[17]The same conclusion is drawn by S. K. Bose and S. N. Biswas, International Centre for Theoretical Physics Report No. IC/66/22. See also E. Ferrari, V. S. Mathur, and L. K. Pandit, to be published.

DIRECT MEASUREMENT OF THE g FACTOR OF THE FREE POSITRON

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We have just completed a determination of the g factor of the free positron. Writing $g=2(1+a)$ where "a" is the g-factor anomaly, we may express the result of our measurement as

$$a=0.001\,168\pm0.000\,011=\alpha/2\pi+(1.2\pm2)\alpha^2/\pi^2.$$

This is in agreement with the theoretical[1] and experimental[2] values of the free-electron anomaly which are given as

$$a_{\text{theor}}=0.001\,159\,615=\alpha/2\pi-0.328\alpha^2/\pi^2+\cdots,$$

$$a_{\text{expt}}=0.001\,159\,622\pm0.000\,000\,027$$
$$=\alpha/2\pi-(0.327\pm0.005)\alpha^2/\pi^2. \qquad (1)$$

The method we used to determine the g factor may be described in the following way: A "bunch" of positrons from a Co^{58} source is sent into a magnetic field (B_t) which is almost uniform but slightly bottle shaped. The positrons are trapped and held in the bottle for a measured length of time, and then ejected into a polarimeter. As emitted by the Co^{58} source, the positron beam is polarized parallel to its momentum. While in the magnetic field, the positrons execute cyclotron orbital motion of frequency ω_c, while their spins precess at frequency ω_s. The polarimeter determines the projection of the final polarization of the positron onto a fixed direction. This is proportional to $\cos[(\omega_c-\omega_s)T+\varphi]$, where T is the length of time spent in the bottle, and φ is a constant. The experiment consists of determining the final component of polarization at a number of trapping times T, and from that result, determining $\omega_c-\omega_s$, which we denote by ω_D. The g-factor anomaly, a, is simply $\omega_D mc/e\bar{B}_{tz}$. $\bar{B}_{tz}$ is the axial component of magnetic field averaged over the time the particle is in it.[3]

The experimental technique follows that already described for the measurement of the g factor of the free electron,[2] except for the way in which the polarization and the analysis of the polarization are accomplished. In the electron measurements, Mott scattering was used for both. In the positron measurements, no polarizing process is necessary. The polarimeter works in the following way: After removal from the trap, the positrons are stopped in a plastic scintillator in a 13-kG magnetic field (B), and measurements on their lifetime before annihilation are made by a delayed coincidence system. The lifetime distribution is dependent upon the component of polarization of the positrons in the direction of the magnetic field in which they are stopped. A more detailed treatment of the polarization analysis will follow the description of the experimental arrangement.

The experimental arrangement is shown in Fig. 1. The solenoid used to produce B_t is 11 ft long, 2 ft in diameter and wound with 8 layers of No. 10 SCHF magnet wire. Small auxiliary coils carrying the main solenoid current shape the field in the trapping and extraction regions. The source of polarized positrons is 600 mCi of Co^{58}. Only positrons with kinetic energy (200 ± 30) keV or $v/c=0.7\pm0.05$ (about 10% of the emission spectrum) are used. They are emitted in a 222-G field and move down the field (south-Fig. 1) with various axial velocities until they enter the trapping region. Here they are trapped by pulsing the inject cylinder 1000-V positive. This causes a loss of axial momentum as the gap is crossed so that a given fraction of the particles reflects from the right-hand magnetic potential hill. The pulse is removed before the reflected positrons recross the gap, so that they do not regain the lost momentum and become trapped. To eject, a positive 1000-V pulse is applied to the left-hand (ejection) cylinder. While the particles are trapped, all surfaces are at ground potential. The cylinders are brass, 12.5 cm in radius, and mounted concentrically within the 15-cm radius vacuum chamber.

After ejection, the positrons must be extracted from the solenoid and brought to the polar-

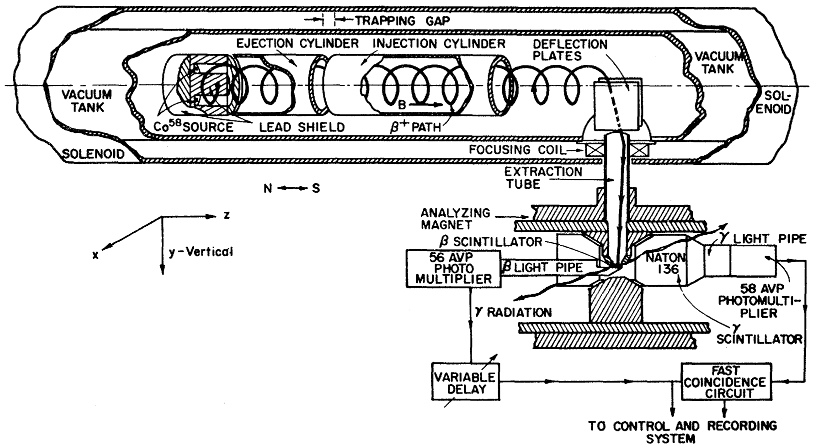

FIG. 1. Schematic diagram of the positron g factor experiment.

imeter. Extraction is accomplished by using the electric field (50 kV/cm) between two parellel deflection plates to cancel the magnetic force on the particles. After leaving the solenoid, a magnetic focusing coil is used to focus the beam into the polarimeter. The polarimeter consists of a magnet, a 1-mm-thick piece of Naton-136 plastic scintillator in which the positrons stop, a $3\times5\times7$-in. slab of Naton 136 to detect the 0.5-MeV annihilation radiation and a fast coincidence circuit. The magnet produces a field of 13 kG over a region 1 cm wide by 1 cm in diameter. This field is oriented approximately antiparallel to the final direction of positron momentum.

Inside the plastic, positrons thermalize within 10^{-10} sec, and about 40% form $n=1$ positronium (Ps), the remainder annihilating directly with molecular electrons.[4] We will show that the fraction of Ps atoms surviving past a given time t varies as $\vec{P}\cdot\vec{B}/|P||B| = \cos\omega_D T$. Thus the coincidence counting rate of a coincidence circuit which counts Ps decays (the 0.5-MeV annihilation γ's are detected) between t_1 and t_2 ($t=0$ is set by the pulse due to the positron stopping), various sinusoidally with T. The period, τ_D, of the cosine curve is calculated by measuring the time (ΔT) between two nodes separated by about 11 μsec and then dividing by the counted number of cycles (in this case $N=8$) between the nodes.

In order to analyze the operation of the polarimeter, consider the case of $\vec{P}$ making an arbitrary angle θ with $\vec{B}$.[5] In the subsequent discussion, spatial dependence of wave functions is suppressed; $\vec{B}$ defines the z (quantization) axis; the first and second arrows indicate positron and electron spin; and all phase factors have been omitted since they cancel when probabilities are computed. Concentrating on spin orientation, the states formed at $t=0$ with their respective probabilities are:

State	$\Psi^{\uparrow\downarrow} = \begin{pmatrix}\cos\frac{1}{2}\theta \\ \sin\frac{1}{2}\theta\end{pmatrix}\begin{pmatrix}-\sin\frac{1}{2}\theta \\ \cos\frac{1}{2}\theta\end{pmatrix}$	$\Psi^{\uparrow\uparrow} = \Psi_{1,1}$	$\Psi^{\downarrow\uparrow}$	$\Psi^{\downarrow\downarrow} = \Psi_{1,-1}$
Probability	$\frac{1}{4}(1+P)$	$\frac{1}{4}(1+P)$	$\frac{1}{4}(1-P)$	$\frac{1}{4}(1-P)$

The fraction of Ps atoms that survive past time t is then given by

$$f(\theta,t) = \int_{\text{space}} d^3\vec{r} \sum_{\text{spins}} \{\tfrac{1}{4}(1+P)[\Psi^{\uparrow\downarrow}(\Psi^{\uparrow\downarrow})^\dagger + \Psi^{\uparrow\uparrow}(\Psi^{\uparrow\uparrow})^\dagger] + \tfrac{1}{4}(1-P)[\Psi^{\downarrow\uparrow}(\Psi^{\downarrow\uparrow})^\dagger + \Psi^{\downarrow\downarrow}(\Psi^{\downarrow\downarrow})^\dagger]\}. \tag{2}$$

The integral may be evaluated for arbitrary t by expanding $\Psi^{\uparrow\downarrow}(t)$, $\Psi^{\downarrow\uparrow}(t)$ in terms of the eigenstates[5] of Ps in a magnetic field. The result is

$$f(\theta,t) = \tfrac{1}{4}\{2\exp(-\Lambda_{1,0}t) + \exp(-\Lambda_{1,0}'t) + \exp(-\Lambda_{0,0}'t) - P\cos\theta(1+x^2)^{-1/2}[\exp(-\Lambda_{1,0}'t) - \exp(-\Lambda_{0,0}'t)]\}, \tag{3}$$

where indices 1, 0 or 0, 0 represent S (total spin), m (z projection of spin) and $x = 4\mu_0 B/(E_{1,0}-E_{0,0}) = 0.35$ at 13 kG. The unperturbed singlet and triplet decay rates ($\Lambda_{0,0}$; $\Lambda_{1,0}$) are about 10^{10} $\sec^{-1}$ and 0.5×10^9 $\sec^{-1}$ while the field perturbed rates $\{\Lambda_{0,0}' = [1+y^2]^{-1}[\Lambda_{0,0}+y^2\Lambda_{1,0}]$, $\Lambda_{1,0}' = [1+y^2]^{-1}[\Lambda_{1,0}+y^2\Lambda_{0,0}]$, $y = x[1+(1+x^2)^{1/2}]^{-1}\}$ are 10^{10} $\sec^{-1}$ and 0.68×10^9 $\sec^{-1}$ at 13 kG. The prompt curve, $Q(t)$, of our coincidence circuit has a full width at half-maximum counting rate of about 4 nsec. The experimental distribution to be expected is given by

$$F(\theta, t_1, t_2) = \int_{t_1}^{t_2} dT \int_{-\infty}^{\infty} f(\theta, t) Q(T-t) dt, \qquad (4)$$

where t_1 is the time delay between positron thermalization and the opening of the coincidence gate. We took $t_1 = 3$ nsec, $t_2 = 16$ nsec. Evaluation of F via graphical integration including an assumed 60% direct annihilation loss yields $F(\theta, t_1, t_2) = (0.4\pm0.1)[1-(0.01\pm0.005)\cos\theta]$ in agreement with our final data.

The asymmetry amplitude in the above is somewhat smaller than would be obtained by a polarimeter employing Mott scattering. However the Ps polarimeter has an enormous advantage in this experiment (where intensity is a severe problem) in that about half of the positrons count. In a Mott scattering polarimeter only one particle in 10^4 or 10^5 is scattered at such an angle as to register. The counting time would be prohibitively long if the present experiment were to be done by using Mott scattering.

The curves in Fig. 2 show the normalized counting rate of the polarimeter versus the time the positrons spend in the trap. Final data to determine two widely separated maxima were obtained in two separate runs at trapping times (T.T.) of about 15 and 26 μsec. By 15 μsec

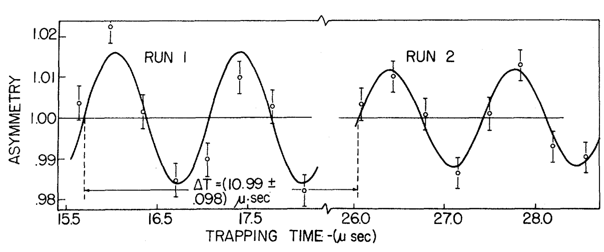

FIG. 2. The asymmetry data and least-squares fit (solid line), of the curve $Y/C = 1+(A/C)\cos(\omega T+\varphi)$. The value of C is about 40 000 counts for each run and the error flags are due to random counting statistics only.

the axial positions of positrons in the magnetic well are completely random, so that all bunching associated with injection is lost, i.e., there is no periodicity of beam rate versus T.T. In order to eliminate systematic variations in the coincidence counting rate not associated with the direction of $\vec{P}$, the T.T. was decreased (injection pulse advanced) by 0.354 μsec (about $\frac{1}{4}\tau_0$) each time 64 positrons were counted. After advancing seven positions the injection pulse returns to its starting point, and the cycle is repeated. The positron-coincidence counts are fed to a different scaler at each T.T. position. In addition, the total time spent at each position and the number of cycles is automatically recorded.

In order to determine ω_D, an IBM 7090 computer was used to obtain the least-squares fit of the form $Y = C + A\cos(\omega T+\varphi)$ to each run. Using the values of φ so obtained, ΔT was computed to be (10.99 ± 0.098) μsec, so that $\tau_D \pm \delta\tau_D = [(10.99\pm0.098)/8] = 1.374(1\pm0.009)$ μsec. Systematic error in ΔT (less than ±0.02 μsec) arises from electronic drifting during runs and from limited visual resolution of the calibrated oscilloscope pictures used to measure the 11.076-μsec change in trapping time between runs 1 and 2. Statistical error in ΔT(±0.096 μsec) is due to the random nature of the counting process which determins φ. The standard deviation in φ, (σ), may be estimated from the method of maximum likelihood[6] as $\sigma = 10$ and 7.5° for runs 1 and 2, respectively. An estimate of σ was also obtained by subdividing the data for each run into 9 consecutive groups, fitting a cosine to each group, and using the standard form

$$\sum_{i=1}^{n=q} (\varphi_i - \bar{\varphi})^2/n(n-1)$$

to estimate σ. This yielded $\sigma = 9$ and 4.5° for runs 1 and 2, respectively. In view of the entirely statistical nature of these uncertainties we have decided to use the so-called "limit of error," 2σ, together with the larger (maximum likelihood) estimates of σ, as the most realistic statement of phase error. The error in ΔT is obtained by taking the square root of the sum of the squares of the independent errors, i.e., $[(0.096)^2+(0.02)^2]^{1/2} = 0.098$ μsec.

Finally, we compute "a" as

$$a = 2\pi(e/m_0c)_{\text{positron}}^{-1}(\bar{B}_t\tau_D)^{-1} = 0.001\,168 \pm 0.000\,011, \qquad (5)$$

where again the square root of the sum of the squares of the independent errors in $\overline{B}_t$ and τ_D is used. Since[7] $(m_0/e)[(e/m_0)_{\text{electron}}-(e/m_0)_{\text{positron}}]=(26\pm71)\times10^{-6}$ and $(e/m_0)_{\text{electron}}$ and c are known to better than ten parts per million, error introduced by these constants is negligible. A limit of error for "a" may be arrived at by adding all independent errors and leads to $a=0.001\,168\pm0.000\,017$. Note that 95% of the error in "a" is due to the 1% statistical error in the counting process which is used to determine τ_D.

As in all of our g-factor measurements, it is crucial to determine the time average of the magnetic field acting upon the particle while it is in the trap, since $a=2\pi(e/m_0c)^{-1}(\overline{B}_t\tau_D)^{-1}$. We will therefore explain in detail how we estimate $\overline{B}_t$.

It can be shown[8] that if the orbits of the particles trapped in the magnetic bottle satisfy the condition $(v_z/v_\perp)(R/B_t)(\partial B_t/\partial z)\ll1$, the particles may be treated as if they are in an electric potential well of potential

$$U(z)=[T_\perp(z_0)/B_t(z_0)][B_t(z)-B_t(z_0)]=\text{constant}-\tfrac{1}{2}m_0v_z^{\,2}. \quad (6)$$

We have taken v_z and $v_\perp$ to represent velocity components parallel and perpendicular to B_t; R the average orbit radius (about 7.5 cm); z_0 an arbitrary reference point at which $U=0$ and $T_\perp(z_0)=\frac{1}{2}m_0v_\perp^2(z_0)[1+\frac{3}{4}(v_0^2/c^2)+\frac{5}{8}(v_0^4/c^4)+\cdots]\cong$ the positron kinetic energy. The constant $T_\perp(z_0)/B_t(z_0)$ is about 920 eV/G.

The time average field, $\overline{B}_t(z)$, seen by positrons trapped at various levels in the well, may be determined by the following equation:

$$\overline{B}_t(z_1,z_2)=[2/T(z_1,z_2)]\int_{z_1}^{z_2}B_t(z)dt=\int_{z_1}^{z_2}B_t(z)v_3^{-1}dz\Big/\int_{z_1}^{z_2}v_z^{-1}dz, \quad (7)$$

where z_1 and z_2 are the axial limits of motion for positrons at a given well depth, i.e., $v_z(z_1)=v_z(z_2)=0$, $B_t(z_1)=B_t(z_2)$, and $T(z_1,z_2)$ is the period of oscillation at this depth. The integral is evaluated graphically after expressing $v_z(z)$ in terms of $\overline{B}_t(z)$ [Eq. (6)]. The results for T and B_t are shown in Fig. 3 for a positron orbit radius of 7.5 cm. Since the injection pulse = 1000 V and the injection hill is 200 eV below the source peak at $z=287$ cm, the minimum well depth of trapped positrons at this radius

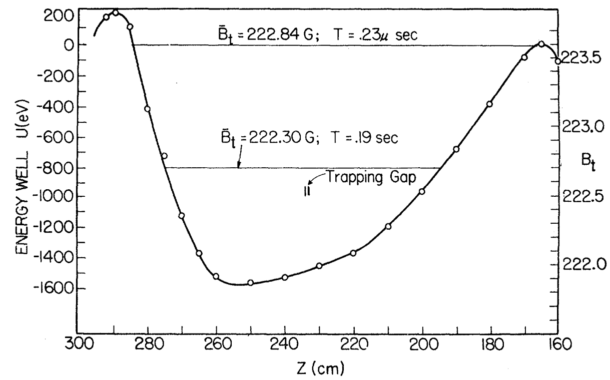

FIG. 3. The magnetic field in the trapping region averaged over eight azimuthal positions (45° intervals) at each value of z. The radius used was 7.5 cm. The values given for $\overline{B}_t$ and T are those corresponding to positrons trapped in the magnetic energy well at the levels indicated by the solid lines. The zero of the z scale corresponds to the south end of the solenoid.

is −800 eV, so $222.30<\overline{B}_t<222.84$, or $\overline{B}_t=222.57\pm0.27$ G. Similar calculations at the maximum and minimum orbit radii (9.5 and 5.5 cm) yield $\overline{B}_t=222.52\pm0.35$ G. The field was mapped and controlled (short- and long-term drift) to better than 0.01% by means of proton resonance devices which are fully described in Ref. 2.

Systematic error will be defined as any error not susceptible to unambiguous estimation. Such error can occur in $\overline{B}_t$ or τ_D. Since $\overline{B}_t$ could be measured and controlled to a hundred times the accuracy of our final result, the effect of systematic error in it such as field drift, the earth's field variation, magnetic contamination of the mapping apparatus, vacuum chamber, etc., is considered completely negligible.

Systematic error in τ_D can arise from any process other than the anomalous magnetic moment which causes the final direction of the polarization to depend on trapping time. Such effects can occur as the particles pass through the electric field of the extraction system, and as they enter the inhomogeneous field of the analyzing magnet, if the average beam trajectory is a function of trapping time. Drifting of the orbit center in the magnetic bottle is a mechanism which causes such trajectory variations. Thus, the beam experiences different deflection electric fields and may enter the field of the analyzing magnet at slightly varying angles as trapping time changes. A detailed analysis of these effects has been carried out[8] and we conclude that their effect on τ_D is less than 0.1%. This has not been included in the error estimate of Eq. (5). Corrections to Eq. (5), which include the effect of a velocity component parallel to B_t, a finite electric dipole moment (E.D.M.) and possible time-average radial electric fields ($\bar{E}_r$), have been obtained by Ford and Hirt (University of Michigan, 1961, unpublished) and are presented in Ref. 2. For zero E.D.M. and negligible ($\bar{E}_r$),[9] these corrections come to less than 0.07% and will therefore be neglected, since our determination of "a" is only accurate to ±1%.

We conclude that although our value of "a" is higher than the theoretical value of 0.001 159 6 for the electron by somewhat more than one standard deviation, we see no basis for assuming that the positron and electron g factors are different. A more accurate experiment now being started should test for a real difference between the anomalies down to the 0.01% level. In the light of recent speculations concerning violations of fundamental symmetry principles, it is interesting to note that if TCP is conserved, $g_{\text{electron}} = g_{\text{positron}}$.

We would like to thank Professor R. R. Lewis, Professor G. Weinreich, and Professor K. M. Case for several stimulating discussions of the theory of the experiment. Dr. D. T. Wilkinson made a very significant contribution during the first stages of the experiment. The mechanical apparatus was constructed with remarkable skill by John Holden and George Miller. The members of our student staff, John Gilleland, Ralph Johnson, Andrew Sabersky, and John Wesley, also made outstanding contributions.

[1]C. M. Sommerfield, Phys. Rev. 107, 328 (1957); A. Petermann, Nucl. Phys. 5, 677 (1958).

[2]D. T. Wilkinson and H. R. Crane, Phys. Rev. 130, 852 (1963).

[3]Throughout this paper $B_t = (B_{tz}^2 + B_{tr}^2)^{1/2}$ will be used in place of B_{tz} since in the trap $B_{tr}/B_{tz} < 10^{-3}$ or $(B_t - B_{tz})/B_{tz} < \frac{1}{2} \times 10^{-6}$. B_{tr} is the radial field component.

[4]A comprehensive review article on the properties of positrons in solids by P. R. Wallace may be found in Solid State Phys. 10, 1 (1960).

[5]See A. Rich and H. R. Crane, in Proceedings of the International Conference on Positron Annihilation (Academic Press, Inc., New York, 1966) for a more complete discussion of the polarimeter. We believe this method of polarization analysis was first suggested by V. L. Telegdi, as cited by L. Grodzins, Progr. Nucl. Phys. 7, 219 (1959). It has since been used by several groups to measure positron polarization.

[6]L. G. Parratt, Probability and Experimental Errors in Science (John Wiley & Sons, Inc., New York 1961), Chap. 3. Since we took data at 92° intervals, the error matrix was almost diagonal. The second derivative of the natural logarithm of the likelihood function was, therefore, equal to $-1/\sigma^2$ to better than 1%.

[7]L. A. Page, P. Stehle, and S. B. Gunst, Phys. Rev. 89, 1273 (1953).

[8]A. Rich, thesis, University of Michigan, 1965 (unpublished); a discussion of constants of the motion for a charged particle moving through a spatially slowly varying magnetic field may be found in, for example, J. D. Jackson, Classical Electrodynamics (John Wiley & Sons, Inc., New York, 1962), Chap. 12.

[9]$\bar{E}_r$ as measured by Wilkinson and Crane (Ref. 2) for geometry almost identical to ours was about −0.002 V/cm. In our experiment an $\bar{E}_r = 0.002$ V/cm causes an apparent anomaly shift of 0.002%, so we have assumed this source of error to be negligible.

ELECTRIC-CHARGE FORM FACTOR ACCORDING TO SL(6, C)

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Any meaningful relativistic generalization of SU(6) symmetry leads to noncompact groups and infinite multiplets, and hence, to practical difficulties in carrying out detailed calculations. This accounts for the long interval that has elapsed between the first proposal[1] to study the group

$$P \cdot \mathrm{SL}(6, C) \tag{1}$$

[semidirect product of the Poincaré group P with SL(6, C)] and the first complete calculation presented in this report. Previously we have calculated the strong Yukawa vertex,[2] but to lowest order in the momenta only. The weak and electromagnetic vertices are much simpler to evaluate, because only two infinite multiplets are involved. In fact, the calculation of all such vertices may be simply reduced to the calculation of a prototype. This latter, which may be tentatively identified with the charge form factor, is evaluated in this paper.

The baryons are assigned to a unitary, irre-