qualitative features of collisional and Landau damping. Using derived experimental parameters (f_p and T_e), there is quantitative agreement with the predicted dispersion.

The results reported provide a previously unavailable empirical foundation for the study of idealized electrostatic plasma waves. These experiments are under continuing developments, but it is felt that the measurements described here form a basis for further study of the damping constants and nonlinear effects which depend closely on phase velocity.

I wish to express my appreciation for the considerable support and interest of Professor P. A. Sturrock and the indispensable advice and encouragement given by Professor G. S. Kino. T. Mantei, H. Ikegami, and R. Harp have been very helpful in respect to experimental problems, and J. MacGowan in the construction of the couplers. Finally, it is a pleasure to acknowledge the independent observations of my colleagues of this Institute, H. Derfler and T. Simonen,[8] who have been obtaining similar results from an entirely different conception at the same time.

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[11]The imaginary part of the wave number, due to collisions, has the well-known variation with frequency

$$k_{ic} \approx \nu_c \left(\frac{d\omega}{dk}\right)^{-1} \approx \nu_c \frac{\omega}{k} \Big/ v_T^2 \text{ for } \frac{\omega}{k} \gg v_T.$$

This result, and the spectral "window" formed before Landau damping becomes dominant, have been detailed in some computer calculations by T. Simonen and H. Derfler (private communication).

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LANDAU WAVES: AN EXPERIMENTAL FACT*

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In 1933 Thomson and Thomson[1] predicted that the electrostatic plasma oscillations observed by Tonks and Langmuir[2] would propagate because of the finite temperature of plasma electrons.

Introducing the formalism of the linearized Boltzmann equation, Vlasov[3] showed that in a plasma with a velocity distribution function $f_0(v)$, frequency ω and wave number k are related by the dispersion relation

$$k^2 = \omega_p^{\,2} \int_{-\infty}^{+\infty} \frac{\partial f_0(v)}{\partial v} \frac{dv}{v - \omega/k}. \tag{1}$$

Landau[4] pointed out an error in Vlasov's interpretation of this integral, and he predicted that electrostatic waves are damped. Neglecting damping, Bohm and Gross,[5] as well as Landau, obtained the dispersion relation

$$\omega^2 \approx \omega_p^{\,2} + (3\kappa T/m)k^2 \tag{2}$$

which is valid for large phase velocities. To our knowledge, the dispersion characteristics of these electrostatic plasma waves have, until now, never been measured experimentally. Though Malmberg and Wharton[6] in a notable experiment established the effect of Landau damping, their measurements were done be-

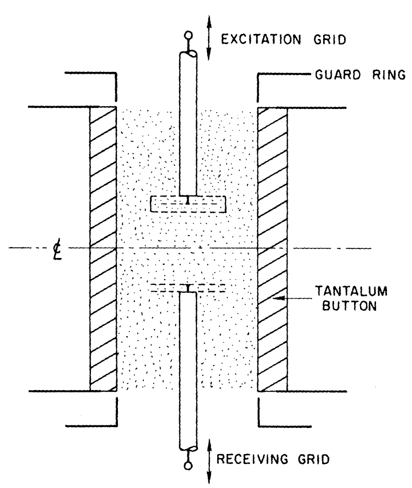

FIG. 1. Diagram of the sodium plasma tube showing the probe arrangement.

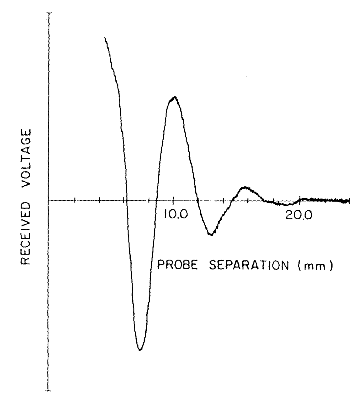

FIG. 2. Interferometer output at 95 Mc/sec as a function of grid separation.

low plasma frequency on a narrow column within a magnetic field. As such they verified damping of Trivelpiece[7] modes rather than Bohm and Gross waves.

We report here preliminary measurements of the dispersion characteristics of electrostatic waves in a one-dimensional plasma in the absence of a magnetic field. The observed waves exhibit all the features of the dispersion relation (1) for a Maxwellian plasma including Landau damping.

A sodium plasma in thermodynamical equilibrium is produced by resonance ionization between two 4.5-cm diam tantalum buttons which are spaced 2.5 cm apart as depicted in Fig. 1. The entire tube is immersed in an oil bath to regulate the sodium vapor pressure. This tube produces a tenuous, virtually collision-free plasma with an electron-rich, thus stable,[8] sheath at the incandescent emitters. As such, it is ideally suited as a carrier for Bohm and Gross waves. Specifically, we used a cathode temperature of 2040°K as measured with an optical pyrometer and an oil-bath temperature of 80°C. The resulting plasma had a density of 2×10^7 electrons/cm^3 and a temperature of 2200°K as measured by a planar guard-ring Langmuir probe. The Debye length was thus 0.7 mm. Since the background pressure was only 2×10^{-6} mm Hg, collisional effects are completely negligible.

Electrostatic waves are excited in the plasma by the parallel plane grids[9] shown in Fig. 1. Direct coupling between grids is reduced to less than −80 dB by completely enclosing the transmitting grid. The $\frac{9}{16}$-in. diameter grids are fabricated from 0.005-in. etched molybdenum mesh having 20 lines per inch. Both grids are attached to mechanical drive mechanisms with 50-Ω molybdenum rf coaxial systems. Typically the grids are biased to plasma potential (+0.5 V above ground) to eliminate the dc plasma sheath around the grid wires. Electrons are thus allowed to pass through the grid assembly very much like in a microwave klystron.

An interferometer having a sensitivity of −90 dB (reference level 1 mW) detects the phase of the received signal. A cw oscillator provides both the local oscillator for the receiving mixer and the rf input to a 1-kc/sec modulator whose output is applied to the transmitting grid. The received signal is detected by the mixer which drives a coherent detector operated by the 1-kc/sec generator. At a fixed frequency, as the transmitting grid is moved, the output of the interferometer is recorded as a function of position on an x-y recorder.

Figure 2 shows a typical curve of the interferometer output when a 400-mV, 95-Mc/sec signal was applied to the transmitting grid. The received signal is down 50 dB at 4 mm. Reducing the input to 40 mV changed only the amplitude of the received signal. From a set of such data taken at various frequencies, the wave numbers k_r (2π/wavelength) and the voltage damping constants k_i have been determined as shown in Fig. 3. Confidence limits indicate the uncertainty in interpreting the raw data. We show for reference the thermal velocity 3×10^8 mm/sec (2040°K). The two curves la-

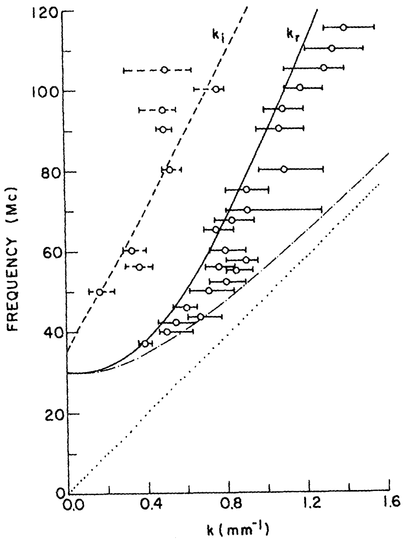

FIG. 3. Dispersion diagram: Circles with bars through them, experimental measurements; k_r and k_i Landau Eq. (1); dotted curves, thermal speed $\omega/k = \sqrt{3\kappa T/m}$; and dash-dotted curves, Bohm and Gross, Eq. (2).

beled k_r and k_i are the real and imaginary components of the wave numbers as computed from the dispersion relation (1) using a Maxwell distribution of electron velocities.[10] The plasma frequency used in computation was 30 Mc/sec, while Langmuir probe data give 34 Mc/sec when the grids are immersed in the plasma. Because of the limited size of the plasma, the longest measurable wavelength was 15 mm at 37 Mc/sec. No waves were observed below this frequency or above 115 Mc/sec.

An interpretation of these experimental results requires discrimination between Landau waves and surface waves, the only other modes which may exist in the absence of an applied magnetic field. The wave number parallel to the electric field $k^2 = k_\perp^2 + k_z^2$ vanishes for surface waves[7,11] so that the associated plasma conductivity[12]

$$\sigma = -i\omega\epsilon_0(\omega_p/v_\theta k)^2 Z'(\omega/v_\theta k) = -i\omega\epsilon_0(\omega_p/\omega)^2 \quad (3)$$

becomes independent of the electron thermal velocity $v_\theta = (2\kappa T/m)^{1/2}$. Consequently, they obey the approximate dispersion relation,[11]

$$(\omega_p/\omega)^2 \approx 2 + 2\ln^{-1}(a/b)(k_z b)^{-2}, \quad (4)$$

where $a = 2.22$ in. and $b = 1$ in. are the wall and plasma radii, respectively. With the geometrical parameters used in our experiment one should see resonances at $\omega \approx \omega_p/\sqrt{2}$ for all $k_z = n\pi/d \geq 0.124\ \mathrm{mm}^{-1}$, where $d = 1$ in. is the separation distance of the emitter buttons. No such resonances were observed nor did we see the exponential variation of the corresponding radial electric field

$$E_r \propto I_1(k_z r) \sim (2\pi k_z r)^{-1/2} \exp(k_z r) \quad (5)$$

in our interferometer traces such as shown in Fig. 2. Having established that surface waves are absent we conclude that the cut-off frequency in our experiment is indeed the plasma frequency ω_p and not $\omega_p/\sqrt{2}$. This statement is further corroborated by the close agreement between the cut-off frequency (30 Mc/sec) and the experimental plasma frequency (34 Mc/sec).

Theoretical studies[9] show that higher order Landau modes become important whenever $\omega \gg \omega_p$ and $x \gg \lambda_D$. In this case Landau damping increases somewhat less than exponential like

$$|E| \propto \exp[-\tfrac{3}{4}(\omega x/\omega_p \lambda_D)^{2/3}]. \quad (6)$$

This seems to be borne out by our experimental data, Fig. 3, for $\omega > 3\omega_p$. The techniques of our measurements are currently being refined considerably to allow a quantitative investigation of these secondary effects. The preliminary results presented here do, however, exhibit all the features of Landau waves with reasonable quantitative agreement between the experimental data and the predominant solution of Landau's dispersion relation.

We gratefully acknowledge the assistance of M. Omura in performing the probe measurements and the help of W. Holmes and J. Schick in designing and constructing the experimental tube. Finally, we are pleased that similar results have been obtained concurrently by our colleague G. Van Hoven using a reflex excitation scheme in an entirely different type of plasma.

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DISPERSION OF ELECTRON PLASMA WAVES*

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The effect of plasma temperature on the dispersion of electron-plasma waves is an old problem. Shortly after the plasma oscillations in an infinite, cold plasma were described by Langmuir and Tonks,[1] the effect of random thermal motion on their dispersion relation was calculated by Thomson and Thomson.[2] They assumed a resonance distribution for the electron velocities and obtained a result differing from that of later work by a numerical coefficient. Vlasov[3] studied the problem using a Maxwellian velocity distribution and obtained, for the long wavelength limit, the dispersion relation

$$\omega^2 \simeq \omega_p^{\,2}[1+3(k\lambda_D)^2], \tag{1}$$

where ω is the wave frequency, $\omega_p=(4\pi ne^2/m_e)^{1/2}$ is the plasma frequency, k is the wave number and λ_D is the Debye length. However, his work has been severely criticized[4] because of the rather cavalier manner with which he dealt with a divergent integral in the derivation. A generally accepted derivation of the dispersion relation has been given by Landau.[4] Landau also found that the waves were damped, even in the absence of collisions, and this part of the theory has been verified in detail experimentally.[5] The physical mechanism responsible for the dispersion and damping of the waves has been elucidated by Bohm and Gross.[6] All these calculations were made for an infinite, uniform collisionless plasma.

In this Letter we report an experiment designed to measure the effect of plasma temperature on the dispersion of electron plasma waves. When the theory is modified to include the effects of finite size of the system and spatial variation in plasma density, it accurately predicts the observed dispersion.

The geometry to be considered is a long column of plasma bounded in the radial direction by a good conductor. The plasma is immersed in a uniform finite magnetic field parallel to the axis of the plasma column. The plasma density is a function of radius, but its temperature is not. We specify the properties of the plasma by the dielectric tensor relating the displacement to the electric field ($\vec{D}=\epsilon\cdot\vec{E}$).[7] In the limit of waves with phase velocity small compared with the velocity of light, $\nabla\times\vec{E}$ may be neglected and the electric field calculated from a scalar potential, Ψ. When the plasma is regarded as a dielectric, there are no free charges and

$$\nabla\cdot\vec{D}=\nabla\cdot(\epsilon\cdot\vec{E})=-\nabla\cdot(\epsilon\cdot\nabla\Psi)=0. \tag{2}$$

The frequency is sufficiently high that the motion of ions may be ignored. The electron gyroradius is assumed small compared with $\Psi(d\Psi/dr)^{-1}$ and $n(dn/dr)^{-1}$, where n is the electron density, so the dielectric tensor is a local quantity. Since n is a function of radial position, ϵ is also. In addition ϵ is a function of the wave frequency and the magnetic field.