

OBSERVATION OF ION PLASMA WAVES  
AT FREQUENCIES HIGHER THAN THE ION PLASMA FREQUENCY

G. M. Sessler

Bell Telephone Laboratories, Murray Hill, New Jersey

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We have generated ion plasma waves in weakly ionized gases and have measured the phase velocity and damping of these waves. Ion plasma waves are the extension of ion acoustic waves to frequencies larger than the ion plasma frequency  $\omega_i$ . For ion plasma waves, the participation of the electrons in the wave motion decreases rapidly with frequency,<sup>1</sup> because of the fact that the electron Debye length is comparable with, or larger than, the wavelength. Previous ion-wave measurements<sup>2-9</sup> have been restricted to ion acoustic waves. Recently, the possibility of excitation of ion plasma waves has been predicted<sup>10</sup> on the basis of the kinetic theory. In agreement with the theory, we found the phase velocity to depend only slightly on frequency; however, the observed damping is substantially lower than expected on the basis of a Maxwellian velocity distribution of the ion gas.

The plasma is generated in a glass tube, 25 cm long and 10 cm in diameter, by a continuous rf discharge at a frequency of 50 MHz. The electron and ion densities,  $n_e$  and  $n_i$ , and the temperature of the electron gas,  $T_e$ , are measured with a Langmuir probe of 0.02-cm<sup>2</sup> area and a floating double probe of 0.4-cm<sup>2</sup> area. In the argon, nitrogen, and hydrogen plasmas used in the experiments, the plasma densities can be chosen around  $10^9$  cm<sup>-3</sup>, and  $T_e$  is about 50 000°K. The temperature of the ion gas,  $T_i$ , cannot be determined by probe measurements.<sup>11</sup> An estimate of the order of magnitude of  $T_i$  can be obtained from the size of dc fields found to be present in the plasma. These fields are due to wall effects introduced by the container, the grids, the probes, etc.; thus, their direction depends on location. The ion drift velocities due to these fields are partly randomized by collisions. Since the fields far from the walls are of the order of 0.1 V per cm,  $T_i$  is expected to be about 1000°K for pressures of about one micron.

The ion waves are generated by applying pulsed wave trains with carrier frequencies between 0.1 and 10 MHz to a pair of grids<sup>7</sup> with grid diameters of 5 cm and a spacing of 0.05 cm between the two grids. The waves are detect-

ed by another pair of grids. As the distance between the two pairs of grids is changed, comparison of the received signal and the excitation signal yields phase velocity and damping of the waves. Ion-wave measurements are possible with this method up to frequencies of about 6 MHz. At higher frequencies, insufficient signal-to-noise ratio prevents accurate measurements of the propagation constants; however, there is evidence that the waves still propagate.

Results of measurements of the real and imaginary parts of the propagation constant  $k = k_r + ik_i$  in argon are shown in Figs. 1 and 2. The ion and electron densities are  $1.3 \times 10^9$  cm<sup>-3</sup>, corresponding to an ion plasma frequency of 1.2 MHz,  $T_e$  is equal to 48 000°K, and the background neutral gas pressure is 0.002 Torr. The estimate for  $T_i$  yields  $T_e/T_i$  equal to about 50.

The velocity measurements, shown in Fig. 1, indicate a slight increase of the phase velocity  $v_p = \omega/k_r$  with frequency. The relative damping,  $k_i/k_r$ , plotted in Fig. 2, is almost frequency independent at the low frequencies and decreases slightly with frequency above  $\omega_i$ . Measurements in a more tenuous argon plasma with plasma densities of  $0.5 \times 10^9$  cm<sup>-3</sup> yield, apart from slightly higher phase velocities and smaller relative damping, the same frequency dependence of  $v_p$  and  $k_i/k_r$ . Similar results were obtained in the hydrogen and ni-

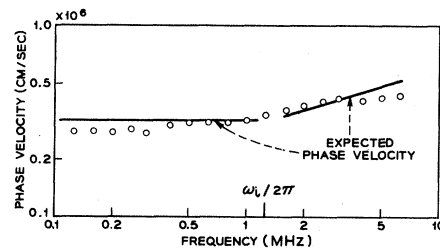


FIG. 1. Comparison of theoretical and experimental phase velocities of ion waves in argon. Experimental conditions: plasma densities,  $n_i = n_e = 1.3 \times 10^9$  cm<sup>-3</sup>; neutral-gas pressure, 0.002 Torr; electron temperature, 48 000°K; temperature ratio,  $T_e/T_i \approx 50$ . Solid lines: low- and high-frequency approximation for the phase velocity, see Eqs. (1) and (6).

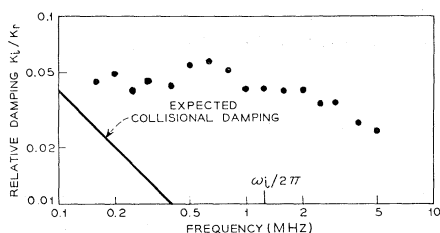


FIG. 2. Comparison of theoretical and experimental data for the ratio of the imaginary and real parts of the propagation constant of ion waves. Experimental conditions see Fig. 1. Solid line: collisional damping from Eq. (2) for frequencies well below  $\omega_i$ . At frequencies above  $\omega_i$ , the predicted damping  $k_i/k_r$  is equal to about 0.58.

trogen plasmas. The experimental errors for the phase-velocity measurements are estimated to be about  $\pm 5\%$  over the entire frequency range. For the attenuation measurements, the errors are about  $\pm 15\%$  at frequencies above 0.5 MHz and increase to about  $\pm 25\%$  at lower frequencies.

Also shown in the figures is the dependence of the phase velocity and the damping on frequency, as predicted by Gould's theory,<sup>10</sup> derived for propagation of ion waves generated by a pair of grids. In Gould's paper, numerical results are only given for  $T_e/T_i = 1$  and 2. This necessitated an evaluation for higher temperature ratios. The calculations<sup>12</sup> show that at frequencies much smaller than  $\omega_i$ , the phase velocity is equal to

$$v_p = v_i \left[ (T_e/2T_i) + \frac{3}{2} \right]^{1/2}, \quad (1)$$

where  $v_i = (2kT_i/m_i)^{1/2}$  is the ion thermal speed,  $k$  is Boltzmann's constant, and  $m_i$  is the ion mass. The damping in this frequency range is, for the gas pressure under consideration, primarily due to ion-neutral collisions and is given by

$$k_i/k_r = v_{in}/2\omega, \quad (2)$$

where  $v_{in}$  is the ion-neutral collision frequency.

At frequencies above  $\omega_i$ , phase velocity and damping depend on the separation,  $s$ , between the two pairs of grids. For separations much larger than the wavelength, the wave potential can be approximated by<sup>12</sup>

$$\Phi \propto z^{2/3} \exp\left[\frac{3}{2}(z/2)^{2/3}(i\sqrt{3}-1)\right], \quad (3)$$

where  $z$  is equal to  $\omega s/v_i$ . Ignoring the factor in front of the exponent, which depends only weakly on  $z$ , and equating the exponent to  $i(k_r + ik_i)s$ , one obtains

$$k_r = \frac{3}{2}(\sqrt{3}/s)(z/2)^{2/3}, \quad (4)$$

$$k_i/k_r = 1/\sqrt{3} \approx 0.58. \quad (5)$$

Equation (4) yields for the phase velocity

$$v_p \approx 0.61v_i z^{1/3}. \quad (6)$$

The phase velocity following from Eqs. (1) and (6) for the measured value of  $T_e$  and for  $T_e/T_i = 50$  is plotted in Fig. 1. For the evaluations, a separation  $s = 4$  cm, corresponding to the center of the measuring intervals, has been substituted in Eq. (6). Obviously, the results depend only weakly on  $T_e/T_i$ , as long as  $T_e$  is constant. The agreement between the theoretical and experimental results in Fig. 1 is good.

The collisional damping following from Eq. (2) is plotted in Fig. 2. At the lowest frequencies, measured and expected damping are in fair agreement. The absorption seems to be primarily collisional. The expected dependence of  $k_i/k_r$  on frequency around  $\omega_i$ , where the predicted damping increases steeply, follows only from detailed numerical calculations<sup>12</sup> and is not shown in Fig. 2. At frequencies above  $\omega_i$ , the observed damping is by about an order of magnitude lower than expected from Eq. (5). This discrepancy is probably due to the fact that the ion velocity distribution is non-Maxwellian. It can be shown<sup>12</sup> that distributions with a sharp cutoff at high velocities will yield much smaller wave damping, while still giving substantially the same phase velocity. It is of interest to note that measurements by Wong<sup>9</sup> in a regime where the predicted damping is similarly high also yielded a considerably smaller damping than expected.

Gould<sup>10</sup> has shown that at large grid separations the electron contribution to the wave potential is much larger than the ion contribution. This, however, does not invalidate the above interpretation of the experiment, since the electron contribution is, because of the larger phase velocity, temporally separated from the ion contribution when pulsed waves are used.

Gould's results are valid if the spacing of the two grids in each pair is small compared with the wavelength. For the highest frequen-

cy under consideration, the wavelength is about 0.07 cm, as compared with a grid spacing of 0.05 cm. Auxiliary experiments with grid spacings between 0.05 and 1 cm show, however, no measurable influence of this parameter on the phase velocity and attenuation.

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### HIGH ION $\beta$ PITCH-ANGLE INSTABILITY\*

Charles F. Kennel and H. Vernon Wong

International Atomic Energy Agency, International Centre for Theoretical Physics, Trieste, Italy  
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In this note we derive a high- $\beta$  plasma instability due to small ion-temperature anisotropies of either sign.

We assume that protons (+) and electrons (-) have temperature distributions ( $T_{\perp}^{\pm}, T_{\parallel}^{\pm}$ ) of the form

$$F^{\pm}(V_{\perp}, V_{\parallel}) = N \left( \frac{M_{\pm}}{2\pi T_{\perp}^{\pm}} \right) \left( \frac{M_{\pm}}{2\pi T_{\parallel}^{\pm}} \right)^{1/2} \times \exp\left(-\frac{M_{\pm} V_{\perp}^2}{2T_{\perp}^{\pm}}\right) \exp\left(-\frac{M_{\pm} V_{\parallel}^2}{2T_{\parallel}^{\pm}}\right), \quad (1)$$

where  $\perp$  and  $\parallel$  denote perpendicular and parallel to the static magnetic field  $B_0$ , and  $M_{\pm}$  denotes the particle masses. Denoting the complex wave frequency by  $\omega$  and the wave number by  $k$ , the dispersion relation for pure right-circularly polarized electromagnetic waves propagating along  $B_0$  in an infinite homogenous plasma is

$$\frac{c^2 k^2}{\omega^2} = 1 - \sum_{+,-} \frac{\omega_{p\pm}^2}{\omega^2} \left\{ \left( \frac{\Delta T}{T} \right)_{\pm} + G^{\pm} \left( 2S - i\pi^{1/2} \frac{k}{|k|} e^{-\alpha_{\pm}^2} \right) \right\}, \quad (2)$$

$$\left( \frac{\Delta T}{T} \right)_{\pm} \equiv 1 - (T_{\perp}^{\pm} / T_{\parallel}^{\pm}),$$

where  $\omega_{p\pm} = (4\pi N e^2 / M_{\pm})^{1/2}$  is the plasma frequency,  $\Omega_{\pm} = \pm e B_0 / M_{\pm} c$  is the gyrofrequency,  $e$  is the electronic charge,  $c$  is the velocity of light,  $N$  is the number density,

$$G^{\pm} = \frac{\omega - (\Delta T / T)_{\pm} (\omega + \Omega_{\pm})}{k (2T_{\parallel}^{\pm} / M_{\pm})^{1/2}}, \quad (3)$$

and

$$\alpha_{\pm} = \omega + \Omega_{\pm} / k (2T_{\parallel}^{\pm} / M_{\pm})^{1/2}, \quad S(\alpha) = e^{-\alpha^2} \int_0^{\alpha} dt e^{t^2}.$$

Asymptotic forms of  $S$  are found in Stix.<sup>1</sup> The left-circular polarization has an analogous dispersion relation with the signs of  $\Omega_{\pm}$  reversed. Instability occurs when  $m\omega = \gamma > 0$ .

When  $\alpha_+ \ll 1$ ,  $\alpha_- \gg 1$ ,  $\omega_{p+}^2 / \omega^2 \gg 1$  and assuming that  $\omega / \Omega_+ \sim 1 / k R_+ \sim (\Delta T / T)_+ < 1$ , where  $R_{\pm} = (T_{\parallel}^{\pm} / M_{\pm})^{1/2} (1 / \Omega_{\pm})$ , we obtain for the oscillation frequency  $\omega_R$  and the growth rate  $\gamma$

$$\omega_R = \Omega_{\pm} \left\{ \left( \frac{\Delta T}{T} \right)_{\pm} + \frac{k^2 R_{\pm}^2}{\beta_{\pm}} \left[ 2 - \beta_{\pm} \left( \frac{\Delta T}{T} \right)_{\pm} \right] \right\} \quad (4)$$

$$(\beta_{\pm} \equiv 8\pi N T_{\parallel}^{\pm} / B_0^2),$$

$$\gamma = \left( \frac{\pi}{2} \right)^{1/2} \frac{\Omega_{\pm}}{k R_{\pm}} \left\{ \left( \frac{\Delta T}{T} \right)_{\pm}^2 - \frac{k^2 R_{\pm}^2}{\beta_{\pm}} \left[ 2 - \beta_{\pm} \left( \frac{\Delta T}{T} \right)_{\pm} \right] \right\}. \quad (5)$$