estimate of Eq. (5). Corrections to Eq. (5), which include the effect of a velocity component parallel to B_t, a finite electric dipole moment (E.D.M.) and possible time-average radial electric fields ($\bar{E}_r$), have been obtained by Ford and Hirt (University of Michigan, 1961, unpublished) and are presented in Ref. 2. For zero E.D.M. and negligible ($\bar{E}_r$),[9] these corrections come to less than 0.07% and will therefore be neglected, since our determination of "a" is only accurate to ±1%.

We conclude that although our value of "a" is higher than the theoretical value of 0.001 159 6 for the electron by somewhat more than one standard deviation, we see no basis for assuming that the positron and electron g factors are different. A more accurate experiment now being started should test for a real difference between the anomalies down to the 0.01% level. In the light of recent speculations concerning violations of fundamental symmetry principles, it is interesting to note that if TCP is conserved, $g_{\text{electron}}=g_{\text{positron}}$.

We would like to thank Professor R. R. Lewis, Professor G. Weinreich, and Professor K. M. Case for several stimulating discussions of the theory of the experiment. Dr. D. T. Wilkinson made a very significant contribution during the first stages of the experiment. The mechanical apparatus was constructed with remarkable skill by John Holden and George Miller. The members of our student staff, John Gilleland, Ralph Johnson, Andrew Sabersky, and John Wesley, also made outstanding contributions.

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[3]Throughout this paper $B_t=(B_{tz}^2+B_{tr}^2)^{1/2}$ will be used in place of B_{tz} since in the trap $B_{tr}/B_{tz}<10^{-3}$ or $(B_t-B_{tz})/B_{tz}<\frac{1}{2}\times10^{-6}$. B_{tr} is the radial field component.

[4]A comprehensive review article on the properties of positrons in solids by P. R. Wallace may be found in Solid State Phys. 10, 1 (1960).

[5]See A. Rich and H. R. Crane, in Proceedings of the International Conference on Positron Annihilation (Academic Press, Inc., New York, 1966) for a more complete discussion of the polarimeter. We believe this method of polarization analysis was first suggested by V. L. Telegdi, as cited by L. Grodzins, Progr. Nucl. Phys. 7, 219 (1959). It has since been used by several groups to measure positron polarization.

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[8]A. Rich, thesis, University of Michigan, 1965 (unpublished); a discussion of constants of the motion for a charged particle moving through a spatially slowly varying magnetic field may be found in, for example, J. D. Jackson, Classical Electrodynamics (John Wiley & Sons, Inc., New York, 1962), Chap. 12.

[9]$\bar{E}_r$ as measured by Wilkinson and Crane (Ref. 2) for geometry almost identical to ours was about −0.002 V/cm. In our experiment an $\bar{E}_r=0.002$ V/cm causes an apparent anomaly shift of 0.002%, so we have assumed this source of error to be negligible.

ELECTRIC-CHARGE FORM FACTOR ACCORDING TO SL(6, C)

G. Cocho,* C. Fronsdal,† Harun Ar-Rashid,‡ and R. White
International Atomic Energy Agency, International Centre for Theoretical Physics, Trieste, Italy
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Any meaningful relativistic generalization of SU(6) symmetry leads to noncompact groups and infinite multiplets, and hence, to practical difficulties in carrying out detailed calculations. This accounts for the long interval that has elapsed between the first proposal[1] to study the group

$$P\cdot\mathrm{SL}(6,C) \tag{1}$$

[semidirect product of the Poincaré group P with SL(6, C)] and the first complete calculation presented in this report. Previously we have calculated the strong Yukawa vertex,[2] but to lowest order in the momenta only. The weak and electromagnetic vertices are much simpler to evaluate, because only two infinite multiplets are involved. In fact, the calculation of all such vertices may be simply reduced to the calculation of a prototype. This latter, which may be tentatively identified with the charge form factor, is evaluated in this paper.

The baryons are assigned to a unitary, irreducible representation of the group (1). As in previous publications,[2-4] we use the representation whose tensorial basis is

$$\Psi^{\dot{B}_1,\cdots,\dot{B}_N}_{A_1,\cdots,A_{N+k}}(x)$$

with $k=3$ and $N=-\frac{9}{2}$. The exact form of the electromagnetic vertex is not yet known. However, except for the possible existence of correction terms with higher derivatives, the charge part of the interaction is related to the current

$$\bar{\Psi}^{A_1,\cdots,A_{N+k}}_{\dot{B}_1,\cdots,\dot{B}_N}(x)\overleftrightarrow{\partial}_\mu Q\Psi^{\dot{B}_1,\cdots,\dot{B}_N}_{A_1,\cdots,A_{N+k}}(x), \quad (2)$$

where Q is the usual SU(6) charge operator. In momentum space this is

$$(1/2m)(p+p')_\mu\bar{\Psi}^{A_1,\cdots,A_{N+k}}_{\dot{B}_1,\cdots,\dot{B}_N}(p')$$

$$\times Q\Psi^{\dot{B}_1,\cdots,\dot{B}_N}_{A_1,\cdots,A_{N+k}}(p).$$

To compute the matrix elements of this current between the physical baryons, we must replace $\Psi(p)$ by its projection onto the 56-dimensional representation of the little group $\mathrm{SU}(6)_p$, and $\bar{\Psi}(p')$ by its projection onto the 56-dimensional representation of $\mathrm{SU}(6)_{p'}$. This is accomplished by the substitutions[3]

$$\Psi^{\dot{B}_1,\cdots,\dot{B}_N}_{A_1,\cdots,A_{N+k}}(p)\to m^{-N}Sp^{\dot{B}_1}_{A_1}\cdots p^{\dot{B}_N}_{A_N}\Psi_{A_{N+1},\cdots,A_{N+k}}(p), \quad (3a)$$

$$\bar{\Psi}^{A_1,\cdots,A_{N+k}}_{\dot{B}_1,\cdots,\dot{B}_N}(p')\to m^{-N}S\bar{\Psi}^{A_1,\cdots,A_k}(p')p'^{A_{k+1}}_{\dot{B}_1}\cdots p'^{A_{N+k}}_{\dot{B}_N}, \quad (3b)$$

where S stands for symmetrization in $A_1,\cdots,A_{N+k}$, and

$$p^{\dot{B}}_A=(p_0-\vec{p}\cdot\vec{\sigma})^\beta_\alpha\delta^b_a,\quad p'^{B}_{\dot{A}}=(p_0'+\vec{p}'\cdot\vec{\sigma})^\beta_\alpha\delta^b_a.$$

The general form of the answer is

$$(1/2m)(p+p')_\mu\Psi^{ABC}[f_0\delta^{A'}_A\delta^{B'}_B\delta^{C'}_C+f_1\delta^{A'}_A\delta^{B'}_BT^{C'}_C+f_2\delta^{A'}_AT^{B'}_BT^{C'}_C+f_3T^{A'}_AT^{B'}_BT^{C'}_C]Q\Psi_{A'B'C'}, \quad (4)$$

where

$$T^B_A=m^{-2}p^{\dot{C}}_Ap'^{B}_{\dot{C}}.$$

We must now calculate the completely contracted product of the two tensors (3a) and (3b). It is sufficient to symmetrize with respect to the upper indices on $\bar{\Psi}^{\cdots}_{\cdots}$. Summing first over the positions of $A_1,\cdots,A_k$ we obtain immediately

$$\binom{N+k}{k}^{-1}\sum_{i=0}^{k}[\bar{\Psi}^{A_1,\cdots,A_k}(p')\Psi_{B_1,\cdots,B_iA_{i+1},\cdots,A_k}(p)]\binom{k}{i}\binom{N}{i}Q_{N\,A_1,\cdots,A_i}^{\;B_1,\cdots,B_i},$$

where

$$Q_{N\,A_1,\cdots,A_i}^{\;B_1,\cdots,B_i}\equiv m^{-2N}[S(pp')^{B_1}_{A_1}\cdots(pp')^{B_N}_{A_N}]\delta^{A_{i+1}}_{B_{i+1}}\cdots\delta^{A_N}_{B_N}, \quad (5)$$

and S stands as usual for symmetrization in the indices. Next we easily see that

$$Q_{N\,A_1,\cdots,A_i}^{\;B_1,\cdots,B_i}=\binom{N}{i}^{-1}S\sum T_{(a_1)A_1}^{\;B_1}\cdots T_{(a_i)A_i}^{\;B_i}Q_{N-a_1,\cdots,-a_i}, \quad (6)$$

where the sum is over all positive integer values of $a_1, \cdots, a_i$, subject to the condition $a_1 + \cdots + a_i \leq N$. [We complete the calculation for positive integral, but otherwise arbitrary, values of N, and continue the result analytically to $N = -\frac{9}{2}$.] Furthermore,

$$T_{(a)A}{}^{B} \equiv m^{-2a}[(pp')^a]_A^B = T_A^B z^{-1}(y^a - y^{-a}) - \delta_A^B z^{-1}(y^{a-1} - y^{-a+1}), \tag{7}$$

where

$$T_A^B = T_{(1)A}{}^{B} \equiv m^{-2} p_A^{\dot{C}} p'_{\dot{C}}{}^{B},$$

and

$$y = m^{-2}\{pp' + [(pp')^2 - m^4]^{1/2}\}, \quad z = y - y^{-1}.$$

Substituting (7) into (6) we obtain

$$Q_{N A_1, \cdots, A_i}^{B_1, \cdots, B_i} = \binom{N}{i}^{-1} \sum_{j=0}^{i} (-)^{i-j} \binom{i}{j} S T_{A_1}^{B_1} \cdots T_{A_j}^{B_j} \delta_{A_{j+1}}^{B_{j+1}} \cdots \delta_{A_i}^{B_i} Q_{N-i+j}^{(i)},$$

where $Q_N^{(i)}$ is defined recursively from Q_N by

$$Q_N^{(i)} \equiv z^{-1} \sum_{j=0}^{N-1} (y^{N-j} - y^{-N+j}) Q_j^{(i-1)}, \quad Q_N^{(0)} = Q_N.$$

The problem is thus reduced to the evaluation of Q_N, defined by (5) when $i = 0$, and this is the difficult part.

We calculated Q_N as follows: First we made a detailed calculation for several low integral values of N, and made an Ansatz for general N. Then we derived the recursion relation

$$Q_N = \frac{3}{N} \sum_{j=0}^{N-1} (y^{N-j} + y^{-N+j}) Q_k,$$

and verified the Ansatz by induction. The calculation of $Q_N^{(i)}$ was then considerably simplified when we discovered that

$$Q_N^{(i+1)} = (1/i+3)(\partial/\partial\omega) Q_N^{(i)},$$

where $\omega = y + y^{-1} + 2$. The result is

$$Q_{-\frac{9}{2}-i}^{(k)} = -\omega^{i-k-\frac{3}{2}} \sum_{\beta=0}^{i=1} (-)^\beta \binom{2i+2-\beta}{\beta} \binom{i+\frac{1}{2}-\beta}{k+2} \omega^{-\beta},$$

and the functions f_j in Eq. (4) are

$$f_j = \binom{N+k}{k}^{-1} \sum_{i=j}^{k} \binom{k}{i} \binom{i}{j} (-)^{i-j} Q_{N-i+j}^{(i)},$$

$$f_0 = (1/80)\omega^{-11/2}[9 - 20\omega + 15\omega^2 - 3\omega^3 - \omega^4],$$

$$f_1 = 3(1/80)\omega^{-11/2}[9 - 14\omega + 5\omega^2 + \omega^3],$$

$$f_2 = 3(1/80)\omega^{-11/2}[9 - 7\omega - \omega^2],$$

$$f_3 = (1/80)\omega^{-11/2}[9 + \omega].$$

Reducing Eq. (4) to a matrix element between proton states we obtain, in terms of the conventional two-component Foldy-Wouthuysen spinors,

$$\frac{1}{2m}(p+p')_\mu \left(1 - \frac{t}{4m^2}\right)^{-9/2} \times \left(\frac{(E+m)(E'+m)}{2E \cdot 2E'}\right)^{1/2} \chi^* \left[1 + \frac{(\vec{p}\cdot\vec{\sigma})(\vec{p}\cdot\vec{\sigma})}{(E+m)(E'+m)}\right] \chi,$$

where $t = (p-p')^2 < 0$ is the invariant momentum transfer.

The current (2) has a formal similarity with the Born approximation in local field theory, and we shall therefore refer to the approximation

$$J_\mu(p,p') = (1/2m)(p+p')_\mu \bar{\Psi}_{\dot{B}_1, \cdots, B_N}^{A_1, \cdots, A_{N+k}}(p') \times Q \Psi_{A_1, \cdots, A_{N+k}}^{\dot{B}_1, \cdots, \dot{B}_N}(p),$$

as the "local limit." This notion of locality is, of course, not the same as the locality of the conventional Dirac theory. The charge form factor $G_E(t)$ is defined by

$$J_\mu^{\text{Dirac}} = \frac{1}{2m}(p+p')_\mu\left(1-\frac{t}{4m^2}\right)^{-1}G_E(t) \times\left(\frac{(E+m)(E'+m)}{2E\cdot 2E'}\right)^{1/2}\chi^*\left[1+\frac{(\vec{p}\cdot\vec{\sigma})(\vec{p}\cdot\vec{\sigma})}{(E+m)(E'+m)}\right]\chi,$$

and Dirac locality means $G_E(t)_1=1$, whereas in our "local limit" $G_E(t)=[1-(t/4m^2)]^{-7/2}$. The principal feature of weak and electromagnetic interactions in a relativistic SU(6) theory is thus the appearance of damping at the vertex, which makes it possible to set up a highly convergent perturbation theory.[5] It turns out that this damping is essentially the same for all the states in the infinite multiplet, and hence for all values of the spin. The usual difficulties of higher spin theories, both of convergence and of positive definiteness, are completely absent.

In Fig. 1 we have shown $G_E(t)$ in both the conventional and the new local limits. The improvement over conventional local field theory is striking, especially at large momentum transfer. Of course, the low-momentum transfer region is known to be strongly affected by the nearby vector-meson poles, and there was no way for the vector meson to insinuate itself into our calculation.

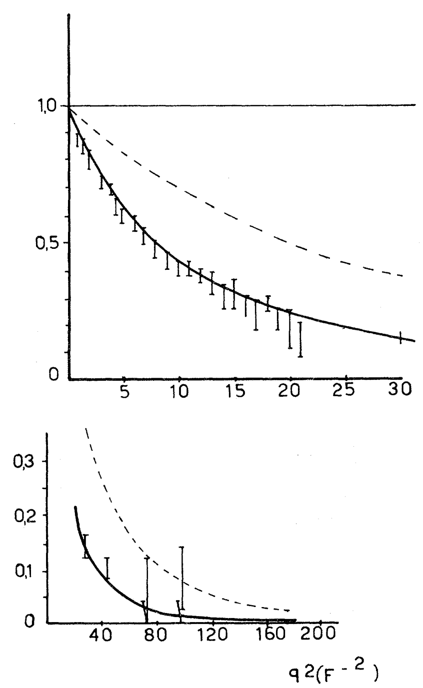

FIG. 1. The horizontal line represents $G_E(t)$ in the conventional local limit. The broken lines represent $G_E(t)$ in the new "local limit" suggested by the current (2). The solid curve is the form factor (8) with the "dynamical" form factor given by (9). Error bars indicate experimental points.[8]

Let us now define the "dynamical-charge form factor" $D(t)$ by writing the effective charge current as follows:

$$J_\mu(p,p') = (1/2m)(p+p')^{\bar{\Psi}\cdots}_{\mu\cdots}(p')Q^{\Psi\cdots}_{\cdots}(p)D(t).$$

Then

$$G_E(t) = [1-(t/4m^2)]^{-7/2}D(t). \tag{8}$$

The first factor is to be regarded as purely kinematical; we therefore attempt to approximate $D(t)$, rather than $G_E(t)$, by the contribution of the nearest vector-meson poles[6]:

$$D(t) = \tfrac{1}{6}\left[\frac{\varphi}{\varphi-t}+2\frac{\omega}{\omega-t}+3\frac{\rho}{\rho-t}\right]. \tag{9}$$

The resulting values of $G_E(t)$ are also shown in Fig. 1. The fit is excellent: thus the kinematical factor $[1-(t/4m^2)]^{-7/2}$ removes the discrepancy between the experimental vector-meson masses and the masses predicted by Frazer and Fulco.[7]

Note added in proof. —The result presented here is not immediately relevant for the process of annihilation through a virtual one-photon state. In fact the direct calculation of the virtual annihilation process gives the following result:

$$\frac{1}{2m}(p-p')_\mu\left(\frac{t}{4m^2}\right)^{-9/2}\left(\frac{E+m}{2E}\frac{E'+m}{2E'}\right)^{1/2} \times\chi_C\left[\frac{\vec{p}\cdot\vec{\sigma}}{E+m}+\frac{\vec{p}'\cdot\vec{\sigma}}{E'+m}\right]\chi,$$

with $t=(p+p')^2>4m^2$, which is perfectly well behaved at threshold. The details of a manifestly crossing symmetric formulation, suitable for the development of a field theory, will be presented in a later publication.

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*On leave of absence from the Instituto de Fisica, Universidad de México D. F., Mexico.

†On leave of absence from University of California,

Los Angeles, California.
‡On leave of absence from Atomic Energy Center, Dacca, East Pakistan.

[1]P. Budini and C. Fronsdal, Phys. Rev. Letters 14, 968 (1965).

[2]C. Fronsdal and R. White, to be published.

[3]C. Fronsdal, in Proceedings of the Seminar on High-Energy Physics and Elementary Particles (International Atomic Energy Agency, Vienna, 1965).

[4]C. Fronsdal, "The Representations of $SL(n, C)$" (to be published).

[5]It is straightforward to write down a Lagrangian with a minimal, gauge-invariant electromagnetic interaction. Due to the strong formal similarity with conventional local field theory it is likewise easy to set up a perturbational scheme of calculation, complete with Feynman diagrams and Feynman rules.

[6]Here the squares of the masses of the vector mesons ρ, ω, and φ are represented by the particle labels. The details of this formula rest on the assumption that the photon is a U-spin scalar and that the ω-φ mixing angle is the standard one of 35.1°.

[7]W. Frazer and J. Fulco, Phys. Rev. Letters 2, 365 (1959); Phys. Rev. 117, 1609 (1960); V. S. Ball and D. Y. Wong, Phys. Rev. 130, 2112 (1963) have shown, with dynamical calculations, that due to the large width of the ρ resonance, the "effective" ρ pole may be shifted as far as to 625 MeV. However, this alone does not fit the data. If in the dynamical form factor (9), we take $m_\rho \simeq 625$-675 MeV instead of the physical value of 770 MeV, the agreement of the form factor (8) with the experimental data is even better.

[8]The experimental points of $G_E(t)$ are taken from E. B. Hughes, T. A. Griffy, M. R. Yearian, and R. Hofstadter, Phys. Rev. 139, B458 (1965); K. W. Chen, A. A. Cone, J. R. Dunning, Jr., S. G. Frank, N. F. Ramsey, J. K. Walker, and R. Wilson, Phys. Rev. Letters 11, 561 (1963); J. R. Dunning, K. W. Chen, A. A. Cone, G. Hartwig, N. F. Ramsey, J. K. Walker, and R. Wilson, Phys. Rev. Letters 13, 631 (1964).

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LINEAR INSTABILITY THEORY OF LASER PROPAGATION IN FLUIDS. K. A. Brueckner and S. Jorna [Phys. Rev. Letters 17, 78 (1966)].

There is an omission on page 80 of this paper. Below Eq. (7), the following line should be added:

$$E_0^2 = E_L^{(0)2} + E_S^{(0)2}, \quad \alpha = k_z/k_x,$$

$$\xi = 1 + 2n_2 k_0/\mu, \quad k^2 = k_x^2 + k_z^2.$$