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ALMOST EXACT SUM RULES FOR NUCLEON MOMENTS FROM AN INFINITE-DIMENSIONAL ALGEBRA*

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Recently there has been a great surge of interest in almost exact sum rules for the magnetic moments of nucleons.[1-4] (By almost exact we mean exact to all orders in the strong couplings but only to the lowest order in electromagnetic and weak couplings.) Besides providing a means for calculation of the magnetic moments on the same level as the calculation of G_A/G_V by Adler[5] and Weisberger,[6] these sum rules, taken together with the Adler-Weisberger sum rule, constitute a useful tool for investigating the nature of the dynamical approximations that underlie higher symmetry schemes.

The purpose of the present note is to report on a set of sum rules which follow from an infinite-dimensional algebra, which contains (and may be regarded as the most natural extension of) an algebra suggested by Gell-Mann.[7] In order to specify the algebra we consider the function

$$M_{\mu\nu}^{\alpha\beta}(k',k)i(2\pi)^4\delta^4(p'+k'-p-k)$$
$$=\int d^4x d^4y\, e^{i(k'\cdot x-k\cdot y)}\langle p'|[T\{J_\mu^{\alpha}(x)J_\nu^{\beta}(y)\}-i\rho_{\mu\nu}^{\alpha\beta}(x)\delta^4(x-y)]|p\rangle. \quad (1)$$

Here α,β are isotopic indices, μ,ν are Minkowski indices, k',k (p',p) are outgoing and incident "photon" (nucleon) momenta, and J_μ^{α} is the conserved isospin current which participates in weak interactions, $\partial^\mu J_\mu^{\alpha}=0$.

The second term in the right-hand side of Eq. (1) is designed to compensate for the noncovariant nature of the T-product,[8] so that $M_{\mu\nu}^{\alpha\beta}$ is a covariant object. The simplest equal-time commutation relations which ensure this covariance are[8]

$$[J_0^{\alpha}(x),J_0^{\beta}(y)]_{x_0=y_0}=i\epsilon^{\alpha\beta\gamma}J_0^{\gamma}(x)\delta^3(x-y), \quad (2)$$

$$[J_0^{\alpha}(x),J_n^{\beta}(y)]_{x_0=y_0}=i\epsilon^{\alpha\beta\gamma}J_n^{\gamma}(x)\delta^3(x-y)+i\partial_m[\rho_{mn}^{\alpha\beta}(x)\delta^3(x-y)] \quad (3)$$

(m, n are 3-space indices). Equations (2) and (3) specify the algebra under consideration. Equation (2), by itself, is the starting point of several interesting conjectures by Gell-Mann.

We note that Eqs. (1)-(3) lead to the divergence conditions

$$k'^{\mu} M_{\mu\nu}{}^{\alpha\beta} = M_{\nu\lambda}{}^{\alpha\beta} k^{\lambda} = i\epsilon^{\alpha\beta\gamma} \langle p' | J_{\nu}{}^{\gamma}(0) | p \rangle. \quad (4)$$

These divergence conditions enable us to derive low-energy theorems for the function $M_{\mu\nu}{}^{\alpha\beta}$, by a straightforward application of techniques invented by Low in connection with Compton scattering.[9] Complications arising from isotopics do not pose any special problem[10]; without further ado, therefore, we state these theorems.

Define

$$e^2 (2\pi)^3 M_{mn}{}^{\alpha\beta}(k,k) = \delta^{\alpha\beta}\{S_1(\omega)\delta_{mn} + S_2(\omega)\tfrac{1}{2}[\sigma_m, \sigma_n]\} + \tfrac{1}{2}[\tau^{\alpha}, \tau^{\beta}]\{A_1(\omega)\delta_{mn} + A_2(\omega)\tfrac{1}{2}[\sigma_m, \sigma_n]\}, \quad (5)$$

where $\omega \equiv k_0$ and Pauli spinors are understood, but not displayed, on the right-hand side. Also, we have specialized to the forward direction in the laboratory frame $\vec{p} = 0$. The theorems are [units: $e^2 = 4\pi\alpha \cong 4\pi/137$]

$$S_1(\omega) = \pi\alpha/M + O(\omega^2), \quad (6)$$

$$S_2(\omega) = \frac{\pi\alpha(\kappa_p - \kappa_n)^2}{2M^2}\omega + O(\omega^2), \quad (7)$$

$$A_1(\omega) = 4\pi\alpha\left\{\frac{(1+\kappa_p-\kappa_n)^2}{8M^2} - \left(\frac{\partial G_E{}^V(q^2)}{\partial q^2}\right)_{q^2=0} - \frac{1}{8M^2}\right\}\omega + O(\omega^2), \quad (8)$$

$$A_2(\omega) = 4\pi\alpha\left(\frac{1+\kappa_p-\kappa_n}{4M}\right) + O(\omega^2). \quad (9)$$

Here κ_p and κ_n are the anomalous moments of proton and neutron, respectively (in units of nucleon magnetons), M is the nucleon mass, and $G_E{}^V$ is the electric isovector Sachs form factor. We have assumed that the neutral component of the conserved isospin current is proportional to the isovector part of the electromagnetic current, and scaled our amplitudes to correspond to the scattering of isovector photons.

Exactly analogous considerations can be advanced for the (simpler) case of isoscalar photons. We start with the conserved hypercharge current and derive the theorems

$$S_1(\omega)^Y = \pi\alpha/M + O(\omega^2), \quad (10)$$

$$S_2(\omega)^Y = \frac{\pi\alpha(\kappa_p + \kappa_n)^2}{2M^2}\omega + O(\omega^2), \quad (11)$$

where we have used an obvious notation for the hyperphoton amplitudes.

The above low-frequency theorems may be converted into sum rules by postulating unsubtracted dispersion relations for the relevant amplitudes.[11] The dispersion relations are easily written down by following the procedure of Gell-Mann, Goldberger, and Thirring.[12] One finds that the amplitudes $S_1(\omega)$ and $S_1(\omega)^Y$ cannot satisfy unsubtracted dispersion relations for exactly the same reasons as in ordinary Thompson scattering.[4] The other amplitudes yield the following sum rules:

$$\frac{\alpha(\kappa_p+\kappa_n)^2}{2M^2} = \frac{1}{\pi^2}\int_0^{\infty} \frac{\sigma_P{}^S - \sigma_A{}^S}{\omega} d\omega, \quad (12)$$

$$\frac{\alpha(\kappa_p-\kappa_n)^2}{2M^2} = \frac{1}{\pi^2}\int_0^{\infty} \frac{\sigma_P{}^V - \sigma_A{}^V}{\omega} d\omega, \quad (13)$$

$$\alpha\left\{\frac{(1+\kappa_p-\kappa_n)^2}{4M^2} - 2\left(\frac{\partial G_E{}^V(q^2)}{\partial q^2}\right)_{q^2=0} - \frac{1}{4M^2}\right\} = \frac{1}{4\pi^2}\int_0^{\infty} \frac{(\sigma_3 - 2\sigma_1)_A{}^V + (\sigma_3 - 2\sigma_1)_P{}^V}{\omega} d\omega, \quad (14)$$

$$\alpha\left\{\frac{1+\kappa_p-\kappa_n}{2M}\right\} = \frac{1}{4\pi^2}\int_0^{\infty} [(\sigma_3-2\sigma_1)_A{}^V - (\sigma_3-2\sigma_1)_P{}^V] d\omega. \quad (15)$$

All the cross sections in Eqs. (12)-(15) are absorption cross sections for photons on protons, the superscripts S and V indicate whether the photon is isoscalar or isovector, the subscripts A and P indicate whether the helicity of the photon is antiparallel or parallel to the proton spin, and the subscripts 3 and 1 imply that the cross section is a partial cross section for absorption in isospin states $\frac{3}{2}$ or $\frac{1}{2}$.

Equation (14) is the well-known sum rule of Cabibbo and Radicati, and has been thoroughly discussed in the literature. Some interesting features of the other sum rules are discussed below.

Remarks.– (a) If one adds Eqs. (12) and (13), one obtains a sum rule for $\kappa_p^2+\kappa_n^2$. This sum rule agrees with the sum rule one would infer for this quantity from the work of Drell and Hearn.[4] Note that we were able to obtain separate sum rules for isoscalar and isovector moments because of our assumption that the isospin and hypercharge were separately constants of the motion. Drell and Hearn only assumed the conservation of total charge.

(b) If one assumes that the cross sections are dominated by the (3, 3) resonance [isobar model], Eq. (12) leads to the relation $\kappa_p=-\kappa_n$, which is in fair agreement with experiment.[13] Substituting this relation in Eq. (13), and staying within the framework of the isobar model, one finds that Eq. (13) is also well satisfied. Note that the numerics here reduces to that in the paper of Drell and Hearn.

These successes of the isobar model should not, however, lead one to infer that the spin-$\frac{3}{2}$ resonances play a decisive role in determining the static properties of baryons.

(c) We find it remarkable that Eq. (13), derived by us as an almost exact sum rule in its own right, appears to follow from a sum rule of Fubini, Segré, and Walecka[14] [based on the algebra of U(12)] if one makes the empirical approximation $\kappa_p+\kappa_n=0$. Clearly, both sum rules cannot simultaneously be regarded as almost exact. Either the difference is of trivial origin, having its roots in the subtraction ambiguities which worried Fubini, Segré, and Walecka, or it points towards some deep relationship between the algebra of U(12) and the infinite-dimensional algebra considered by us. We do not, at the moment, have a clear-cut answer.

(d) It has been noted elsewhere[10] that the sum rules (14) and (15) do not admit of even approximate saturation by the (3, 3) resonance. We feel, therefore, that they provide a good testing ground for representation-mixing theories of the type recently proposed.[15]

(e) By evaluating the above sum rules and their U-spin and V-spin counterparts, in an approximation suggested by Alessandrini, Bég, and Brown,[16] it is possible to derive many of the attractive results of SU(6). We hope to report on this in the near future.

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Note added in proof.– It is important to realize that a no-subtraction *Ansatz*, in the context of dispersion theory, need not be compatible with perturbation theory to any finite order if the coupling constants and masses are regarded as free parameters. Equation (15) provides an obvious illustration of this point. In the event of such a conflict, one may choose to assert either that a subtraction is mandatory, or that naive perturbative arguments are not a reliable guide in strong interactions and that the no-subtraction *Ansatz* may well be a constraint respected by nature. Neither viewpoint is logically compelling.

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CURRENT-COMMUTATOR CALCULATION OF THE K_{l4} FORM FACTORS*

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In the past few years we have learned how to use the hypothesis of a partially conserved axial-vector current and the current commutation relations (PCAC and CCR) to obtain matrix elements for emission or scattering of low-energy pions.[1] One of the most impressive results of this sort is a formula for a K_{l3} form factor, derived by Callan and Treiman,[2] which agrees very well with experiment. However, the same authors also attempted a calculation of the decay $K \to 2\pi + l + \nu$ ($l = e$ or μ), and concluded that the final-state π-π interaction makes the K_{l4} form factors vary too rapidly to allow a meaningful confrontation of PCAC and CCR with present experimental data on K_{e4} decay.

This note will show that Callan and Treiman were too pessimistic, and that in fact using the full strength of PCAC and CCR allows an unambiguous calculation for all K_{l4} form factors, with results in excellent agreement with experiment. Final-state interactions are neglected throughout, so the success of this calculation not only scores another victory for PCAC and CCR, but also strongly supports the recent prediction of small π-π scattering lengths.[3]

The axial-vector K_{l4} form factors F_1, F_2, and F_3 are defined by[4]

$$\langle \pi_{qa}\pi_{pb}|A_n^{\lambda}(0)|K_{km}\rangle = i(2\pi)^{-9/2}(8q^0p^0k^0)^{-1/2}(g_V/m_K)\{(q+p)^{\lambda}F_1 + (q-p)^{\lambda}F_2 + (k-p-q)^{\lambda}F_3\}. \quad (1)$$

Here k^{λ} and q^{λ}, p^{λ} are the momenta of the K meson and π mesons, $m = \pm\frac{1}{2}$ is the I_3 value of the K meson, a and b are the pion isovector indices, A_n^{λ} is the $\Delta S = -1$, $\Delta I_3 = -n$ axial-vector current, and $g_V = 1.02\times10^{-5}m_p^{-2}$ is the weak coupling constant. The form factors F_i are functions of $k\cdot q$, $k\cdot p$, $p\cdot q$, a, b, m, and n. Treiman and Callan let one of the pion momenta leave the mass shell and approach zero, and found that the value of F_3 (and, they presumed, of F_1 and F_2) depends sensitively on whether $q\to0$ or $p\to0$.

In order to see what is really going on here we must exploit PCAC and CCR more systematically, by taking both pions off the mass shell.[5] We will extend the form factors to functions of q^2 and p^2, as well as the other scalars and the isospin indices, by defining

$$F_{\pi}^{-2}m_{\pi}^{-4}(q^2+m_{\pi}^2)(p^2+m_{\pi}^2)\int d^4x\,d^4y\,e^{-iq\cdot x}e^{-ip\cdot y}\langle 0|T\{\partial_{\mu}A_a^{\mu}(x), \partial_{\nu}A_b^{\nu}(y), A_n^{\lambda}(0)\}|K_{km}\rangle$$
$$\equiv i(2\pi)^{-3/2}(2k^0)^{-1/2}(g_V/m_K)[(q+p)^{\lambda}F_1 + (q-p)^{\lambda}F_2 + (k-p-q)^{\lambda}F_3], \quad (2)$$

where $A_a^{\mu}(x)$ is the $\Delta S = 0$ axial-vector current, and F_{π} is the pion decay amplitude, defined by

$$\langle 0|\partial_{\mu}A_a^{\mu}(0)|\pi_{pb}\rangle \equiv F_{\pi}m_{\pi}^2(2\pi)^{-3/2}(2p^0)^{-1/2}\delta_{ab}. \quad (3)$$

This definition of the off-mass-shell form factors ensures[6] that, when $q^2 = p^2 = -m_{\pi}^2$, the F_i reduce exactly to the physical form factors appearing in Eq. (1). The PCAC assumption states that, despite the factor $(q^2+m_{\pi}^2)(p^2+m_{\pi}^2)$ in Eq. (2), the form factors defined by (2) vary slowly with q^{μ} and p^{ν}. We will calculate the