CURRENT COMMUTATION RELATIONS, UNIVERSALITY, AND NONLEPTONIC BARYON DECAYS

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The purpose of this note is to extend Cabibbo's successful phenomenological form of universal weak interaction for leptonic decays to nonleptonic s- and p-wave decays of baryons assuming the validity of the results obtained by Sugawara[1] and Suzuki[2] and Hara, Nambu, and Schechter[3] for the parity-nonconserving and parity-conserving amplitudes, respectively. These results are based on the assumptions (a) partial conservation of axial-vector current, (b) commutation relations of the current components,[4] (c) unsubtracted dispersion relations, and (d) CP-even Cabibbo weak Hamiltonian with the currents transforming like the components of an octet. To achieve a relation between the leptonic and nonleptonic decays, we introduce a complete set of intermediate states and exhaust these contributions by keeping only the single-baryon (B) and one-meson, one-baryon states. The latter are approximated by the decuplet resonances (B^*). This procedure immediately enables us to obtain the nonleptonic decay amplitudes in terms of the leptonic ones and the electromagnetic form factors.[5] The form factors associated with $\langle B^*|J_{A,V}|B\rangle$ are taken from the calculation of Albright and Liu[6] for $\nu+B\rightarrow B^*+l$ reaction cross section. Thus the only unknown parameter in the present calculation is the D/F ratio for weak and strong interactions. With these as input and a universal D/F ratio, we conclude that Cabibbo's form of weak current is capable of explaining both leptonic and nonleptonic decays of baryons.[7]

S-wave amplitudes. — Following Sugawara[1] and Suzuki,[2] the s-wave nonleptonic decay amplitude may be written as

$$A(B^{\nu_1}\Pi^{\nu_3}|B^{\nu_2})=\langle B^{\nu_1}\Pi^{\nu_3}|H_{\text{p.v.}}{}^{\nu_4\nu_5}(0)|B^{\nu_2}\rangle=\frac{\mu^2}{c(2k_0)^{1/2}}\left[\left(\begin{array}{cc|c}8&8&8f\\ \nu_3&\nu_4&\nu_6\end{array}\right)\langle B^{\nu_1}|H_{\text{p.c.}}{}^{\nu_5\nu_6}(0)|B^{\nu_2}\rangle+(\nu_4\leftrightarrow\nu_5)\right]. \quad (1)$$

With the approximation scheme outlined above we write down the s-wave amplitude as[8]

$$\begin{aligned}A(B^{\nu_1}\Pi^{\nu_3}|B^{\nu_2})=\frac{\mu^2}{c(2k_0)^{1/2}}\Bigg[&\left(\begin{array}{cc|c}8&8&8f\\ \nu_3&\nu_4&\nu_6\end{array}\right)\times\sum_{\nu}\Bigg\{\left(\begin{array}{cc|c}8&8&8f\\ \nu_5&\nu&\nu_1\end{array}\right)\left(\begin{array}{cc|c}8&8&8f\\ \nu_6&\nu_2&\nu\end{array}\right)\langle B^{\nu_1}\|V_\mu{}^{\nu_5}\|B^{\nu}\rangle\langle B^{\nu}\|V_\mu{}^{\nu_6}\|B^{\nu_2}\rangle\\&+\left(\begin{array}{cc|c}8&10&8\\ \nu_5&\nu&\nu_1\end{array}\right)\left(\begin{array}{cc|c}8&8&10\\ \nu_6&\nu_2&\nu\end{array}\right)\langle B^{\nu_1}\|V_\mu{}^{\nu_5}\|B^{*\nu}\rangle\langle B^{*\nu}\|V_\mu{}^{\nu_6}\|B^{\nu_2}\rangle+\left(\begin{array}{cc|c}8&8&8d+8f\\ \nu_6&\nu&\nu_1\end{array}\right)\left(\begin{array}{cc|c}8&8&8d+8f\\ \nu_5&\nu_2&\nu\end{array}\right)\\&\times\langle B^{\nu_1}\|A_\mu{}^{\nu_6}\|B^{\nu}\rangle\langle B^{\nu}\|A_\mu{}^{\nu_5}\|B^{\nu_2}\rangle+\left(\begin{array}{cc|c}8&10&8\\ \nu_6&\nu&\nu_1\end{array}\right)\left(\begin{array}{cc|c}8&8&10\\ \nu_5&\nu_2&\nu\end{array}\right)\langle B^{\nu_1}\|A_\mu{}^{\nu_6}\|B^{*\nu}\rangle\\&\times\langle B^{*\nu}\|A_\mu{}^{\nu_5}\|B^{\nu_2}\rangle+(\nu_5\leftrightarrow\nu_6)\Bigg\}+\left(\begin{array}{cc|c}8&8&8f\\ \nu_3&\nu_5&\nu_6\end{array}\right)\times\sum_{\nu}\{(\nu_4\leftrightarrow\nu_5)\}\Bigg]. \end{aligned}\quad (2)$$

The various reduced matrix elements in (2) are

$$\langle B(p_1)\|V_\mu+A_\mu\|B(p_2)\rangle=\bar{u}(p_1)\left[\gamma_\mu(F_1{}^V+\gamma_5F_1{}^A)+\frac{i\sigma_{\mu\nu}q_\nu}{2m_1}F_2{}^V+\frac{iq_\mu\gamma_5}{2m_1}F_2{}^A\right]u(p_2), \quad (3)$$

$$\langle B_\lambda{}^*(p_1)\|V_\mu+A_\mu\|B(p_2)\rangle=\bar{\psi}_\lambda(p_1)\left[\delta_{\lambda\mu}(G_1{}^A+G_1{}^V\gamma_5)+\frac{ip_{1\lambda}\gamma_\mu}{M_1}(G_2{}^A+G_2{}^V\gamma_5)\right]u(p_2). \quad (4)$$

Here, we have assumed that all the form factors occurring in (3) and (4) have the following structure[9]:

$$F^{V,A}(q^2)=F^{V,A}(0)/(1+q^2/b)^2, \tag{5}$$

where $b=37.4\ \mu^2$. The static values of these form factors are[5,6]

$$F_1^V(0)=1.0,\quad F_2^V(0)=1.85,\quad F_1^A(0)=-1.2,$$
$$F_2^A(0)=0,\quad G_1^V(0)=3.5,\quad G_2^V(0)=-1.5,$$
$$G_1^A(0)=-0.83,\quad G_2^A(0)=0. \tag{6}$$

Using Eqs. (3) and (4) for the reduced matrix elements in (1) and performing the summation over the intermediate spins and momenta, we obtain the s-wave amplitudes. We find in our approximation scheme that the $|\Delta I|=\frac{1}{2}$ rule is valid and the Sugawara sum rule,[1]

$$2A(\Lambda\pi^-|\Xi^-)-A(p\pi^-|\Lambda)=(\tfrac{3}{2})^{1/2}A(n\pi^-|\Sigma^-), \tag{7}$$

also holds. With $D/F=-1.43$ for axial-vector current[10] our results (Table I) compare very well with the experimental results[11] and the recent theoretical predictions of Ref. 3. We may note here that the relative contributions of decuplet intermediate states are of the order of a few percent (<5 %) of the total. We may mention that in contrast with the assumptions of Ref. 3 we have not made use of the octet-dominance hypothesis for the weak-interaction Hamiltonian.

P-wave amplitudes.—Extending the work of Sugawara[1] and Suzuki[2] and using the method of Nambu and Shrauner,[12] the p-wave decay amplitude $B(B^{\nu_1}\Pi^{\nu_3}|B^{\nu_2})$ can be written down as (see Ref. 3)

$$\frac{c}{\mu^2}(2k_0)^{1/2}B(B^{\nu_1}\Pi^{\nu_3}|B^{\nu_2})=\sum_{\substack{n=\text{baryon}\\ \text{octet}}}[\langle B^{\nu_1}|\int d^3x A_0^{\nu_3}(x)|B^n\rangle\langle B^n|H_{\text{p.c.}}(0)|B^{\nu_2}\rangle$$
$$-\langle B^{\nu_1}|H_{\text{p.c.}}(0)|B^n\rangle\langle B^n|\int d^3x A_0^{\nu_3}(x)|B^{\nu_2}\rangle], \tag{8}$$

where the matrix elements $\langle B^\nu|H_{\text{p.c.}}(0)|B^{\nu'}\rangle$ can easily be related to the s-wave amplitudes and $\langle B^\nu|\int d^3x A_0^{\nu_3}(x)|B^{\nu'}\rangle$ to strong interaction coupling constants. Using Eq. (1) the p-wave decay amplitudes can now be related to the various s-wave amplitudes as follows[13]:

$$B(p\pi^-|\Lambda)=\frac{m_pG_A}{\sqrt{2}}\left[\frac{\sqrt{2}}{m_p}A(n\pi^0|\Lambda)+\frac{1}{m_\Lambda}\frac{2d}{\sqrt{3}}A(p\pi^0|\Sigma^+)\right], \tag{9}$$

$$B(n\pi^0|\Lambda)=-\frac{m_pG_A}{\sqrt{2}}\left[\frac{1}{m_p}A(n\pi^0|\Lambda)+\frac{1}{m_\Lambda}\frac{d}{\sqrt{3}}A(n\pi^-|\Sigma^-)\right], \tag{10}$$

$$B(\Lambda\pi^-|\Xi^-)=\frac{m_pG_A}{\sqrt{2}}\left[\frac{\sqrt{2}}{m_\Xi}(d-f)A(\Lambda\pi^0|\Xi^0)\right], \tag{11}$$

$$B(p\pi^0|\Sigma^+)=-\frac{m_pG_A}{\sqrt{2}}\left[\frac{1}{m_p}A(p\pi^0|\Sigma^+)+\frac{2f}{m_\Sigma}A(p\pi^0|\Sigma^+)\right], \tag{12}$$

$$B(n\pi^+|\Sigma^+)=-\frac{m_pG_A}{\sqrt{2}}\left\{\frac{\sqrt{2}}{m_p}A(p\pi^0|\Sigma^+)+\frac{1}{m_\Sigma}\left[\frac{2d}{\sqrt{3}}A(n\pi^0|\Lambda^0)-fA(n\pi^-|\Sigma^-)\right]\right\}, \tag{13}$$

$$B(n\pi^-|\Sigma^-)=-\frac{m_pG_A}{\sqrt{2}}\left[\frac{f}{m_p}A(n\pi^-|\Sigma^-)+\frac{1}{m_\Sigma}\frac{2d}{\sqrt{3}}A(n\pi^0|\Lambda)\right]. \tag{14}$$

Table I. Comparison of the results of the present calculation with the available experimental data and the theoretical values of Ref. 3.

		Λ_-^0	Ξ_-^-	Σ_0^+	Σ_+^+	Σ_-^-
$A\times 10^7$	(i) Expt.[a]	3.3 ± 0.1	$+4.4\pm0.1$	-1.9 ± 0.3 -3.8 ± 0.3	-0.1 ± 0.2	4.2 ± 0.1
	(ii) b	3.3 (3.2)	+4.4 (−4.5)	−3.2 (−3.3)	0 (0)	4.5 (4.7)
	(iii) Present calculation	3.3	+4.1	−4.8	−2.8	+4.0
$\lvert\Delta p/\Delta m\rvert B\times 10^6$	(i) Expt.[a]	1.2 ± 0.1	-0.9 ± 0.1	3.7 ± 0.3 1.5 ± 0.2	4.2 ± 0.2	-0.5 ± 0.6
	(ii) b	1.2 (1.2)	−0.9 (0.9)	1.7 (2.8)	1.8 (4.0)	−0.6 (−0.1)
	(iii) Present calculation	0.83	−0.98	1.1	0.25	−0.92

[a]Ref. 11. [b]Ref. 3

It should be noted that these relations have been obtained without the assumption of octet dominance of the Hamiltonian. While for the s-wave decays our results are consistent with the $|\Delta I|=\frac{1}{2}$ rule, we find for the p-wave decays that the $|\Delta I|=\frac{1}{2}$ rule is satisfied if $A(n\pi^+|\Sigma^+)$ is zero,[14] in agreement with the results of Ref. 3. However, the Ξ decay modes satisfy the rule exactly in our model. The amount of violation of the $|\Delta I|=\frac{1}{2}$ rule is small for the Σ decays, as is evident from Table I, while for the Λ decays we get[15]

$$\Lambda_-^0/\Lambda_0^0=-2.1.$$

For the ratio p_-/s_- for Λ decay, we get 0.3, to be compared with the experimental value[16] $p_-/s_-=0.35\pm0.03$, and for p_0/s_0 we get 0.2, the experimental value being $p_0/s_0=0.30^{+0.2}_{-0.12}$. The various s- and p-wave decay amplitudes are given in Table I.

We thus find that the phenomenological form of the Cabibbo current with accepted value of the Cabibbo angle $\theta=0.25$, universal Fermi coupling constant $G_V=10^{-5}/m_p^2$, and with a reasonable $d/f=-1.43$ having the same value for both the weak and strong interactions, it is possible to unify[17] both leptonic and nonleptonic decays of baryons through the use of current commutation relations.

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[1]H. Sugawara, Phys. Rev. Letters 15, 870, 997(E) (1965).

[2]M. Suzuki, Phys. Rev. Letters 15, 986 (1965). [Nonleptonic decays of K mesons have also been investigated using similar procedures by C. G. Callan and S. B. Trieman, Phys. Rev. Letters 16, 153 (1966); and S. K. Bose and S. N. Biswas, Phys. Rev. Letters 16, 330 (1966)].

[3]O. Hara, Y. Nambu, and J. Schechter, Phys. Rev. Letters 16, 380 (1965).

[4]M. Gell-Mann, Phys. Rev. 125, 1067 (1962); Physics 1, 63 (1964).

[5]In writing down the weak vector form factors, the conserved-vector-current hypothesis has been taken for granted. The axial form factors are assumed to be the same as the vector ones [N. Mistry, thesis, Columbia University, 1963 (unpublished)].

[6]C. H. Albright and L. S. Liu, Phys. Rev. Letters 13, 673 (1964).

[7]The accepted value of the Cabibbo angle ($\theta=0.25$) has been used here.

[8]The indices ν occurring in the reduced matrix elements in Eq. (2) have been used for convenience in our dynamical calculations.

[9]The other form factors in (3) are found to be negligible (see Ref. 6). The various kinematic factors and weak-coupling constant ($G_V=10^{-5}/m_p^2$) have been suppressed in writing Eqs. (2) and (3).

[10]We have normalized axial $D+F=1.2(-G_A/G_V)$, also $C=-2m_pG_A\mu^2/\sqrt{2}\,G$, where $G^2/4\pi=14.7$.

[11]R. H. Dalitz, Properties of the Weak Interactions, Lectures at the International School of Physics at Varenna (Oxford University Press, New York, 1965) (see p. 112, Table III).

[12]Y. Nambu and E. Schrauner, Phys. Rev. 128, 862 (1962); J. Schechter and Y. Ueda, Phys. Rev. 144, 1338 (1966).

[13]In evaluating the sum rules (9)-(14), we encounter the s-wave weak decay $\Sigma^0\to n+\pi^0$. Using Eq. (2) we can obtain $A(n\pi^0|\Sigma^0)=\frac{1}{2}(n\pi^-|\Sigma^-)$. This relation has been used in deriving the sum rules (9)-(14).

[14]The s-wave Σ_+^+ amplitude in the formalism of the current algebra goes via the 27-plet weak Hamiltonian only, but in our calculation it is dominated by the octet intermediate state contributions.

[15]In evaluating the p-wave amplitudes, the d/f ratio for strong vertices is taken to be the same as for the axial D/F.

[16]See Ref. 11, p. 74 for the experimental results.

[17]The same conclusion is drawn by S. K. Bose and S. N. Biswas, International Centre for Theoretical Physics Report No. IC/66/22. See also E. Ferrari, V. S. Mathur, and L. K. Pandit, to be published.

DIRECT MEASUREMENT OF THE g FACTOR OF THE FREE POSITRON

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We have just completed a determination of the g factor of the free positron. Writing $g = 2(1+a)$ where "a" is the g-factor anomaly, we may express the result of our measurement as

$$a = 0.001\,168 \pm 0.000\,011 = \alpha/2\pi + (1.2 \pm 2)\alpha^2/\pi^2.$$

This is in agreement with the theoretical[1] and experimental[2] values of the free-electron anomaly which are given as

$$a_{\text{theor}} = 0.001\,159\,615 = \alpha/2\pi - 0.328\alpha^2/\pi^2 + \cdots,$$

$$a_{\text{expt}} = 0.001\,159\,622 \pm 0.000\,000\,027$$
$$= \alpha/2\pi - (0.327 \pm 0.005)\alpha^2/\pi^2. \quad (1)$$

The method we used to determine the g factor may be described in the following way: A "bunch" of positrons from a Co^{58} source is sent into a magnetic field (B_t) which is almost uniform but slightly bottle shaped. The positrons are trapped and held in the bottle for a measured length of time, and then ejected into a polarimeter. As emitted by the Co^{58} source, the positron beam is polarized parallel to its momentum. While in the magnetic field, the positrons execute cyclotron orbital motion of frequency ω_c, while their spins precess at frequency ω_s. The polarimeter determines the projection of the final polarization of the positron onto a fixed direction. This is proportional to $\cos[(\omega_c - \omega_s)T + \varphi]$, where T is the length of time spent in the bottle, and φ is a constant. The experiment consists of determining the final component of polarization at a number of trapping times T, and from that result, determining $\omega_c - \omega_s$, which we denote by ω_D. The g-factor anomaly, a, is simply $\omega_D mc/e\bar{B}_{tz}$. $\bar{B}_{tz}$ is the axial component of magnetic field averaged over the time the particle is in it.[3]

The experimental technique follows that already described for the measurement of the g factor of the free electron,[2] except for the way in which the polarization and the analysis of the polarization are accomplished. In the electron measurements, Mott scattering was used for both. In the positron measurements, no polarizing process is necessary. The polarimeter works in the following way: After removal from the trap, the positrons are stopped in a plastic scintillator in a 13-kG magnetic field (B), and measurements on their lifetime before annihilation are made by a delayed coincidence system. The lifetime distribution is dependent upon the component of polarization of the positrons in the direction of the magnetic field in which they are stopped. A more detailed treatment of the polarization analysis will follow the description of the experimental arrangement.

The experimental arrangement is shown in Fig. 1. The solenoid used to produce B_t is 11 ft long, 2 ft in diameter and wound with 8 layers of No. 10 SCHF magnet wire. Small auxiliary coils carrying the main solenoid current shape the field in the trapping and extraction regions. The source of polarized positrons is 600 mCi of Co^{58}. Only positrons with kinetic energy (200 ± 30) keV or $v/c = 0.7 \pm 0.05$ (about 10% of the emission spectrum) are used. They are emitted in a 222-G field and move down the field (south-Fig. 1) with various axial velocities until they enter the trapping region. Here they are trapped by pulsing the inject cylinder 1000-V positive. This causes a loss of axial momentum as the gap is crossed so that a given fraction of the particles reflects from the right-hand magnetic potential hill. The pulse is removed before the reflected positrons recross the gap, so that they do not regain the lost momentum and become trapped. To eject, a positive 1000-V pulse is applied to the left-hand (ejection) cylinder. While the particles are trapped, all surfaces are at ground potential. The cylinders are brass, 12.5 cm in radius, and mounted concentrically within the 15-cm radius vacuum chamber.

After ejection, the positrons must be extracted from the solenoid and brought to the polar-