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EFFECT OF SURFACE PATCH FIELDS ON FIELD-EMISSION WORK-FUNCTION DETERMINATIONS*

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Two types of measurements have previously been employed in field-emission single-crystal plane work-function determinations: (1) Relative work functions of various planes have been obtained from the relative slopes of Fowler-Nordheim (FN) plots for emission from these planes,[1,2] (2) absolute work functions have been calculated by combining FN and field-emission energy-distribution measurements from a plane.[2,3] All results depend, of course, on the validity of the FN equation. The following work involves (1) discussion of electrostatic fields at the surface due to patches (regions of different work function), (2) correction to FN work-function determinations due to the patch field, (3) derivation of energy-distribution method for determining work function, and (4) a method for determining work function independent of both patch and applied surface fields.

Surface patch fields.—An application of Laplace's equation shows that differences in work function (or surface potential) on adjacent regions of a single-crystal emitter give rise to substantial electrostatic fields at the surface. Surprisingly, none of the previous work has taken these fields into account. For example, either the (110), the (100), or the (112) plane, together with its surrounding regions, can be considered, in first approximation, as a disc at a potential equal in magnitude to the negative of its work function, surrounded by a wide annular ring, or even an infinite plane, at a potential equal in magnitude to the negative of the average work function of tungsten, say 4.50 eV. If a disc of radius R at potential V_0 is located in an infinite plane held at zero potential everywhere outside of the disc, the field on the disc axis is given by[4]

$$F_0 = V_0 R^2/(R^2+z^2)^{3/2}, \tag{1}$$

where V_0 is the difference in potential between the disc and the surrounding plane, and z is the distance above the surface of the disc along the disc axis. The field at the surface of the disc is $F_0 = V_0/R$. A typical (110) plane with a radius of 25 to 50 Å, assumed work functions of 6 eV,[1,2] and surrounded by a region with a work function of 4.5 eV would have a patch field of from 6 to 3 MV/cm. This is in addition to the applied field of about 40 MV/cm. The patch field decreases somewhat over the 10-Å distance above the surface, which contributes to the potential barrier and increases with distance off the axis.

Fowler-Nordheim technique for measuring work function.—The current density from the portion of the single crystal plane which is being measured is given by the FN equation[5]

$$J = \frac{1.54\times10^{-6}F^2}{\varphi t^2(y)} \times \exp[-6.83\times10^7 \varphi^{3/2} v(y)/F] \text{ A/cm}^2, \tag{2}$$

where φ is the work function in eV of the portion of the surface being measured, F the field strength in V/cm, $v(y)$ and $t(y)$ are slowly varying elliptic functions,[5] and $y = 3.79\times10^{-4}F^{1/2}/\varphi$. The relationship between voltage and field strength is defined as

$$F = F_0 + \beta V, \tag{3}$$

where F_0 is the static field at the plane due to patch fields and V is the applied voltage. The collected current is $i = JA$, where A is the emitting area. Substituting Eq. (3) into Eq. (2) and noting that β and A are independent of V, the slope of the FN curve is given by

$$S_{\rm FN}=\frac{\partial\log(i/V^2)}{\partial(1/V)}$$
$$=\frac{0.87V}{(1+\beta V/F_0)}-\frac{2.96\times10^7\varphi^{3/2}s(y)}{\beta(F_0/\beta V+1)^2},\qquad(4)$$

where $s(y)$ is a slowly varying elliptic function.

If the field distribution over the emitter in the absence of the patch field is assumed to be constant (β = constant), then the ratio of the work function of a plane a (with patch field F_0) to plane b (no patch field) predicted from the slopes of the FN plots, neglecting the first term, is

$$\frac{\varphi_a}{\varphi_b}=\left[\frac{S_{{\rm FN}_a}}{S_{{\rm FN}_b}}\left(\frac{F_0}{\beta V}+1\right)^2\right]^{2/3}.\qquad(5)$$

Calculations show that the presence of the patch field does not cause a serious curvature in the FN plot over the range of currents usually measured in this type of experiment. Since the patch field at the (110) plane is in the direction of the applied field, if the above corrections are appled to the previously measured[1,2] 6-eV work function of the (110) plane, assuming a 100-Å diameter plane surrounded by a 4.5-eV work-function surface, a value of 7 eV is obtained for the corrected work function (see Fig. 1). The low work-function planes, which comprise the greater part of the emitter surface, will experience a much smaller field in the opposite direction.

Energy distribution combined with Fowler-Nordheim theory with patch field correction.—Energy-distribution measurements have been combined with FN data so that two independent relationships between work function and field may be obtained. In this way the work function and field associated with the (134) plane of tungsten was determined.[2] Van Oostrom has employed this technique extensively.[3]

In the following discussion the broad exponential character of the integrated current is used to overcome problems resulting from energy-analyzer resolution and the difficulties of extracting energy distributions from the integrated curves.[6]

The zero-temperature distribution in total energy of field-emitted electrons is given by[7]

$$P(E)dE=(4\pi md/h^3)\exp(-c-\zeta/d)\exp(E/d),\qquad(6)$$

where $c=[4(2m\varphi^3)^{1/2}/3\hbar eF]v(y)$, $d=[\hbar eF/2(2m\varphi)^{1/2}t(y)]$, E is the total energy of the emitted electrons, m and $-e$ are the electronic mass and charge, h is Planck's constant, and ζ is the Fermi energy $=-\varphi$. Retarding-potential analyzers measure the integral of the current up to an energy E, as a function of E. Expressed as a fraction of the total current, Eq. (6) gives

$$\frac{i(E)}{i_0}=\frac{\int_E^\zeta e^{E'/d}dE'}{\int_{-\infty}^\zeta e^{E'/d}dE'},\qquad(7)$$

where i_0 = total current. Therefore,

$$\frac{i_0-i(E)}{i_0}=\exp\frac{E-\zeta}{d}.$$

The "slope" of the energy-distribution curve is defined as

$$S_E=\frac{d\log[i_0-i(E)]}{d(E)},$$

and it follows that

$$S_E=\frac{0.434\times2(2m\varphi)^{1/2}t(y)}{\hbar eF}.\qquad(8)$$

Substituting Eq. (3) into Eq. (8) and dividing by Eq. (4) gives

$$\varphi=-\frac{3}{2}\frac{S_{\rm FN}}{S_E}\frac{t(y)}{s(y)}\frac{1}{V}\left(\frac{F_0}{\beta V}+1\right).\qquad(9)$$

When a value has been obtained for φ using Eq. (9), this value can be substituted into Eq. (8) to obtain the field strength at the surface. In the final analysis an iterative process must be employed in assigning values to F_0, $t(y)$,

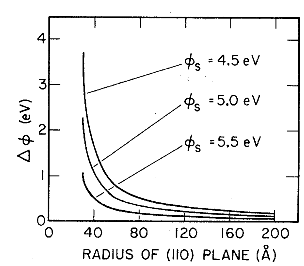

FIG. 1. Work-function correction [Eq. (5)] versus radius of measured plane, with work function of surrounding region, φ_S, as a parameter. Values typical of the (110) plane (i.e., $\varphi_{\rm meas}$ = 6.0 eV, R = 30 to 200 Å) were employed. Since the resolution of the field electron microscope is at best 20 Å, the corrections would be of questionable value for radii less than about 30 Å.

$s(y)$, and β. The complications introduced by the patch field would seem to make precise determination of φ and F by this technique difficult or impossible.

Field-independent determination of φ.—If Eq. (8) for the "slope" of the energy-distribution curve is solved for F, the true field at the surface, and substituted into the FN equation, we obtain for the current density

$$J=(3.04\times10^9/S_E^{\,2})\exp[-1.54\varphi S_E v(y)/t(y)]. \quad (10)$$

Since $t(y)$ is extremely slowly varying, it can be considered to be independent of S_E, and it follows that

$$\frac{d(\log i_0 S_E^{\,2})}{d(S_E)}=-0.668\varphi\frac{s(y)}{t(y)}. \quad (11)$$

By measuring the dependence of the "slope" of the energy-distribution curve, S_E, on total current (by varying the applied field), it is possible to obtain accurate values of φ regardless of patch fields. The fact that the energy-distribution measurement gives the ratio of the square root of the work function to the true surface field makes this approach feasible. From this information the true surface field can then be calculated.

Discussion.—It is clear that patch fields exist and must be corrected for[8] in FN and past energy-distribution-type work-function measurements. This fact is consistent with the annealing-temperature dependence of the uncorrected (110) work-function values (φ decreases as annealing temperature increases) as measured by Müller,[1] since plane size decreases with increasing annealing temperature. The use of the field-independent analysis [Eq. (11)] obviates this problem.

The above work grew out of recent energy-distribution measurements in this laboratory which indicated (110) work functions in excess of 7 or 8 eV[9,10] (before patch-field correction), depending on (110) plane size. Patch-field correction would further increase this work-function value. Should these values prove to be correct, a unique surface with novel electronic and perhaps chemical properties is available for surface investigations.

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