \tag{1}$$

where I is the current transmitted transverse to the axis of rotation, m is the mass of the helium atom, x is the distance along the direction of the applied field, and Ω is the angular velocity of rotation. The quantity $2m\Omega h^{-1}$ is the number of vortex lines per cm^2. σ has been found to be a function of temperature and applied electric field,[8] and to exhibit a cutoff at about 1.7°K at the vapor pressure. This cutoff (or "lifetime edge") is well described by

$$\sigma = \sigma_0 \exp(-pt), \tag{2}$$

where σ_0 is the cross section well below cutoff, p is the probability of escape from the line, and t is some characteristic time for the ion to remain within the experimental region. The temperature interval over which σ falls rapidly is about 0.1°K.

From the foregoing, we are led to predict that increasing applied pressures will decrease the radius of the negative ion, thus moving the lifetime edge of the cross section to lower temperatures. Figure 1 shows that this is indeed the case: It is found that a 1 Å change in radius depresses the lifetime edge by about 0.1°K.

The calculation of radii from Fig. 1 was performed as follows: The pressure and temperature coordinates of points such that $\sigma = \frac{1}{2}\sigma_0$, i.e., $pt = 0.693\,15$, can be taken from the figure. Then using the value $R = 15.96$ Å, p, and hence t at the vapor pressure, can be found by a calculation similar to that in Ref. 7. To find R at any other pressure certain assumptions have to be made about t. If t remains constant throughout, the P, T coordinates from Fig. 1 yield the upper curve in Fig. 2. A more reasonable assumption is that t varies inversely with the free-ion mobility, which gives the middle curve in Fig. 2. The lower curve in Fig. 2 is derived in the same manner except that the core radius of a vortex is assumed to vary as the square root of the density. It is known that t does not vary as fast as the inverse of the free-ion mobility,[9] so that the upper and lower curves in Fig. 2 are, respectively, upper and lower limits on the radius as a function of pressure.

The superfluid density which enters in these calculations is a function of pressure. This has been allowed for in the calculations of R by calculating $\rho_S(P)$ using the Landau model

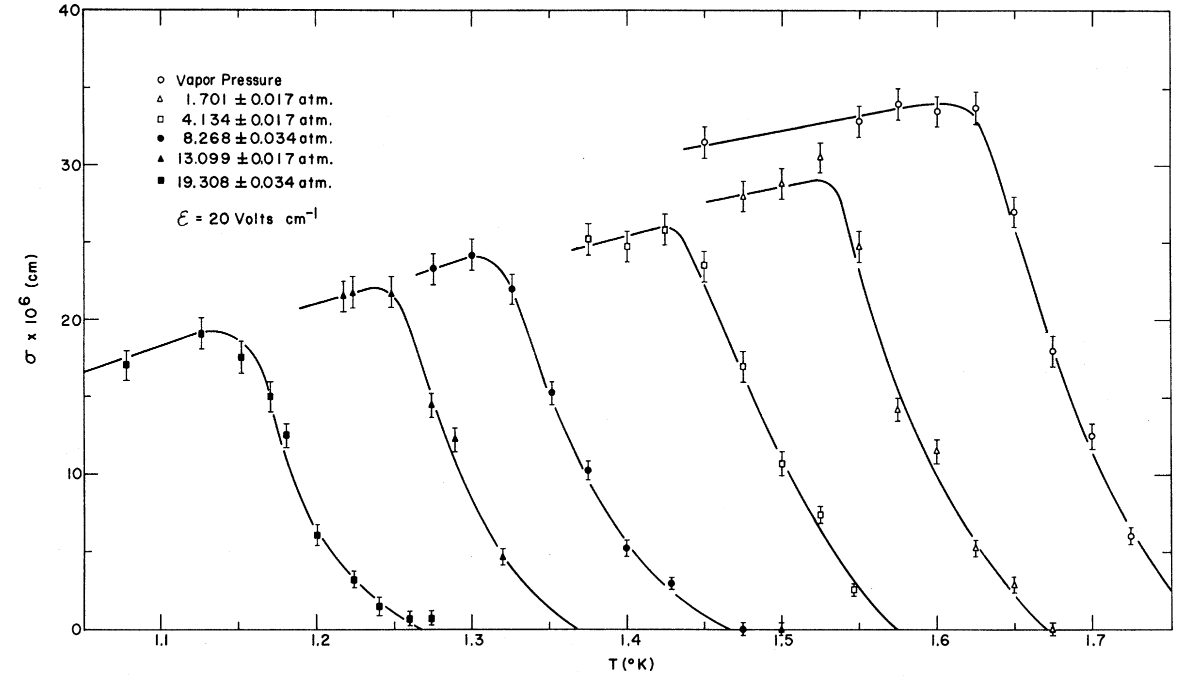

FIG. 1. Lifetime edges for capture cross sections of negative ions at various pressures.

and the parameters determined from the neutron data of Yarnell et al.,[10] and Henshaw and Woods,[11] the first-sound measurements of Atkins and Stasior,[12] and the density measurements of Keesom and Keesom.[13] The values of ρ_S so found deviate by less than 2% from those measured directly at the vapor pressure.[14] Second-sound,[15] and specific-heat and entropy measurements[16] give good agreement at other pressures. The maximum uncertainty in R due to uncertainties in ρ_S is shown by the vertical line in the upper right corner of Fig. 2.

Mobility measurements were made using the same apparatus. The method employed was to apply a square wave which switches the current on and off. In this case the transmitted

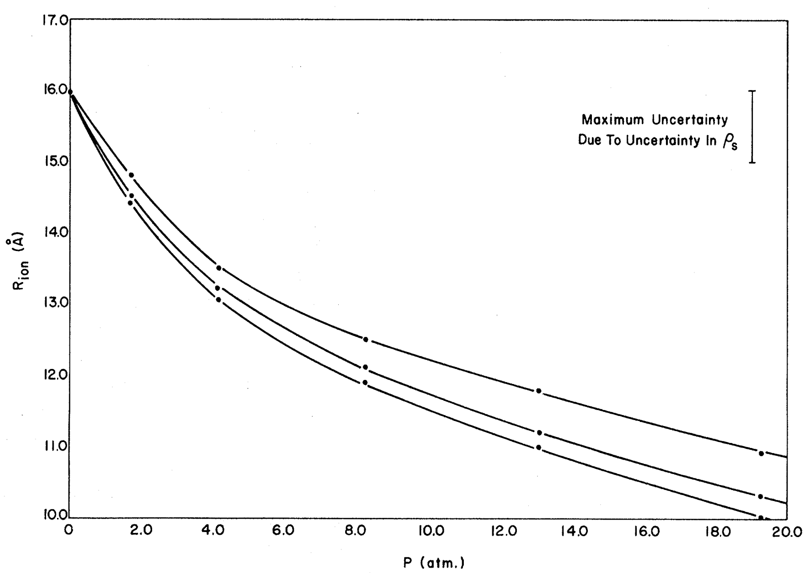

FIG. 2. Variation of ion radius with pressure derived from Fig. 1.

current is

$$I(\nu)=I(0)[\tfrac{1}{2}-x^2\nu/\mu V], \qquad (3)$$

where I is the transmitted current, x is the length of the beam, ν is the square-wave frequency, V is the square-wave amplitude, and μ is the mobility. $I(\nu)$ reaches zero when $\nu=\nu_c=\mu V/2x^2$.

According to Meyer and Reif[17] a peak in the negative-ion mobility occurs at a pressure of a few atmospheres. The solid curve in Fig. 3 shows the mobility as a function of pressure at 1.025°K in good agreement with the results of Ref. 17. The limiting slope of the mobility curve in Fig. 3 is 7.25×10^{-3} atm^{-1}, agreeing well with 7.54×10^{-3} atm^{-1} obtained from neutron data.[11] Our data were obtained by measuring the mobility every $\frac{1}{2}$ atm; the points are not shown in interests of clarity (the estimated error in this curve is $\pm1\frac{1}{2}\%$).

We interpret the mobility data by kinetic theory. A Stokes-law interpretation should not be expected to hold at these temperatures because mean free paths are comparable to the size of the ion. In fact, putting the viscosity measurements of Brewer and Edwards[18] and the present values of R into the expression $\mu=e/6\pi\eta R$ does not agree with our values of μ. The work of Meyer and Reif shows that, at these temperatures and pressures, rotons limit the ion drift velocity. The mobility may be written as

$$\mu=e/MvN_\gamma\sigma, \qquad (4)$$

where M is a reduced mass taking account of persistence of velocity, v is the mean relative velocity of ions and rotons, N_γ is the number density of rotons, and σ is the ion-roton collision cross section. This expression may be divided into two parts. e/Mv is not expected to be very sensitive to pressure; $(N_\gamma\sigma)^{-1}$ is essentially the ion-roton mean free path and is shown as a function of pressure by the dashed curve in Fig. 3. N_γ is calculated from the Landau model using the sources for the parameters quoted above, and σ is taken as $\pi(R+4)^2$ Å^2. As in Ref. 2, this value for the cross section is used because ion-roton scattering is "high-energy" scattering in the sense of Mott and Massey,[19] and 4 Å is the effective radius of a roton as estimated by Landau and Khalatnikov.[20] As can be seen, the present values of negative-ion radius adequately explain the negative-ion mobility on the basis of ion-roton scattering.

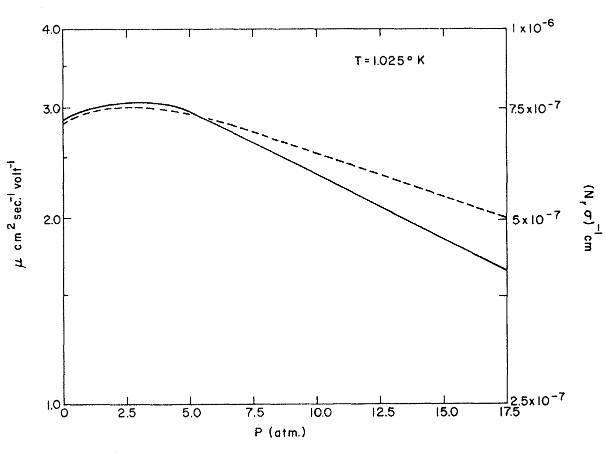

FIG. 3. Comparison of ion mobility (solid curve) and the expression $(N_\gamma\sigma)^{-1}$ (dashed curve) as a function of pressure. Note the coincidence of the maxima.

If, in the factor M, one used the hydrodynamic mass of the ion, an extra factor R^3 would appear which destroys the correspondence shown. The hydrodynamic mass determines the impulse required to accelerate the ion in the superfluid. It does not necessarily determine the momentum exchange in a collision.

Finally, in Ref. 4 it is stated that a particle-in-a-box model is not a bad approximation for the bubble. A calculation using this model was made, and it was found that the variation with pressure is about the same as that of Fig. 2; e.g., the ratio of the radius at 20 atm to the radius at the vapor pressure is 0.64 ± 0.07 experimentally and 0.732 on the model.

We are greatly indebted to Mr. P. E. Parks for assistance with the numerical calculations.

†Research supported by the National Science Foundation under Grant No. GP-5302, and by the Air Force Office of Scientific Research under Grant No. AF-AFOSR 785-65. We have also benefited from the use of facilities provided by the Advanced Research Projects Agency for materials research at the University of Chicago.

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THERMAL AND MAGNETIC PROPERTIES OF DILUTE SOLUTIONS OF He^3 IN He^4 AT LOW TEMPERATURES*

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The prediction by Edwards *et al.*[1] that there should be no phase separation down to $T = 0°K$ in dilute solutions of He^3 in He^4 for concentrations of less than about 6% He^3 was confirmed at least in part through heat-capacity measurements by Anderson, Roach, Sarwinski, and Wheatley[2] for a concentration of about 5% down to a temperature of nearly 10 mdeg K. The idea[3] that the He^3 quasiparticles in the He^4 at low temperatures constituted a weakly interacting Fermi fluid was also confirmed in these experiments. This Letter describes the results of a set of experiments at saturated vapor pressure on the thermal and magnetic properties of two dilute solutions of He^3 in He^4. It shows that the He^3 in dilute solutions does indeed have several of the properties associated with a normal Fermi fluid. Dilute solutions of He^3 in He^4 are particularly advantageous in studying the weakly interacting Fermi fluid since the Fermi momentum may be varied by changing concentration. Hence the momentum dependence of the quasiparticle interactions may be tested. Moreover, the present experiments form a quantitative basis for theories of the quasiparticle interactions and for predictions of a low-temperature cooperative state.[4]

In the present work the earlier heat-capacity measurements[1,2] have been confirmed and extended to lower temperatures, and measurements have also been made of the spin-diffusion coefficient and nuclear susceptibility of the *same* two dilute solutions, of nominal concentrations 1.3 and 5.0%. It is important to measure both heat capacity and magnetic properties on the same concentration since parameters such as effective mass and Fermi temperature determined by the former are used in interpreting the latter measurements.

Only a few brief remarks can be given here on the experimental method. The refrigeration problem and methods used for thermal and magnetic measurements are given by Abel, Anderson, Black, and Wheatley.[5] The method of thermal isolation while using superfluid He^4 is discussed by Vilches and Wheatley.[6] We had one important experimental difficulty. We normally cool He^3 by means of powdered cerium magnesium nitrate (CMN) which passes through an NBS 40 sieve (particle size less than 0.42 mm). CMN power prepared in this way cools pure He^3 anomalously well.[7] In the present work it was necessary to pass the CMN through an NBS 400 sieve (particle size less than 37 μ) to obtain an adequately low thermal time constant at low temperatures. The background heat capacity for the thermal measurements was obtained with pure He^4 in the cell. Above about 5 mdeg K the resulting molar heat capacity (attributed entirely to the CMN) was the same for different powder sizes (varying over a factor of 40 in particle size) and also the same as determined by Abel, Anderson, Black, and Wheatley[8] by a difference method