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[14]Assuming two-body breakup, the same laboratory angular distribution, and correcting for the photon spectral shape and kinematic compaction.

[15]M. Gell-Mann, M. Goldberger, and W. Thirring, Phys. Rev. 95, 1612 (1954), give $\sigma_{\text{int}} = 60(NZ/A)[1 + 0.1A^2/NZ]$ MeV mb.

PHYSICAL SIGNIFICANCE OF OPTICAL-MODEL PARAMETERS*

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A recent optical-model analysis of 30-MeV proton-scattering data[1] indicated that the radius parameter for the spin-orbit interaction was approximately 10% less than that for the real central interaction. A similar result has been noted at 10,[2] 14,[3] 18,[4] and 40 MeV.[5] At 30 MeV the averaged radius and diffuseness parameters for nuclei with A from 40 to 208 were 1.20 F, 0.7 F for the real central potential, and 1.10 F, 0.7 F for the spin-orbit potential, using a Saxon-Woods form and a Thomas form, respectively.

The difference between the radii for the real central potential and the spin-orbit potential of the optical model can be interpreted in terms of the interaction of the incident proton with the nuclear matter distribution via the two-body nucleon-nucleon force. To do this it is necessary to recognize the particular components of the two-body force giving rise to the two potentials and to adopt an appropriate folding procedure. In a first approximation neglecting target polarization and exchange effects, the folding procedure for the real central potential consists essentially of adding mean-square radii[6] with the dominant contribution coming from the "direct" (spin- and isospin-independent) part of the nucleon-nucleon potential. Phenomenological two-body potentials which are commonly accepted have mean-square radii for the attractive part of the "direct" component in the range 2.5-3.5 F^2. The precise value within this range is not critical for the present purpose and a value of 3 F^2 is taken which is the mean-square radius appropriate to a two-pion exchange mechanism.[7]

An indication that the approximations involved here are reasonable can be obtained from a consideration of alpha-alpha scattering where a great deal has been done using the resonating-group formalism.[8] In this case, using fully antisymmetrized wave functions, the effective interaction between the two alpha clusters is given by a direct term and an exchange term. The direct term represents a local potential which arises from the direct part of the nucleon-nucleon potential and has a mean-square radius equal to the sum of the mean-square radii of the two alpha particles and the mean-square radius of the two-body potential. The exchange term, on the other hand, represents a nonlocal potential with a kernel which is l dependent.

These results have been used by Ali and Bodmer[9] to construct phenomenological alpha-alpha potentials for $l = 0$, 2, and 4 which fit the relevant phase shifts for center-of-mass energies up to about 20 MeV. These potentials consist of an attractive and a repulsive part. The attractive part is l independent and of significantly longer range than the repulsive part which depends upon the l value. Furthermore, the tail of the attractive part of the alpha-alpha potential corresponds to a central spin- and isospin-independent part of the nucleon-nucleon force with a range close to that for a two-pion exchange mechanism.

The resonating-group formalism, upon which these results are based, neglects the effects of mutual distortion of the alpha particles. This is not a serious limitation as far as the mean-square radius of the alpha-alpha potential is concerned since Herzenberg and Roberts[10] have shown that the polarization potential resulting from this mutual distortion has a range similar to that of the exchange part (i.e., of shorter range than the direct term) and is relatively small in magnitude.

Thus for the alpha-alpha potentials the direct term has an appreciably longer range than the exchange and polarization terms. It seems reasonable to expect that a similar circumstance exists in the nucleon-nucleus case where, in addition, it has been estimated by Drell[11] that the magnitude of these terms is also rather small. Since the primary concern in the present study is with mean-square radii which depend mainly on the long-range part of the potential, a relatively crude treatment of the exchange and polarization potentials should not seriously affect the conclusions of this investigation, and the folding procedure outlined above should be satisfactory.

The spin-orbit part of the optical potential can arise from both the two-body spin-orbit and the tensor components of the nucleon-nucleon force. However, calculations in both nuclear matter[12] and finite nuclei[13] indicate that the tensor contribution to spin-orbit splitting is small and may be neglected in first approximation. Phase-shift analyses show the two-body spin-orbit force to be of very short range which is in agreement with an origin in a vector-meson exchange term.[14] The mass of the vector meson is known to be about 800 MeV, yielding a mean-square radius about 10% of the two-pion term. It is therefore reasonable, for the present purpose, to take zero range for the two-body spin-orbit force. In these circumstances it has been shown that the appropriate one-body spin-orbit force is of a Thomas derivative type using the matter distribution.[15]

Writing the mean-square radii for (1) the matter distribution, (2) the optical-model real central potential, and (3) the Saxon-Woods shape used for the spin-orbit term as $\langle R_M^2\rangle$, $\langle R_R^2\rangle$, and $\langle R_{SO}^2\rangle$, respectively, the following relations should be a good approximation:

$$\langle R_R^2\rangle = \langle R_M^2\rangle + 3, \quad \langle R_{SO}^2\rangle = \langle R_M^2\rangle .$$

Rather than using the averaged geometries of Ref. 1, which in the present view must be an approximation, the radius parameters were optimized for each element. This was done by searching on the optical-potential strengths for a χ^2 minimum using a range of values for the radius parameters, keeping the diffuseness values constant at 0.7 F. The optimum was taken to be that corresponding to the minimum in the curve of χ^2 versus radius parameter for each element. Table I gives the results of these searches, and columns 5 and 6 give $\langle R_R^2\rangle - 3$ and $\langle R_{SO}^2\rangle$ which are both equal to $\langle R_M^2\rangle$ in the present view. With the exception of Ca^{40}, the agreement between the two columns is quite satisfactory. The errors quoted in Table I (columns 5 and 6) are derived from an examination of the sharpness of the minima in the plots of χ^2 versus radius parameter. In no case (excluding Ca^{40}) is the difference between the two values for the matter mean-square radius greater than 40% of the error expected for independent measures of the same quantity with the quoted errors, and, on average, the difference is only 20% of the error. This implies a very close correlation between the optimum real central mean-square radius and the spin-orbit mean-square radius of the optical potential for each element. The disagreement for Ca^{40} is not surprising, since, in all analyses using Ca^{40} data, the optical-model fits are relatively unsatisfactory; it is therefore not a suitable case for study in this way.

Analyses of electron-scattering data provide values for the nuclear-charge mean-square radii. The proton distribution $\langle R_p^2\rangle$ is readily obtained from these by subtracting the proton mean-square radius (0.6 F^2). Table II compares the proton root-mean-square radius values with the average of the root-mean-square matter-distribution radii determined from the

Table I. Comparison of the mean-square matter radii as obtained from the optical-model real central and spin-orbit potential shapes. The radius parameters (columns 2 and 3) are the optimum values using a diffuseness parameter of 0.7 F.

Nucleus	r_R (F)	r_{SO} (F)	$\langle R_R^2\rangle$ (F^2)	$\langle R_M^2\rangle$ (F^2) $\langle R_R^2\rangle - 3$	$\langle R_{SO}^2\rangle$	$\langle R_M^2\rangle_{av}^{1/2}$ (F)
Ca^{40}	1.19	1.152	16.71 ±0.9	13.71 ±0.9	16.08 ±1.0	3.86 ±0.1
Ni^{58}	1.19	1.015	19.50 ±0.9	16.5 ±0.9	16.03 ±1.7	4.04 ±0.12
Co^{59}	1.19	1.057	19.64 ±0.9	16.64 ±0.9	16.93 ±1.7	4.10 ±0.12
Ni^{60}	1.19	1.049	19.79 ±0.9	16.79 ±0.9	16.89 ±1.7	4.10 ±0.12
Sn^{120}	1.22	1.172	28.50 ±1.6	25.50 ±1.6	26.82 ±3.0	5.12 ±0.17
Pb^{208}	1.21	1.135	37.61 ±2.5	34.61 ±2.5	33.9 ±5.0	5.86 ±0.23

Table II. Comparison of root-mean-square matter and proton radii, in fermis. The matter radii are obtained from the optical model (see text) and the proton radii are deduced from the electron-scattering radii.[a] For Ni^{60} and Sn^{120} the proton radii for Ni^{58} and Sb^{122} are used.

Nucleus	$\langle R_M^2\rangle_{av}^{1/2}$	$\langle R_p^2\rangle^{1/2}$	$\langle R_M^2\rangle_{av}^{1/2} - \langle R_p^2\rangle^{1/2}$	$\frac{N-Z}{A}$
Ni^{58}	4.04 ± 0.12	3.86 ± 0.12	0.18 ± 0.17	0.035
Ni^{60}	4.10 ± 0.12	3.86 ± 0.12	0.24 ± 0.17	0.069
Co^{59}	4.10 ± 0.12	3.74 ± 0.10	0.36 ± 0.16	0.085
Sn^{120}	5.12 ± 0.17	4.57 ± 0.13	0.55 ± 0.21	0.167
Pb^{208}	5.86 ± 0.23	5.37 ± 0.15	0.49 ± 0.27	0.211

[a]L. R. B. Elton, Nuclear Sizes (Oxford University Press, London, 1961).

optical-model parameters (column 7, Table I) for the five cases with $A = 58\text{-}208$. It is seen that there is a significant difference between $\langle R_M^2\rangle^{1/2}$ and $\langle R_p^2\rangle^{1/2}$, a difference which increases with the neutron excess of the nuclei. This is interpreted as indicating that the neutron distribution in medium to heavy nuclei extends beyond that of the protons. Figure 1 shows a plot of the difference between the matter and proton root-mean-square radii as a function of $(N-Z)/A$.

The arguments presented here indicate a close connection between optical-model mean-square radii. The normal parametrization of this model involves radius and diffuseness parameters associated with Saxon-Woods form factors. A variety of parameter sets, for the real central potential, exist which fit various data at different energies. It is a noticeable feature of these parameter sets that where a smaller radius parameter is used, a larger diffuseness parameter is needed, and vice versa; this tends to keep the mean-square radius more nearly constant than the radius parameter itself. A preliminary study has been made in which optimum fits to the 30-MeV experimental data were obtained for a range of values of r_R; the results indicate that for equivalent fits, the variation of $\langle R_R^2\rangle^{1/2}$ is significantly less than the variation of r_R. This is currently being explored in more detail.

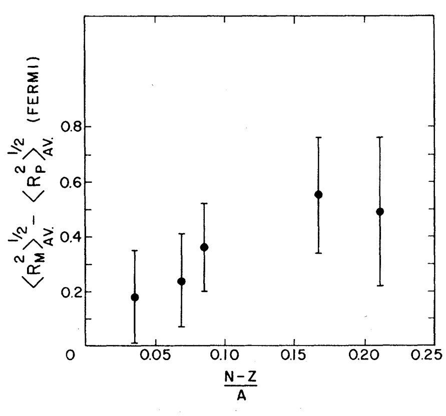

FIG. 1. The variation of the difference between the matter and proton root-mean-square radii with $(N-Z)/A$.

The approximation used here should be increasingly valid as the incident energy increases up to energies of around 100 MeV.

In conclusion, it is suggested that (1) the difference between the parametrization required for the optical-model real central and spin-orbit potentials is readily interpreted in terms of the two-body force range; (2) nuclear matter radii can be determined from accurate elastic-scattering and polarization measurements using protons (or neutrons) in the energy range 30-100 MeV; (3) the neutron distribution of nuclei extends beyond the proton distribution; (4) the significant quantity determined by optical-model analyses of nucleon-scattering data is the mean-square radius rather than the shape of the potentials.

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SPIN-SPIN INTERACTION IN THE TOTAL CROSS SECTION FOR POLARIZED 7.85-MeV NEUTRONS ON POLARIZED Ho^{165}†

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It has been suggested[1] that the optical potential describing the interaction between a proton or neutron and a heavy nucleus should include the term[2] $-V_{SS}F(r)\vec{I}\cdot\vec{\sigma}/I$, where $\vec{I}$ is the nuclear spin and $\vec{\sigma}$ the spin of the incident nucleon. Davies and Satchler[3] have given a theoretical investigation of the effects of such a term using the distorted-wave-Born-approximation (DWBA) approach, and Khan[4] has attempted to explain a systematic difference in neutron s-wave strength functions for even- and odd-A nuclei by invoking such a term. As yet, no direct experimental evidence exists for the inclusion of a spin-spin interaction in the optical potential, although Wagner et al.[5] have established an upper limit for V_{SS} by a measurement of the total cross section for polarized neutrons of energy $E_n = 0.350$ MeV incident on polarized Ho^{165} nuclei.

We report here on a search for the spin-spin interaction in the total cross section for 7.85-MeV neutrons on Ho^{165}. This investigation is a natural extension of previous work with a polarized Ho^{165} target and an unpolarized neutron beam, and much of the experimental apparatus has been described.[6] This includes the SCONT cryostat, the neutron-detection system, and the Ho^{165} sample. Figure 1 shows the present experimental arrangement.

The reaction $Be^9(\alpha, n)C^{12}$ was used as a source of polarized neutrons with average energy 7.85 MeV and energy spread 0.33 MeV. At $\theta = 45°$, the average neutron polarization[7] was $P_n = 0.34 \pm 0.02$. Magnet M in Fig. 1 was designed to produce a 90° precession of the neutron spins. Shielding around the pole pieces provided collimation to a half-angle of 1.0°. The polycrys-

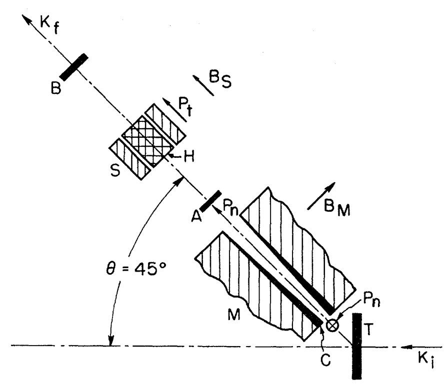

FIG. 1. Experimental arrangement. K_i, incident alpha-particle direction; T, Be^9 target; C, copper collimator; M, iron magnet; A, 80-mm^2×2-mm Si(Li) detector; B, stack of two 200-mm^2×2-mm Si(Li) detectors; S, superconducting solenoid; H, polycrystalline Ho^{165} cylinder; P_n, neutron polarization; P_t, Ho^{165} polarization; B_M, field in magnet M; B_S, field in solenoid S; K_f, neutron direction. The figure is not drawn to scale.