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[6]J. C. Phillips and B. O. Seraphin, Phys. Rev. Letters 15, 107 (1965).

[7]Our analysis only requires separability of the x_3 component of the potential. This is an automatic consequence of applying the adiabatic approximation to motion in the x_3 direction.

[8]For potentials which are finite at the origin this result follows from the argument in the text. For potentials which diverge at the origin but which lead to a regular singular point there (e.g., the $1/x_3$ potential) the result is still true because the differential equation admits only odd solutions with finite potential energy.

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[10]F. W. J. Oliver, in Handbook of Mathematical Functions, edited by M. Abramowitz and I. A. Stegun (National Bureau of Standards, Washington, D.C., 1964), p. 355.

[11]See, e.g., R. G. Newton, J. Math. Phys. 1, 319 (1960) for the exponential potential, and L. D. Landau and E. M. Lifshitz, Quantum Mechanics (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1958) for the harmonic-oscillator and x^{-1} potentials.

[12]In this one-dimensional model, the first nonzero strength function corresponds to taking three derivatives in the complete expression for the strength function.

[13]See, e.g., A. Messiah, Quantum Mechanics (John Wiley & Sons, Inc., New York, 1963), Vol. II, pp. 781-793. This approximation is valid for our applications only sufficiently near the M_1 critical point that $(m_\perp T_3/m_3 E_\perp) \ll 1$ where $m_\perp$ is the larger of m_1, m_2; T_3 is $\hbar^2 k_3{}^2/2m_3$; and $E_\perp$ is the binding energy of the bound state for the x_1, x_2 coordinates.

[14]The cusp-like structure of the effective potential near $x_3 = 0$ is readily shown to be a consequence of the r^{-1} divergence of the Coulomb potential by applying the theorem $\partial E_n/\partial x_3 = (\partial H/\partial x_3)_{nn}$ to the x_1-x_2 Hamiltonian. This cusplike behavior does not occur for the potential $e^2/\epsilon_0(r^2+\alpha^2)^{1/2}$, for example.

ANOMALOUS ULTRASONIC ATTENUATION IN SUPERCONDUCTING SINGLE-CRYSTAL Hg†

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It has been pointed out[1-3] that the properties of strong-coupling superconducting metals, Pb and Hg, deviate from the predictions of the BCS theory. Recently Deaton[4] reported anomalous results in the ultrasonic attenuation in pure Pb. This paper describes apparent anomalies in the bulk superconducting properties of oriented single-crystal Hg measured by ultrasonic techniques. Figure 1 shows α_s/α_n (where α_s and α_n are, respectively, the attenuation coefficients in the superconducting and normal states) as a function of reduced temperature, $t = T/T_c$, for longitudinal waves propagated along the [111] direction at four frequencies from 10 to 250 Mc/sec; similar curves were obtained for propagation along the [$\bar{1}$10] direction. The lowest frequency data fall on a BCS curve characterized by $2\Delta(0) = 3.5kT_c$ [where $2\Delta(0)$ is the energy gap at $T = 0$, k is Boltzmann's constant, and T_c is the critical temperature].[5] As the frequency is increased,

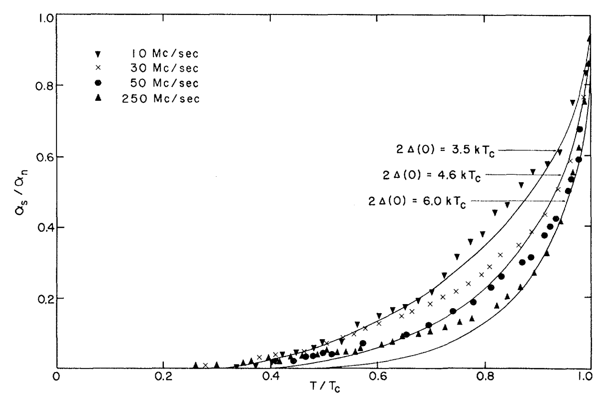

FIG. 1. Experimental values of α_s/α_n as a function of T/T_c for four longitudinal wave frequencies propagated along a [111] axis in Hg. The solid lines are BCS curves. Data were also obtained at 110, 190, and 210 Mc/sec. In order to keep the plot legible, these points are omitted; they all lie within experimental scatter of the 250-Mc/sec data.

the attenuation just below T_c falls off more rapidly, apparently reaching a limiting curve at a frequency between 50 and 90 Mc/sec. The limiting curve shape and frequency appear to be similar in the [111] and [$\bar{1}$10] directions. Approximate values of l, the electronic mean free path, in the Hg crystals used for the [111] measurements were determined using Deaton's[6] high-field attenuation criterion. These l and ql values (where q is the ultrasonic wave number), given in Table I, are rough, but presumably represent an average over the same parts of the Fermi surface as those responsible for the superconducting attenuation.[4] It is noteworthy that at frequencies where $ql \lesssim 1$ over the entire temperature range, BCS behavior is observed, while at frequencies where $ql > 1$ over the entire range, the limiting curve is obtained.

Deaton's curves for Pb,[4] up to his maximum frequency of 112 Mc/sec, do not show evidence of a limiting curve similar to that reported here, although the ranges of ql involved are comparable; further Pb experiments at higher frequencies would be of interest. Such limiting curve behavior may depend upon T_c/Θ_D (where Θ_D is the Debye temperature), which is larger in Pb than in Hg[3]. On the other hand, detailed differences in the phonon spectra[7] of these two strong-coupling superconducting metals may give rise to α_s/α_n behavior which is basically dissimilar. Hg departs sharply from a Debye spectrum; Θ_D rises from 50°K at T_c to 80°K at very low temperatures.[3]

Early ultrasonic attenuation experiments in polycrystalline Hg at 10 Mc/sec by Mackinnon and Myers[8] show good agreement with a BCS curve shape for $2\Delta(0) = 3.5kT_c$. Higher frequency experiments by Chopra and Hutchison,[9] still using polycrystal Hg, show a fall-off below T_c which is qualitatively the same as in Fig. 1. Independent work by Shaw and co-workers[10] on single-crystal Hg also indicates a non-BCS behavior. Infrared absorption[3] and tunneling experiments[7] on polycrystalline Hg show BCS behavior for $2\Delta(0) \approx 4.6kT_c$. These latter experiments may not measure truly bulk properties of the metal. Thus, these experimental results are not inconsistent with the present work. Though an explicit relation to the ultrasonic attenuation behavior is not clear,[11] departure from BCS behavior has been observed[2] in the thermal conductivities of both Hg and Pb, and discussed in terms of a strong-coupling

Table I. Approximate l and ql values in Hg for longitudinal wave propagation along [111].[a]

T (°K)	$t = T/T_c$	l (cm)	ql (10 Mc/sec)	ql (30 Mc/sec)	ql (50 Mc/sec)	ql (110 Mc/sec)	ql (190 Mc/sec)	ql (210 Mc/sec)	ql (250 Mc/sec)
1.1	0.26	1.9×10^{-2}	3.4	10	17	38	65	72	85
1.79	0.431	1.5×10^{-2}	2.7	8.2	14	30	52	57	68
2.13	0.513	9.5×10^{-3}	1.7	5.1	8.5	19	32	36	43
2.60	0.626	4.6×10^{-3}	0.8	2.5	4.1	9.1	16	17	21
3.57	0.860	1.7×10^{-3}	0.3	0.9	1.5	3.5	6.0	6.6	7.8
3.75	0.903	1.6×10^{-3}	0.3	0.9	1.4	3.2	5.6	6.2	7.3

[a] q values were determined from our measurements of the longitudinal sound velocity in the [111] direction, $v_s \approx 3.5\times10^5$ cm/sec.

theory by Ambegaokar and co-workers.[12]

The samples, $\frac{1}{16}$ in. thick, were grown with flat parallel faces from 99.999 99% pure Hg[13] and were not subjected to lapping or polishing. Our experiments were carried out using the conventional pulse-echo technique.[14] Attenuation measurements were made in both the normal and superconducting states. The echo pulse was calibrated against a comparison pulse subjected to the same electronic circuitry. At the lower temperatures, magnetoacoustic oscillations were observed in the normal attenuation; however, these were very small compared to the difference in attenuation between the superconducting and normal states. Hence no attempt was made to currect for them; α_n was measured for each temperature at the critical (transverse) field. No amplitude effects[15] were observed in present experiments, although several pulse voltages were used at various frequencies.[16]

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MICROWAVE ABSORPTION STUDIES OF SUPERCONDUCTORS–MAGNETIC-FIELD-INDUCED ANISOTROPY AND THE EFFECT OF IMPURITIES*

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The effect of a static magnetic field on the electromagnetic absorptivity of superconductors has been studied extensively both experimentally[1-6] and theoretically.[6-8] These studies fall into two groups: those in the frequency region such that the photon energy is very small compared with the energy gap (i.e., $h\nu \ll 2\Delta$)[2-4, 6-8]; and those where the photon energy is of the same order of magnitude as the energy gap,[1,5] so that absorption can occur with excitation of a quasiparticle across the gap. In this paper we present experimental results on the effect of electron free path on the magnetic-field-induced anisotropy of the superconducting energy gap as measured by high-frequency ($h\nu \sim 2\Delta$) microwave absorption in silver-doped aluminum. These results, along with results for a similar experiment on pure aluminum,[1] will be shown to help substantiate the basic physical postulate used by Maki[8] in his theory of the magnetic-field dependence of the low-frequency microwave absorption in superconductors.

In the work on high-frequency absorption in pure superconducting aluminum with a static magnetic field parallel to the surface,[1] it was